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Prof. **K. Yamagawa**, Ph. B., *Rigakuhakushi*, Director of the College (*ex officio*).

Prof. **E. Divers**, M. D., F. R. S., etc.

Prof. **D. Kikuchi**, M. A., *Rigakuhakushi*.

Prof. **K. Mitsukuri**, Ph. D., *Rigakuhakushi*.

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All communications relating to this Journal should be addressed to the
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The Manufacture of Calomel in Japan.

By

Edward Divers, M. D., F. R. S., *Prof.*

Imperial University.

(With Plates I-III)

Introductory.—Calomel, in the form common in England and all countries under Western civilisation, is now extensively used and is even manufactured in Japan, under the name of *kankō*. But mercurous chloride is also largely used here, under the name of ‘light powder,’ *keijūn* (Chinese, *kingjūn*), in another and very much older form, which is of signal purity, and made by a simple process as yet quite unknown in Europe. I witnessed this interesting process from beginning to end some years ago, and now make this publication of it with full permission of the proprietor of the works I visited, Mr. H. Kokubu, who has aided me in every way he could, and notably with drawings, some of which illustrate this paper.

Historical.—According to Terajima Hōkyō and Ono Ranzan, writers who lived in the last century, the first-named perhaps a little earlier, calomel was known in Japan as far back, at least, as the beginning of the eighth century, having then been presented to the Empress Gen-miyō ; but their authority is the *Zoku Nihongi*, reference to which, Professor Haga, F.C.S., informs me, makes it clear that mercury itself, not its chloride, was the thing presented. In the time of the writers above named, mercurous chloride was well-known and

was manufactured in Japan, not only at Isawa, a village in Isé, where it is still made, but also in the city of Osaka and in a town near it, called Sakai. Mr. H. Kokubu, manufacturer, tells me that records exist at Isawa-mura of his family having carried on the manufacture of *keifun* there for the last three hundred years.

Far earlier, namely, in the tenth century Minamoto-no-Shitagō, in his work entitled *Wamyō-Ruijūshō*, makes mention of a mercurial preparation named *kōfun* or 'powder of mercury.' It is, however, questionable whether this was mercurous chloride or mercuric oxide, and therefore whether calomel was known or not at this time. But since calomel, under the name of *keifun*, is mentioned by Chinese writers even earlier than this it may be safely accepted that Japanese knowledge of this body is older than that in Europe. The Western knowledge of chloride of mercury dates from the first half of the sixteenth century, but the distinction between calomel and corrosive sublimate was not recognised till near the end of that century.

Literary.—The literature on Japanese calomel is meagre. Japanese writers of the old school have contented themselves for the most part with translating Chinese writings. Ono Ranzan mentions that the Japanese method differs from the Chinese in making use of water in place of alum and other chemicals, in which he came near the truth. The late Dr. Geertz, who in the Government service did much in establishing Western pharmacy in Japan, treated of *keifun* in some metallurgical contributions he made to the *Transactions of the Asiatic Society of Japan*. What he wrote is contained in vol. iv (1875), and consists of information almost exclusively about Chinese calomel, and gained more from Chinese and Japanese writings than from any experience of his own. Concerning Chinese calomel English readers have the *Notes on Chinese Materia Medica*, among the *Science Papers* by the late Daniel Hanbury, F.R.S., edited by J. Ince. Hanbury

mentions, as the result of his own observation, the characters of *kingfun* and its great purity but for the presence of minute, transparent, acicular crystals of calcium sulphate. He refers to Porter Smith's *Contributions towards the Materia Medica and Natural History of China* for an account of the manufacture. Smith, however, takes his information solely from Pearson's account on p. 59, vol. iii of Sir J. Davis's work *on the Chinese*. I have not seen this book, but it is clear from Porter Smith that Pearson, again, has only derived his information from the Chinese *Materia Medica*, *Pun-tsaou-kung-muh*, and not from his own observation, and it amounts to this :—Common salt and mercury, of each one oz.; alum, 2 ozs.; or, salt, mercury, copperas, and saltpetre, in some such proportions ; are rubbed together and put into an iron bowl which is then covered with a roomy earthen dish well luted down. This is exposed to the heat of a strong charcoal fire for four or five hours, when water is thrown on the cover and the cover taken off. On its inner surface the calomel is found adhering in the form of a beautiful, feathery, white sublimate. Ten parts of mercury are said to yield about eight parts of calomel. Dr. Geertz's paper, already referred to, contains essentially the same account translated from the Japanese version of the Chinese work.

Lastly, there is a paper, in the Japanese language, on the manufacture of *keifun* at Isé, which is the forerunner of the present one. That paper appeared in 1887 in the *Journal of the Tōkyō Chemical Society*, by Mr. T. Shiinidzu, M.E., F.C.S., my former pupil and colleague, and it was his description to me of what he had seen that led to my own visit to Isé in company with Professor Haga in the following year. In one or two points I have availed myself of this paper to make my own account more complete.

Of the specific properties of keifun.—*Keifun* is in very thin minute scales, lustrous, transparent, and white or faintly cream-

coloured. It might be described as micaceous calomel. To the touch it is soft and smooth. Measured in bulk, dry, it is four times as voluminous, more or less, as the ground calomel prepared by the European process, and can be readily scattered by a puff of the breath. Rubbed hard in a porcelain mortar it gives the brown resinous streak characteristic of calomel and the evidence therefore, according to pharmaceutical authorities, of its freedom from corrosive sublimate. Exposed to bright sun-light it gradually assumes a light brown colour, a colour, that is, having no affinity to grey or black. Moisture does not seem to favour this change which is certainly not owing to any reduction to metal. European calomel suffers a similar change. *Keijun* is free from corrosive sublimate, and from metallic mercury.

Hanbury found selenite in Chinese calomel, and Geertz found calomel of this form generally adulterated with selenite and mica, but whether what he examined was ever Japanese and not always imported Chinese calomel he does not show. I have found *keijun*, as it came direct from Isé, quite free from adulteration, and have not met with any adulterated.

Of the material used in making calomel in Isé, Japan.—The materials for making Japanese calomel are—mercury, an arenaceous red clayey earth, bay-salt, bittern or salt-mothers, and air. The mercury is imported from Europe, but in old times is said to have been found in the neighbourhood of Isé as cinnabar.

The earth, called *mitsuchi* ('seed-earth'), is all taken from a neighbouring hill, Shunakayama, and according to Mr. Kokubu, many other clays have been tried in place of it, always with bad results. It is of a rather light bright red colour, which changes to a duller and somewhat brown red on drying and gently heating the earth, and to a light ordinary brick red by a strong heat. As mined,

the earth is seen to consist largely of colourless quartz grains. Besides the quartz a very little biotite is seen sparkling through it. The fresh damp earth does not form a compact mass, but a slightly cohering aggregate of damp crumbs. This texture appears to be due to the earth being a mass of quartz in small grains from the size of a hemp seed down to that of impalpable particles, held together by plastic clay. For use that which does not contain coarse quartz grains too abundantly is selected, and is made into briquettes and moderately baked on the hearth of the fire-place under the calomel pots. These briquettes are then as light and porous as the prepared porous clay used in Fletcher's gas-furnaces. The raw earth air-dried is readily rubbed into its constituents by the fingers ; and the baked briquettes very easily and rapidly reduce to a soft powder, quartz grains and all, in the agate mortar. The larger grains of quartz in the raw earth are also very brittle. I have treated thus fully of the mechanical characters of the earth, because probably much of its efficiency is due to them. But its chemical character also calls for notice. As baked ready for use it contains in the thoroughly air-dry condition still 5 per cent. and more of water. Before ignition it is almost entirely decomposed by sulphuric acid, either in some days in the cold or quickly by heat. It is also largely acted upon by hot hydrochloric acid, and heated in sealed tubes with this acid to 120-150,° it is almost as fully decomposed as by sulphuric acid. It contains practically no silica soluble in hot sodium-carbonate solution, but after acid treatment yields, of course, much silica to this reagent. The composition of the earth, as found in use at the works, but rendered anhydrous is as follows :—

Quartz	38.4
Combined silica	24.2
Alumina	26.3
Ferric oxide... ..	10.5
Magnesia	0.2
	<hr/>
	99.7

The magnesia is only got by fusion of the finely ground earth with alkali carbonate, and belongs to the particles of mica scattered, through it. Only doubtful traces of phosphoric acid could be found and, what is specially to be noted, no lime whatever. The earth is thus nothing but quartz, kaolin, ferric hydroxide, and a very little biotite, and is probably valuable to the calomel maker not only for its highly porous texture, but also for its negative chemical qualities.

The bittern and even the rough bay-salt contain magnesium chloride, and this rather than sodium chloride must be the source of hydrochloric acid in the process. That air finds a graduated entrance to the other materials by diffusion during the process, will become evident from a consideration of the set-up of the apparatus.

Of the plant.—The apparatus for making Japanese calomel consists of a table-furnace supporting sixty cast-iron pots lined and surmounted with the Shumakayama earth, on which rest, as covers, and condensers and receivers of the calomel, unglazed clay cups bottom upwards. In Pl. I. the furnace as it is when in action is seen from the working side; ten pots on the left side are shown still to be charged and covered. The wooden step in front is to enable the workman to reach over the table easily when charging the pots or emptying them. In Pl. II, fig. 1 the furnace is seen from the back, or firing side, and with the walls broken away to show its interior and the method of firing. In Pl. II, fig. 2 the mounting of the pots is shown in three stages by sectional plans of the table.

On a smoothed clay hearth the walls of the furnace are raised in clay, building, in the three stones which frame the stoke hole (Fig. 2). The walls are 2.6 ft. high and the enclosure is 7.6 ft. by 4.7 ft. measured outside. The stoke hole is 1 ft. by 1 ft., but a little wider than this at the base, and is without door. The table of pots and roof of the furnace is constructed (Pl. I and Pl. II, fig. 2) by laying a square iron rod on each of the long walls, and on these eleven cross rods also square on which are to rest the flanges of the pots. The pots are then put in position as close together as possible, hanging by their flanges, in ten rows of six each, and plastic clay pressed into the openings left between the flanges and the rods, and the rods and flanges covered in so that only the mouths of the pots remain visible, as shown by the middle rows in Fig. 2. The furnace clay being thoroughly dry, it is deeply laid over with the red earth mixed with a little bay salt and moistened with bittern in small quantity. The pots are also filled with the same moist red earth, except a central cylindrical shaft (see the left side of the furnace-table in Pl. I or the right side in Pl. II, fig. 2) reaching to the bottom of the pot, which is left bare. The pot is 0.5 ft. deep inside, and across its mouth, inside, is 0.45 ft. It is shown in Pl. III, fig. 4. The shaft or cavity left in the filling is 0.18 ft. in diameter, and is shaped by resting a wooden core on the bottom of the empty pot, and then pressing-in the moist earth round it, smoothing-off the top, and dropping a perforated board over the projecting core to hold down the earth while withdrawing the core which is then removed by its handle. The furnace is now ready for work. It should have been mentioned that after the clay walls of the furnace are built they are framed-in with wood to increase their stability and to give support to a wooden back and to shelving above the table, as seen in Pl. I.

Of the firing.—The firing the pots is kept far below what are

usually regarded as furnace heats. The fuel employed is wood, and the Japanese are certainly clever in the use of this, in the old style of furnaces, economically and effectively in firing pots for boiling, evaporating, distilling, or subliming. The method of heating is seen in Pl. II, fig. 1. Five lengths of fire-wood are ranged along the back and front walls on the hearth, generally raised at one end by resting on a lump of clay. In the ordinary working of the furnace, as I saw it, the heat from previous work is sufficient to kindle the fresh wood. The flames rise up the sides and run over the bottoms of the pots, leaving the central space in the chamber free from flame. The air enters by the lower part of the stoke hole and the products of combustion escape, invisible, by its upper part, so perfect is the combustion. At the time of first lighting the fire and of irregularities in stoking, some smoke is unavoidable, and to keep the top and table free from this smoke and from ashes, a wooden back is put in above the table as shown in the figures. There is also a wooden hood and flue above the stoke hole, to carry off any smoke; this is not shown in the figures, but is similar to those put up in England and elsewhere, over the working doors of furnaces to protect the workmen from arsenic, sulphur, or other noxious fumes. The heating is so well effected that the pots two feet above the burning logs are made sufficiently hot, barely red-hot at the bottom, and yet the wooden frame on the outside of the furnace is not charred, and the work-room is not unpleasantly warmed. About three bundles or 40 lbs. of wood serve for one firing, and it is remarkable to see so little fuel working so many pots.

Of the working.—A compost of burnt *mitsuchi* with about a fourth of its weight of bay salt is made up with bitteru into lumps the size of large chestnuts. The furnace being hot enough, the *mitsuchi* surface of the table is once for all freely wetted by a watering-pot, perhaps half a

gallon of water being used, all of which is absorbed ; a lump or two of compost is dropped by tongs or by hand into each pot in rapid succession ; a very small spoonful of mercury poured into each pot, the total charge for the sixty pots being somewhat less than one pound avoirdupois. (more exactly $\frac{5}{6}$ of a lb.), and a clay cup, bottom up, placed over each pot and adjusted by gently pressing, and turning it round slightly. The cup is thus made to fit neatly on the earthen top without adhering to it in the least. The cups are thick and unglazed, but become very smooth inside by use. They have an inside diameter at the mouth of 0.5 ft. and a depth of 0.25 ft.

Thus arranged, the pots and cups are left for three hours, and during the latter part of this time the furnace is cooling. When cool enough, each cup is lifted in turn, and, with two turns of a feather, the *keyim* or calomel, which fills it in the form of a sparkling network of delicate crystalline scales, is transferred to a sheet of paper held under it, and the cup, mouth downwards, placed on a shelf of the furnace ready for use in the next operation.

The calomel, it will thus be seen, forms no adherent cake in the cup, but particles only loosely hanging together. So obtained it is ready for the market, needing only to be packed in small wooden boxes for sale.

The spent lumps of earth and salt are lifted out of the pots by the tool shown in Pl. III, fig. 5, and when the furnace has become still cooler, the fire is made up again, the furnace top freely wetted, and a new operation set going as before. The furnace is worked twice each day. Nothing could well be carried out with greater simplicity and less expenditure of labour, time, and fuel.

Of the yield.—The loss I am told is about sixteen per cent. of the theoretical amount of calomel which is, I believe, about twice as much as is lost in the Western process.

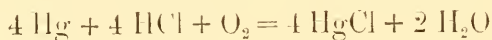
Experimental.—If one of the cups is removed a few minutes after the operation has been started, much of the mercury is found in it as a sublimate of fine globules, mixed with only a little almost amorphous calomel, from which it would appear that the formation of the calomel results from reaction in the cup between the mercury in vapour and the active gases.

On dipping into the pot, uncovered during the process, a glass rod with a drop of water hanging to it and then withdrawing it and testing the remainder of the water with potassium iodide and starch no chlorine can thus be detected. The same is true when a drop of solution of potassium hydroxide is used. Nor can the slightest odour of chlorine be detected in the vapours issuing from the uncovered pot.

On passing air containing a little hydrochloric acid gas through a tube in which mercury is freely boiling, sparkling calomel is formed close to and mixed up with the mercury.

Red earth which has been used in the process turns moist red litmus-paper blue, while fresh red earth is neutral.

Theoretical.—The nature of the materials used and the observations gained by the preceding experiments are sufficient to establish that the calomel is formed by a reaction between mercury vapour, oxygen, and hydrochloric acid gas, in which along with mercurous chloride, water is formed :—



and that the formation takes place at a temperature near—above or below—the boiling point of mercury and much below that at which calomel freely volatilises. The source of the hydrochloric acid is certainly the magnesium chloride of the bittern and bay salt, which heated in a moist atmosphere, even in the presence of sodium chloride,

is, as is well known, partly converted into magnesia and hydrochloric acid. Hence the alkalinity of used *mitsuchi*.

The porosity of the walls of the apparatus particularly of the layer of *mitsuchi* on which the clay cup rests, must be more than sufficient to allow enough air to enter during the working. I had supposed that the hot hydrochloric acid and air might, in contact with the earth, have yielded a little chlorine, but the temperature reached in the process, normally worked, seems never to reach that required for the liberation of chlorine.

The Chinese process, if *correctly described*, differs from that used in Japan in several material points, one of which is that the mercury is intimately ground up with the other materials, and one fails to see what reaction can take place between it and the two others, namely, salt and alum. Heated, the mixture must give off mercury and hydrochloric acid, and then these as in the Japanese process will with air give the calomel ; but this is independent of the previous intimate mixture of the mercury with the salt and the alum. Another point is that in place of the magnesium chloride of sea-water as the source of hydrochloric acid, the Chinese are said to use alum, or copperas, which with the salt will react to give hydrochloric acid. A third point is that the cover is said to be closely luted to the iron pot, which must nearly exclude the air, without which it is impossible to explain the formation of the calomel. Perhaps this is the reason why the Chinese process is said to take four or five hours firing, since this may give time enough for the needed oxygen to diffuse through the cup and luting : it will also account for the fact, if it is one, that the yield of calomel is markedly less in weight than the mercury used. Again, there are to be noticed the statements that the iron pot is exposed for hours to the strong heat of a charcoal fire, and that the hot clay cover is cooled by throwing cold water on it, statements which must be

regarded as greatly exaggerated, if not erroneous. Lastly, the calomel-giving vapours are allowed to remain in contact with the hot iron of the pot instead of being kept from it by the thick lining of earth provided in the Japanese process, a contact which such vapours could not stand without destruction. I think, therefore, that we may be fairly doubtful whether any reliable description has yet been given concerning the Chinese process, which we may expect to find to differ little, if at all, from the Japanese process, except in salt and alum being used in place of the mother-liquor of sea-salt, or 'water' as Ono Ranzan supposed it to be.

One thing, to which attention may be called, is that the Chinese are stated to add some nitre to a similar mixture when employed to give corrosive sublimate. For that being the case, it is seen that free chlorine, which would here be developed from the salt, nitre, and alum, is necessary for the production of the higher chloride, and that air and hydrochloric acid can only yield the lower chloride, a difference of much theoretical interest, and indeed of practical moment also.

It is known that re-sublimation of calomel generates some corrosive sublimate, and, although authorities are not quite agreed as to whether reaction occurs between gold leaf and calomel vapour, it is hardly to be doubted that such reaction does occur. Now I have found that if in the Japanese apparatus the temperature of the cover is raised sufficiently to volatilise much of the calomel, the remaining calomel is no longer free from corrosive sublimate. It must therefore be borne in mind that the calomel formed in the Japanese process is not, and cannot be, the result of true sublimation, but of precipitation as fast as formed from the three gaseous bodies which give rise to it. At the temperature at which mercury boils, calomel is either quite fixed or at most has a vapour of exceedingly small tension. In the two facts, that the three gases do not react to yield corrosive sublimate

and that the calomel is not heated to its volatilising point, lie the explanation and, at the same time, the assurance, that Japanese (and Chinese) calomel contains no corrosive sublimate.

Summary.—The Japanese prepare calomel pure, above all things free from corrosive sublimate. They heat balls of porous earth and salt, soaked in bittern, along with mercury, in iron pots lined with earth. The heat forms hydrochloric acid from the magnesium chloride in the bittern, and the mercury sublimes into the closely-fitting but unattached clay covers of the pots. Air enters by diffusion, and oxygen and hydrochloric acid gas act together in the hollow cover on the vapour given off from the sublimate of mercury there formed. The cover thus becomes filled with a network of micaceous particles of calomel, precipitated, at a temperature below its subliming point, at the moment of its formation.

This investigation of an interesting product of Japanese industry has been carried out under the authority of the Imperial University of Japan. I cannot acknowledge fully enough the indispensable assistance I have received from my colleague, Assistant-Professor Haga, F.C.S.

Description of Pl. III.

- Fig. 1. Tongs for lifting and dropping the lumps of wet compost into the pots.
,, 2. Brass mercury spoon.
,, 3. Clay cup condenser.
,, 4. Iron furnace-pot.
,, 5. Peel for lifting compost ball out of pot.
,, 6. Quill feather for emptying enps.
,, 7. Wooden water-pot, for wetting furnace top.
,, 8. Wooden core and board for lining pots.
,, 9. Rectangular iron bars for pots.

Oximidosulphonates or Sulphazotates.

By

Edward Divers, M. D., F. R. S.

and

Tamemasa Haga, F. C. S.

By treating a solution of potassium nitrite and hydroxide with sulphur dioxide, Fremy, in 1845, discovered a series of salts of complex composition, of which, without succeeding in getting any definite insight into their chemical constitution, he was yet able to indicate to some extent a classification into three groups: the *sulphazates*, the *sulphazotates*, and the *sulphammonate*. He attached, and justly so, special importance to the sulphazotates, the salts which are the subject of the present paper, and from which by hydrolysis are derived the *oxyamidosulphonates* treated of in an earlier communication by us to this Journal (**3**, 211, where will be found references to the papers of Fremy, Claus, and Raschig, which will serve for the present paper).

A slight difference in procedure gave Fremy one or other of two salts, which he named respectively *basic sulphazotate* and *neutral sulphazotate*, because from their chemical composition and their ready passage into each other he held them to be salts of the same acid. He also described still more basic sulphazotates of potassium with barium and with lead.

Claus in 1871 worked upon the two sulphazotates, and gave an account very different from Fremy's of their composition, properties, and relations to each other. Finding them inconvertible into each other, he referred them to different acids and renamed them. He brought to light the sulphonic constitution of these salts (which had been, however, foreshadowed by Fremy), and consequently named the *neutral* sulphazotate of Fremy *disulphhydroxyazate*, while for the latter's *basic* sulphazotate he retained the name *sulphazotate* simply.

The sulphazotates were again examined in 1887 by Raschig, who established their constitution as derivatives of hydroxylamine* and made discovery of a potassium salt still more alkaline than Fremy's *basic* salt. He, in his turn, differed greatly from Claus and found Fremy's account of the salts in some respects more in accordance with the facts, but, on other grounds than those Claus had taken, retained the distinction made by this chemist between the neutral sulphazotate, which he renamed *hydroxylamine-disulphonate*, and the *basic* sulphazotate, which he agreed with Claus in calling simply *sulphazotate*. To his own discovered third potassium salt he gave the name of *basic sulphazotate*.

In the present contribution to the subject the existence of *sodium* oximidosulphonates is established; these and salts of ammonium, calcium, strontium, barium, and lead are described; much-needed methods, definite and productive, for preparing both sodium and potassium oximidosulphonates are given; the reversion of these salts to sulphite and nitrite made known; and the interrelations of the salts

* Claus had pointed out the hydroxylamine derivation of oxyamidodisulphonates but had decided against such a derivation for the (neutral) sulphazotate. In a foot-note to our preliminary paper on the *Reaction between sulphites and nitrites*, *J. Ch. S.* **51**, 659, we erroneously represented Raschig to be not quite accurate in stating that Claus had so decided. We regret our error. The facts are that while in one place, overlooked by us, in his several long papers Claus expressly makes this decision, he in another place, indicated in our note, gives the formulæ— $\text{ONH}(\text{SO}_3\text{K})_2$ and $\text{HON}(\text{SO}_3\text{K})_2$ —as alternative, without deciding between them.

classed apart by Claus and by Raschig shown to be such as to demonstrate the unity of their constitution as oximidosulphonates, or Fremy's sulphazotates.

Preparation of sodium and potassium oximidosulphonates.

In practice two courses are open by which to proceed in preparing oximidosulphonates from nitrites : * to mix together solutions of the nitrite and sulphite ; and to treat a solution of the nitrite and hydroxide or carbonate with sulphur dioxide. The question, already discussed by Claus, whether these methods are in principle identical, need not here be considered, and will be taken up in a future paper.

Freymy succeeded only by the second of these methods in getting oximidosulphonates. Claus found both successful but the second to be much more productive. Raschig recommended (*Annalen*, **241**) the first method as being the more convenient and productive when sodium salts are worked with, neutral potassium oximidosulphonate and also an alkaline potassium-sodium oximidosulphonate being then got by double decomposition with potassium chloride. Only incidentally, in discussing Claus's views, did he mention that the second method of getting the potassium oximidosulphonates is occasionally successful. He included, however, this method in the specification of his patent (1887)** for the manufacture of hydroxylamine. Both methods have been investigated by us, and in what follows it will be seen that we have found the second method to be much the better one, whether for the potassium or for the sodium salt, but that the first can be made far more successful than it has hitherto proved to be.

* It has been shown by Raschig that oximidosulphonates are also obtainable from nitric oxide, namely, by first converting it into nitrososulphonate, Pélouze's salt, and then letting this decompose in alkaline solution.

** Our references concerning the patent are abstracts in the *Berichte*, *J. Ch. S.*, and *J. S. Ch. Ind.*

*Oximidosulphonates prepared by the direct use
of sulphite.*

Introduction.—Claus supposed himself to use the normal or dipotassium sulphite but, it is clearly evident, used in reality mainly the metasulphite. The ‘neutral’ (or meta) sulphite therefore is to be taken in the proportion of ‘less than four’ molecules, or $2 \text{ K}_2\text{S}_2\text{O}_5$, to one of the nitrite, KNO_2 ; with more than four molecules only nitrilo-sulphonate— $\text{N}(\text{SO}_3\text{K})_3$ —separates, and even with less than four much of this salt is produced; both salts are almost insoluble, but the loose fine needles of the nitrile can be mechanically washed away from the large crystals of the oximide; the solutions used are not to be too dilute; the yield of oximidosulphonate is very small. We can confirm Claus’s account of this very unproductive process.

To prepare the neutral salt Raschig runs into a mixture of the solution of one molecular proportion of sodium nitrite with ice, a solution of two molecular proportions of sodium disulphite (*i. e.*, one mol. of metasulphite), adds two molecules of potassium chloride in cold saturated solution, and leaves the whole to crystallise. Nitrile forms, as well as oximide, and the two salts are to be separated by elutriation, as directed by Claus. The yield of potassium salt is stated to be about half the calculated quantity. No mention is made in the memoir in the *Annalen*, or in the papers in the *Berichte*, of a similar proceeding with potassium nitrite and sulphite to get the potassium oximidosulphonate direct, the above indirect process obviously being resorted to as superior to it. But in the specification for the patent, which of course preceded the publication of the memoir, Raschig did give the direct method. His process has proved successful in our hands but less so than in his.

The same investigator records the preparation, from sulphite, of a potassium-sodium oximidosulphonate having the basicity of Fremy's basic potassium sulphazotate. Having on one occasion poured commercial solution of sodium bisulphite upon cold sodium nitrite he found the mixture become boiling hot, and by addition of much potassium chloride to yield much potassium-sodium nitrilosulphonate. Then, during the twenty four hours following, the mother-liquor deposited the new salt together with some potassium nitrilosulphonate. We have not been able to confirm his experience. The potassium-sodium salts we can get, but according to our own observation, the addition of alkali hydroxide, of which he says nothing, is requisite to form an alkaline oximidosulphonate. And from Raschig's own sound criticism (*op. cit.*, p. 222) of Claus's method of getting the alkaline potassium salt, it might be supposed that he too believed in this necessity of adding hydroxide in order to form any basic oximide.

Preparation of the neutral potassium oximidosulphonate from metasulphite.—To prepare the potassium oximidosulphonate by using metasulphite we ourselves work from the first with potassium salts. The solutions of potassium nitrite and metasulphite are used with ice floating in them, the mixture is kept in ice for a day or more, and the ice-cold mother-liquor is then drained off from the crystals which will have formed. To allow, as Raschig would, as we understand from abstracts of the specification of his patent, the temperature to rise even up to 40°, greatly interferes, and gives such a result as Claus obtained,—very much nitrile and very little oximide. Some nitrile forms even in the ice-cold mixture and has to be washed away from the oximide crystals; and when the mother-liquor is afterwards allowed to acquire the common temperature it produces and deposits more nitrile.

The proportion of the quantities of the salts to be taken is somewhat more than that of three molecules of metasulphite to two

molecules of nitrite, instead of one to one as when Raschig's directions are followed. In the mother-liquor remains much sulphite but only insignificant quantities, at most, of nitrite. In order to measure with some degree of accuracy the quantities of nitrite and metasulphite taken, it is best to use good commercial nitrite, the real strength of which in this salt is known from a previous assay ; and to prepare the metasulphite, shortly before it is wanted, by passing-in sulphur dioxide just to the point when the solution becomes neutral to methyl-orange or to lacmoïd paper. Some inconvenience in preparing the ice-cold solutions is caused by the sparing solubility of the metasulphite, which entails the cooling of large volumes of solution. The yield of washed crystals of the oximide is about 60 (63) per cent. of the calculated quantity, and such therefore as seems not to have been even remotely approached by previous workers, although greatly surpassed by that of the sulphur-dioxide method described on pp. 27 *et seq.* By working at temperatures some degrees below 0° the production of nitrile seems to be further lessened and that of oximide somewhat increased.

It is besides almost certain that the formation of the nitrile increases relatively to that of the oximide as the nitrite remaining grows less, the sulphite having more and more already-formed oximide to work upon along with less and less nitrite. This view of the matter is supported by experience. For when the metasulphite is added to an excess of nitrite, the crystals of oximide produced amount, when washed from the little nitrile accompanying them, to 80 per cent. of the quantity theoretically possible from the quantity of metasulphite taken.

Formation of the neutral sodium oximidodisulphonate from metasulphite.—By treating one molecule of sodium nitrite with two, or a little less than two, of sodium metasulphite, the nitrite may be wholly

sulphonated, principally into oximide but partly into nitrile. To prevent heating up the solutions are best mixed ice-cold, but the mixture may then be exposed to the ordinary temperature and left so for a day. This process though successful is of little practical value because the very soluble sodium oximidosulphonate cannot be separated from the sulphite and nitrilosulphonate which accompany it. The actual success of the process in forming much oximidosulphonate has been ascertained therefore only by analysis: the excess of sulphite having been precipitated as barium salt, the solution has been hydrolysed, and the hydroxylamine estimated. In this way evidence has been got that there is formed about 83 per cent. of the oximidosulphonate equivalent to the nitrite.

The solution of the sodium salt thus obtained can be used to get the potassium salt by double decomposition, like that prepared after Raschig's directions, than which it is markedly more productive and gives a purer product. Raschig used only half as much metasulphite as we use and therefore, by our finding, left one-third of the nitrite undecomposed in the solution. He also added the potassium chloride just after mixing the sodium salts, and did not, it would seem, preserve the coldness of the solutions necessary in presence of potassium salts. By his method, consequently, much more nitrile is got than by ours, in which the solution of the sodium salts is only treated with the potassium chloride after it has stood a day, so as nearly to complete the reaction, and then been cooled again in ice, to prevent as much as possible the formation of nitrile which otherwise goes on, especially in the potassium salt. Following this method the oximide is obtained with so little nitrile accompanying it as only to become visible during recrystallisation of the product. But these points are now of no importance as regards the preparation of the potassium salt, since there are according to our experience, the much better direct processes for

its preparation, one just given (p. 19) and the other now to follow (p. 27).

Oximidosulphonates prepared by the use of sulphur-dioxide.

Introduction.—Fremy's process for the *neutral* potassium salt is to pass sulphur dioxide into a suitably concentrated solution of potassium nitrite and hydroxide, until the salt separates. Should the solution, however, be sufficiently concentrated, the *basic* oximidosulphonate will make its appearance instead, he found ; and with high concentration the *sulphazate*, and occasionally the *metasulphazate* will first form. Then water must be added, enough to dissolve up any of these salts, and the passage of sulphur dioxide be continued until the neutral salt begins to crystallise.

Claus found Fremy's prescription too indefinite and uncertain, and therefore modified it, mainly so far as to employ equivalent quantities of nitrite and hydroxide. He could get no such simple succession of salts as Fremy had obtained, and he had always much nitrile to deal with, a salt which gave no trouble to Fremy. Keeping the mixture cool during the entry of the sulphur dioxide, he got a solution which, not immediately, as Fremy had found, but only on standing, gave crystals of the neutral oximidosulphonate. Letting the mixture grow hot by the action of the sulphur dioxide, crystals separated, very much like Fremy's sulphazate, and these treated with enough water left much insoluble nitrile and gave a solution which slowly deposited crystals of the neutral oximide along with those of the basic oximide.

Raschig, in the specification of his patent, appends to the account of the process he gives for the preparation of the neutral oximidosulphonate a statement to the effect that this may also be obtained by passing sulphur dioxide in excess into a solution of alkali nitrite and

its equivalent of either hydroxide or carbonate. But in his memoir in the *Annalen*, published later, he makes no mention of this, and condemns Fremy's and Claus's similar processes as being frequently unproductive, although Claus had also used equivalent quantities of nitrite and hydroxide. Evidently he thought little of this method because of its uncertainty; yet, as we shall show, it is, when properly modified, regularly very productive in the case of both sodium and potassium salts, and indeed is the only one by which sodium oximidosulphonate can be isolated.

Fremy prepared his *basic* salt nearly in the same way as his neutral salt, varying this only to the extent of working with more concentrated solutions, and so regulating the passage of sulphur dioxide (assisted, it would seem, by addition of more potassium hydroxide), that separation of crystals should take place with the solution still strongly alkaline. Some care was required as to the degree of concentration, which he adjusted by beginning with solutions strong enough to deposit sulphazate, and then adding just sufficient water to redissolve this before passing in more sulphur dioxide. The yield of salt was large. A secondary method of his was to make sulphazate and salts allied to it and then treat these with water, when the alkaline oximidosulphonate slowly crystallised from their solution.

Claus in following Fremy's main process could get the alkaline salt only in admixture with the neutral salt and other compounds. By fractional crystallisation it could indeed be separated from these, but he preferred a modification of Fremy's secondary process, in which salts nearly the same as Fremy's sulphazate, with their adhering alkaline mother-liquor, are boiled with two or three times their volume of water, the solution filtered hot, and when barely cold decanted from the crystals that have formed of the basic salt. This Fremy-Claus

process is no longer, we consider, of any value, the two alkaline salts being readily obtainable from the neutral salt.

Preparation of the neutral sodium oximidosulphonate by the sulphur-dioxide method.—The salts are to be in the proportion of two molecules of sodium nitrite to one of sodium carbonate or to two of sodium hydroxide, but with about a tenth extra of the carbonate or hydroxide, (called for by the unavoidable* conversion of about a tenth of the nitrite into nitrile). In our experience this proportion gives about the highest yield of oximide along with decomposition of all the nitrite. Suppose the nitrite used to be 96 per cent. pure, and the quantity taken 50 grams. Then this is put with 110 grams of sodium carbonate crystals, or 30.8 grams of real sodium hydroxide, a piece of lacmoïd paper, and 150 cub. cents. of water when carbonate is taken, or 200 cub. cents. when hydroxide is taken, into a 500 cub. cents. flask, fitted with cork carrying inlet and exit tubes and, if convenient, a thermometer. The inlet tube dipping into the solution is connected with the sulphur-dioxide apparatus, and the exit tube with a washing bottle of water, by caoutchouc tubes of adequate length to allow of free movement of the flask. The stream of sulphur dioxide may be rapid (in the case of the carbonate being used, very rapid), but not so as to cause any fumes either white or red in the flask kept in active motion and immersed in water with ice floating in it. Hardly any sulphur dioxide at all will escape consumption. In about 70 minutes the solution will have become acid or nearly so, if the sulphur dioxide have been passed in at a good rate. Notice of the remote approach of neutralisation of the solution is given, when carbonate is being used, by the disappearance of the last of the sodium hydrogen carbonate which has been precipitated previously by the sulphur dioxide and, when sodium hydroxide has been used, by the disap-

* No longer unavoidable: March, 1894.

pearance of the precipitated sulphite. By effecting neutralisation very slowly at the last until acidity to lacmoïd paper is just reached, the finish of the process may be so hit off that the solution is left free not only from nitrite but also from more than traces of sulphite. In practice we find it better to enter the gas quickly up to distinct acidity and then at once to stop its flow, for although the solution is then found to contain a not inconsiderable quantity of sulphite and to give off a small quantity of nitrous fumes, this hardly affects the yield of oximide, and there is great saving of time and attention. When the method of slow finishing has been followed it is usually necessary to add a drop of dilute sulphuric acid at last, to destroy nitrile.

In the solution, either way prepared, the nitrile present now suffers hydrolysis into imidosulphonate and acid sulphate. To make sure of the disappearance of every trace of this very unstable and therefore injurious salt, the solution is well cooled after the rapidly occurring hydrolysis, and left in its acid condition for ten or fifteen minutes, while in the case of its containing sulphur dioxide a free stream of air is blown through it to carry this away. For fifteen minutes or so in such a solution, the oximidosulphonate proves able to resist the hydrolysing action of a little sulphuric acid. To the solution deprived of all nitrile and almost all sulphur dioxide, concentrated solution of sodium carbonate is added until alkalinity is reached. For this purpose about 10 grams of the carbonate will be wanted but more should be at hand to at once arrest hydrolysis should it happen to have set up in the oximide.

The liquor is now a solution of about 150 grams oximidosulphonate in about 220 grams of water, along with 15 grams of disodium imidosulphonate, and 22 of hydrated sodium sulphate. After filtration from any impurities derived from the nitrite it has to be evaporated and this cannot be safely effected at a steam-heat. It has

therefore to be done either at a gentle heat in the air, or in a vacuum over sulphuric acid. When the solution has been somewhat concentrated, say to a weight of 300 grams, it is cooled in ice and after some hours strained, still in the refrigerator, from a transparent magma of the crystals which have separated of sodium sulphate. Should the oximidosulphonate be wanted for the preparation of oxyamidossulphonate or, through this, of hyponitrite (*This Journal*, **3**, 211), the solution at this stage is serviceable without further preparation.

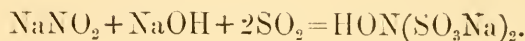
The solution, deprived of much of its sulphate by cooling, soon yields when restored to the vacuum-evaporator much of the sodium oximidosulphonate. 90 or 100 grams of this having crystallised out, its mother-liquor may be deprived of more sulphate by cooling and then yields a further crop of oximide crystals, nearly pure, by evaporation.

Even as first obtained, disodium oximidosulphonate is nearly pure, being an anhydrous salt in hard and dense small thick prisms deposited as a thick crust on the bottom of the crystallising pan. It may be recrystallised from its solution, made with a little hot water rendered slightly alkaline by ammonia, by evaporation.

Titration of some of the original liquor after hydrolysis of the salt into hydroxylamine, shows the yield to be 90 per cent., more or less, of the quantity calculated from the nitrite, but the crystals obtained do not amount to much more than 70 per cent., because the magma of sulphate retains much of the solution, which is very concentrated.

The theory of the formation of the oximidosulphonates is contained in the theory of the reaction between nitrites and sulphites, and this we hope to discuss in a future paper. It will be enough to point out here that one molecule of nitrite, one of hydroxide, and two of

sulphur dioxide are by calculation convertible into oximidosulphonate and nothing else :—



It will therefore be seen that, with these proportions, were sulphite used instead of hydroxide or carbonate to begin with, the use of sulphur dioxide could still not be dispensed with. Cooling during the preparation of the solution guards against premature hydrolysis and lessens the production of nitrile. The quantity of water is limited because it has to be evaporated afterwards in the cold. Much less than the quantity prescribed will not be enough because then so much acid-carbonate or normal sulphite, as the case may be, may separate out in the middle of the process as to thicken the solution so that it cannot be sufficiently rapidly agitated with the sulphur dioxide to prevent the injurious action of local excess of the latter.

It is generally more convenient to work with sodium carbonate than with the hydroxide ; and on a large scale especially or on a moderately large scale, much more rapid working is possible with the carbonate than with the hydroxide, because of the much greater heat to be dealt with when the latter is used.

Preparation of the neutral potassium oximidosulphonate by using sulphur dioxide.—The process just described for getting sodium oximidosulphonate, but modified as to temperature, is eminently successful, and even simpler than when used for that salt, when employed for getting potassium oximidosulphonate from potassium nitrite and either carbonate or hydroxide. Other processes, already published, cannot approach it in certainty and in purity of the salt yielded. Over the process described in this paper (p. 19), in which are mixed somewhat more than three molecules of potassium metasulphite with two of nitrite, it has the same advantage, although in a less degree, and in

addition that of not requiring voluminous solutions to be worked with. For it is still more certain than that process, and gives a yield in crystals of nearly 97 per cent. of the calculated quantity of the salt when carbonate has been used, and nearly 95 per cent. when hydroxide has been used, and with not enough nitrile with it to be visible, even on recrystallising. It is the small solubility of the potassium salt which makes its preparation simpler than that of the sodium salt, and also which, by throwing it out of solution removes most of it from the influence of the sulphite and sulphurous acid, and thus lessens the formation of nitrile. Our success by this method has been so much greater than that reached by either Claus or Raschig mainly because we have taken the nitrite and the hydroxide in the right proportion, besides guarding against local excess of sulphur dioxide, and being careful to preserve a low temperature.

Potassium nitrite, assayed for real salt, and either powdered potassium acid-carbonate or concentrated solution of potassium hydroxide in equal molecular proportions are put, with a weight of water about eight times that of the real nitrite, in a roomy flask kept at 0°, or even better at 2° or so below zero, by means of a brine bath with ice floating in it. To preserve this temperature during the passage of the sulphur dioxide the flask must be kept in active motion, with a thermometer in it for observation. By a wide inlet tube terminating high above the surface of the solution, in order to guard against its being choked, the sulphur dioxide is rapidly passed in until the carbonate has all dissolved and effervescence has almost ceased, or until the hydroxide, if that has been used, is nearly neutralised, and then passed slowly until the solution has become neutral to lacmoid paper. The oximidosulphonate separates during the latter part of the process as a crystalline powder. The mother-liquor retains a little of it and contains, besides, some nitrile along with small quantities of both

nitrite and sulphite. The precipitated oximidosulphonate can be dissolved in hot water containing for safety a little ammonia, and from the solution be got in good and characteristic crystals of great purity and comparative stability. The limit to the quantity of water to be used is given by the condition that there must be enough not to be thickened by the separation of the insoluble oximidosulphonate. When passing the sulphur dioxide into the flask closed with a cork holding inlet and outlet tubes and the thermometer, and with the inlet tube ending seven or eight centimeters above the surface of the solution, and even when the alkali is used as carbonate so that carbon dioxide is freely escaping, it is remarkable to observe the almost perfect absorption of the sulphur dioxide by the well-agitated solution.

Preparation of alkaline oximidosulphonates from the neutral salts.

Alkaline sodium oximidosulphonates.—The alkaline sodium oximidosulphonate, $\text{Na}_3\text{NS}_2\text{O}_7$, corresponding to Raschig's *basic potassium sulphazotate*, can be got by dissolving the neutral salt in water, adding just the calculated quantity of sodium hydroxide, and evaporating to the crystallising point in a vacuum desiccator. The possibility of preparing the salt in this way is of theoretical importance, but the salt, being less soluble in presence of sodium hydroxide, can also be prepared at once as a crystalline precipitate by adding the sodium hydroxide in excess to a concentrated solution of neutral sodium oximidosulphonate. The crystals are drained on a tile and recrystallised once or more by dissolving and evaporating.

A *less alkaline sodium salt*, $\text{Na}_3\text{H}(\text{NS}_2\text{O}_7)_2$, is obtained when a solution of the neutral oximidosulphonate is evaporated with somewhat less sodium hydroxide than is needed to form the above salt,

such as half or more of that quantity. It can be purified by recrystallisation from water. It does not correspond in composition to Fremy's *basic sulphazotate* of potassium, being more alkaline than that salt.

Alkaline potassium oximidosulphonates.—The alkaline potassium salt, $K_3NS_2O_7$, discovered by Raschig, was obtained by him by dissolving the neutral salt in hot alkalisied water, cooling rapidly, adding to the supersaturated solution cold concentrated potassium-hydroxide solution until it caused a troubling, and finally leaving the mixture to deposit the new salt in crystals. He also prepared it by adding an excess of very strong potassium-hydroxide solution to Fremy's basic salt already dissolved in a little warm water, and precipitating by alcohol.

Not only is excess of alkali useful here, as when preparing the sodium salt, by diminishing the solubility of the salt in its aqueous mother-liquor, it is also necessary, which is not the case with the sodium salt, to preserve the salt from decomposition by water. But when alcohol is to be used as the precipitating agent, the excess of alkali used by Raschig is uncalled for, equally as in the case of the sodium salt, and only the calculated quantity of potassium hydroxide having been added, a sufficiency of alcohol precipitates the salt. Indeed the best way to purify the salt from adhering alkali is to dissolve it in water and precipitate by alcohol, repeating the process once or twice.

The less alkaline potassium salt, $K_5H(NS_2O_7)_2$,—Fremy found that the neutral salt in the solid state combines at once with potassium hydroxide when treated with an excess of it in solution, and that the product dissolved up in hot water deposits his *basic sulphazotate* in crystals on cooling. We have fully confirmed Fremy's experience although from the fact of both salts being very sparingly soluble, the change of the

powdered neutral salt into the alkaline one is not very apparent to the eye. Yet the potassium hydroxide is withdrawn by the powdered salt rapidly from the solution, which consequently loses all its causticity to the tongue. When used concentrated the alkali causes, as Fremy noted, a heating up, while when it is dilute it causes, we find, a fall of temperature amounting to about three degrees, due no doubt not to dissolution of salt so much as to liquefaction of three-fourths of the water of crystallisation of the neutral salt transformed.

Claus denied altogether that the neutral salt could be converted into the alkaline salt, and Raschig found it necessary for success to modify Fremy's process. The latter uses twice the calculated quantity of potassium hydroxide and in concentrated solution, heats to boiling, and crystallises the salt by cooling. The salt is purified by recrystallisation from water. Weak alkaline solutions of the neutral salt did indeed yield him crystals of the alkaline salt, but in order to do so had to be left to stand for some weeks. According to our experience, however, any excess of alkali is unnecessary and best avoided, as then the alkaline salt is at once obtained pure. Neither need the solution of the alkali be concentrated; nor the mixture be heated to boiling; nor time be given, more than is usual for crystallising out salts. When crystallisation happens to be slow in setting in, a particle of the solid salt, previously obtained, at once determines it when dropped into the solution.

Alkaline potassium sodium oximidosulphonates.—There is some difficulty in preparing potassium-sodium salts having a basicity as great as that of the more alkaline sodium or potassium salt, but several less alkaline can be obtained by mixing in solution the neutral potassium salt with sodium hydroxide, or the neutral sodium salt with potassium hydroxide, and either evaporating or adding alcohol.

Alkaline barium, strontium, and lead salts.—Addition of barium

hydroxide or of oxy-lead acetate to a solution of neutral potassium, sodium, or ammonium oximidosulphonate causes precipitation of an alkaline or 'basic' salt. Strontium hydroxide gives a precipitate of alkaline salt with potassium oximidosulphonate almost immediately ; it gives no precipitate with sodium or ammonium oximidosulphonate, but causes an alkaline salt to crystallise out in a few hours.

Decomposition of chlorides and nitrates by the neutral oximidosulphonates.—The neutral potassium salt in solution with potassium chloride, potassium nitrate, or sodium chloride, yields when treated with strong ammonia-water, alkaline oximidosulphonates either at once or after evaporation, ammonium chloride or nitrate being formed.

Decomposition of carbonates and of acetates by the neutral oximidosulphonates.—When a warm solution of sodium or potassium carbonate is saturated with the neutral potassium oximidosulphonate, the less alkaline potassium-salt or an alkaline potassium-sodium salt crystallises out on cooling. Neutral sodium or potassium oximidosulphonate evaporated with either sodium or potassium acetate gives off acetic-acid vapours and yields a highly alkaline solution (of alkaline oximidosulphonate).

*Reconversion of alkaline into neutral and less alkaline
oximidosulphonates.*

The conversion of the more alkaline back into the alkaline, and of these again into the neutral oximidosulphonates, described in this section, and the conversion of neutral into alkaline oximidosulphonates, described in the preceding section, demand particular attention as being facts opposed to the view maintained by Claus and by Raschig that Fremy's sulphazotates include the salts of two distinct acids.

Solutions of the more alkaline and the neutral sodium oximidosul-

phonates evaporated together, best in a vacuum desiccator, are converted into the less alkaline salt in crystals. Two equivalents of the former combine with one of the latter, and this happens even when only one equivalent of the former salt is taken, excess of the latter then remaining in the mother-liquor. We find also that, when a concentrated solution of the more alkaline potassium salt is mixed with a cold supersaturated solution of the neutral potassium salt in equivalent quantity, crystallisation of the less alkaline salt soon sets in, or else can be at once determined by adding a crystal of this salt.

Conversely, in a manner, as Fremy found, the less alkaline salt is resolved, when treated with the solution of a lead or a barium salt, into the neutral salt and the more alkaline salt, the latter becoming by the reaction an insoluble salt of lead or barium with potassium. Indeed, this conversion is the basis of one of Fremy's methods of getting the neutral potassium salt. Fremy's observations were emphatically discredited by Claus and ignored by Raschig.

Weak acids—including sulphurous acid (*cf.* Fremy)—and even strong acids diluted and cautiously used, convert the alkaline salts into the neutral, when added not in excess so as to hydrolyse the oximidosulphonate. Claus strongly denied this to be the case, but Fremy was right. Metasulphites also, added not in excess, remove alkali from the alkaline salts, they themselves becoming thereby normal sulphites, as shown by the mixed solutions being neutral to rosolic acid.

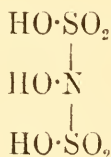
Several metallic salts such as those of zinc and manganese behave like acids with the alkaline salts, converting them into the neutral salts, and depositing their own metal as hydroxide, as was observed by Fremy. Water alone when present in large quantity suffices to convert the more alkaline potassium salt into the less alkaline salt

along with potassium hydroxide (*cf.* Raschig), but has little if any action upon the sodium salt.

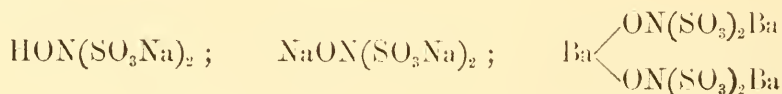
Constitution.—Unity of oximidosulphonates and sulphazotates.

The constitution of the oximidosulphonates as sulphonic derivatives of hydroxylamine follows from the fact of their formation by the union of nitrite with sulphite, and from that of their hydrolysing in stages into hydroxylamine and sulphate. From the account we have given—and Fremy before us—of the conversion of the neutral into the alkaline, and of the alkaline into the neutral salts, it might also be accepted without hesitation that they are all the salts of one acid, were it not for the dissent of Claus and of Raschig from this conclusion.

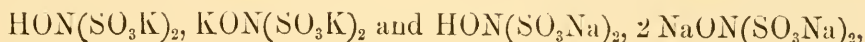
All the compounds described in this paper seem to us to be as clearly salts of one acid as are the acid, neutral, and alkaline orthophosphates all salts of one phosphoric acid. This acid, Fremy's sulphazotic acid, is oximidosulphonic acid, a tribasic acid with the constitutional formula :—



and unknown as the wholly hydrogen salt. Of this acid the salts with neutral or slightly acid reaction are monohydrogen salts, and those with three monad atoms of metal (or their equivalent) which we have hitherto designated *the more alkaline* salts are normal salts. For example :—



The less alkaline salts, called in the case of potassium, *basic sulphazotate* by Fremy and *sulphazotate* by Claus and by Raschig, are double salts of the normal and the acid or hydrogen salts, thus :—



pentapotassium bi-oximidosulphonate and *octasodium ter-oximidosulphonate*.

Nothing is wanting in the evidence to the truth of the views here set down. Any acid serves to replace in the normal salts the third atom of metal by hydrogen ; many salts, such as zinc sulphate, do the same (p. 33) ; potassium or sodium hydroxide directly reacts with the hydrogen salts to replace some or all of the hydrogen by metal, and just as readily as it does with phosphoric acid, which, as is well known, does not exchange all its hydrogen for metal unless the alkali is used in good excess.

That the pentapotassium and octasodium compounds are double salts is a conclusion in accordance with all that is known of them. They are formed by the simple union of the hydrogen salt with the normal salt in solution, even when the quantities of these salts deviate to a not inconsiderable extent from their proportions in the compound salts, and they have an action on litmus, phenolphthaleïn, etc., the same as that of the normal salts alone. In reaction they decompose into their component salts, one only being active. Fremy observed that lead and barium salts added to a solution of the pentapotassium salt gave precipitates more basic than it and left in solution the neutral potassium salt ; but as he did not know of the existence of the normal potassium salt, and got complex basic precipitates, he could not represent the change quite as we do. Even now it still remains open to say with him that a basic salt precipitating, the acid of the precipitant converts another portion of the alkaline potassium salt into the neutral one, though to do so is to

pay no consideration to the fact that normal salts are known and that the barium salt is one of them and the lead salt another, though in this the lead is half as hydroxide. It seemed of interest to examine the precipitation by barium chloride quantitatively. Adding this salt in moderate excess to a solution of the pentapotassium salt, the proportion of oximidosulphonic radical precipitated was found to be eight-nineteenths of the whole, a result which, after allowing for the solubility of barium oximidosulphonate, may be accepted as proof that the pentapotassium salt had acted as tripotassium salt united with indifferent dipotassium salt.

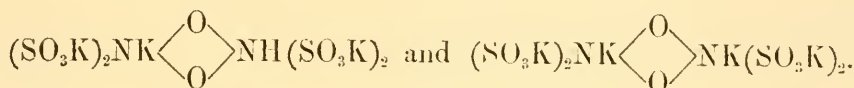
One thing more is to be pointed out in support of the double-salt character of the pentapotassium oximidosulphonate. Although this salt so readily and frequently crystallises from solution as to have become the best known of the three potassium salts, matters are quite otherwise with the sodium salts among which the pentasodium salt cannot even be prepared at all; the salt intermediate to the normal sodium and the disodium salts which can be formed is troublesome to get; and the normal salt itself is that which readily crystallises out from a sufficiently alkaline solution.

There is a strikingly characteristic test for these salts in common, from which it may be assumed that they have a common constitution. Both neutral and alkaline oximidosulphonates, and no other salts, yield in solution, at once and in the cold, an intensely coloured salt when gently oxidised in the absence of acids, the reaction being one of the most remarkable in the domain of inorganic chemistry (p. 46). This reaction was described by Fremy and his accuracy fully confirmed by Raschig.

Failing entirely to convert Fremy's two potassium sulphazotates into each other, or believing that he had failed, Claus held them to be salts of distinct acids and gave them the distinctive formulæ—

$\text{NHO}(\text{SO}_3\text{K})_2$ and $\text{NH}(\text{SO}_3\text{K})_3 \cdot \text{NO}(\text{OK})(\text{SO}_3\text{K})$ —with the nitrogen quinquevalent. He believed that he had evidence of the latter salt being always formed from the nitrile, $\text{N}(\text{SO}_3\text{K})_3$, which for him was trisulphammonate, $\text{NH}_2(\text{SO}_3\text{K})_3$. Claus's observations and conclusions were examined at length by Raschig and satisfactorily refuted by him: they need not, therefore, be discussed by us.

Having rejected Claus's views, Raschig gave the hydroxylamine constitution to the neutral salts, but proposed another constitution for the two alkaline salts, in which like Claus he made the nitrogen quinquevalent :



The character of this constitution need not be noticed here : we shall probably return to it in our paper on the *sulphazilates*.

One argument given by Raschig for regarding as distinct the acids of the neutral and alkaline salts was, that although acids at once convert the latter salts into the former, alkalis effect the opposite change only with difficulty. Weak alkali, he found, requires weeks and to be in excess in order to effect the conversion ; while a concentrated solution of alkali must be, in order to act quickly, boiling hot and also in excess. Our experience, recorded on page 31, lends no support to this finding. The delay he observed in the appearance of crystals of the salt, a fact to which he attached weight, must have been due to the salt remaining in supersaturated solution, although he himself dismissed this explanation of the matter as inapplicable, averring that this salt when obtained does not show the phenomenon of supersaturation, without however testing the matter by adding a bit of the salt to excite crystallisation. The salt does not indeed supersaturate its solutions to the degree shown by the neutral potassium

salt, but nevertheless its hot strong solution in simple water can be rapidly cooled and then kept for an hour or more without crystallising, and nothing can be objected to allowing that it may show still greater supersaturation in presence of the other salts of its mother-liquor. Mixtures made to prepare it, yield it at once if cold when a crystal of it is dropped in or when alcohol is added, free alkali remaining in solution in the latter case, only when in excess of that required to form the normal salt, and this salt occurring in the alcoholic precipitate when the alkali is in excess of that necessary to the composition of the pentapotassium salt. But apart entirely from this matter of supersaturation, there is in proof that cold dilute alkali converts the neutral salt at once into the alkaline salt, Fremy's experiment, repeated by us, of acting upon the solid neutral salt with potassium hydroxide in aqueous solution (p. 30). The neutral sodium salt also passes readily and at once into the normal salt on adding sodium hydroxide to it.

Raschig gave as a chemical property distinguishing between the neutral and alkaline salts, the activity of alkali sulphite upon the former and inactivity upon the latter. We have not succeeded ourselves in finding anything in Claus's writings upon this subject, but Raschig adduced Claus in evidence of his statement. Fremy's writings, however, are sufficient to show that the above distinction, so far as it holds good, serves only to mark the alkalinity of the alkaline salt, for he points out that sulphurous acid first converts the basic sulphazotate to the neutral sulphazotate and then acts upon this salt. To this we may add that metasulphite and alkaline oximidosulphonate change together into normal sulphite and neutral oximidosulphonate, and that when enough metasulphite is added all oximidosulphonate is quickly destroyed.

There yet remains for consideration one other ground taken by

Raschig for separating the alkaline sulphazotates from the neutral salts. According to him, there is a salt isomeric with Fremy's basic sulphazotate which, unlike this, cannot be recrystallised from water, its alkaline solution depositing neutral oximidosulphonate. It, therefore, rather than Fremy's basic sulphazotate, must be held to be half-basic oximidosulphonate. The Fremy-Claus process for preparing neutral oximidosulphonate had yielded him the new salt but this was only once, and many repetitions of the process failed to reproduce it.

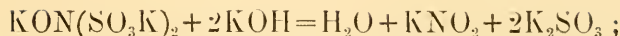
The difficulties in the way of accepting the existence of this salt as proved are very great. As just stated it could not be got again by the process which had apparently yielded it. Moreover it cannot be got by treating the neutral salt with potassium hydroxide, and it is hard to see why, if it is a basic derivative of that salt decomposable by water. Neutral oximidosulphonate recrystallised from water holding potassium hydroxide is strongly alkaline, so that Claus found the percentage of potassium to become some units higher than in the pure salt, and such a preparation behaves with water like Raschig's salt. But, according to Fremy and us, there is here only the production of some basic sulphazotate, and recrystallisation from water separates the two salts. Now, the process which gave Raschig his salt may, according to Claus, give not only the neutral salt but also some basic salt, and therefore a combined product behaving with water like Raschig's preparation.

But if we accept the results of the analysis of the salt, it could not have been other than a new salt, for Fremy's basic sulphazotate does not decompose with water into the neutral salt. We are strongly of opinion, and venture here to suggest that some mistake such as may happen to any one at times, crept into Raschig's work or the record of his work. He made but one analysis, and in that determined only the potassium and the sulphur. Of these two the sulphur content

happens to be a factor of no account in the present case, for the percentage of sulphur is nearly the same in the neutral and the basic sulphazotate (or its isomer), namely, 20.97 and 21.53, and Raschig's preparation could not from the circumstances claim any high degree of purity. There is, however, a wide degree of difference between the percentage of potassium in the neutral and that in the basic salt, and that got by Raschig agrees well with that for the basic sulphazotate with its one molecule of water of crystallisation. But here Raschig himself comes, as it were, to our assistance, and this emboldens us to ask for some distrust of his potassium number, for he actually discusses (though only to reject on experimental grounds) the possibility of his salt being Fremy's sulphazate, which does not differ 'so very much' from it, he says, in the amount of sulphur,—although that difference is really $3\frac{1}{2}$ per cent. Now, let such a lowering as this be made in the potassium percentage and the sulphur be allowed to stand, and we should have a sulphazotate mixture or compound resolving itself, as Raschig's preparation did when dissolved in water, into the two salts. In concluding this criticism we may add that Raschig ends by affixing a note of interrogation to the formula of his salt.

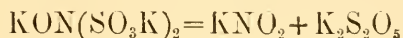
Reversion of oximidosulphonates to nitrite and sulphite.

Sodium or potassium oximidosulphonate digested in the cold for some hours with highly concentrated solution of sodium or potassium hydroxide, or boiled with it for a few minutes, is largely converted into nitrite and sulphite. Some of the latter salt crystallises out, and the solution evolves sulphur dioxide and oxides of nitrogen when acidified, or when neutralised and mixed with barium chloride gives a precipitate of barium sulphite, leaving nitrite in plenty in solution, easy then to detect. The following equation expresses this change:—



but the reversion is attended with some other changes resulting in the escape of nitrous oxide and perhaps nitrogen, a consideration of which will be found following the paragraphs on hydrolysis (p. 42).

The normal salts in solution, or as solids not quite free from moisture, soon suffer some reversion in the cold, and very much more when the solutions are boiled, without any alkali being added :—



The five-sixths normal potassium salt, although when well prepared and preserved in a dry atmosphere it can be kept for months without suffering noticeable change, is when damp unstable, as found by Claus and Raschig. But even the best preparations of it prove at last unstable, and we have found that this is due to the reversion which slowly goes on in it :—

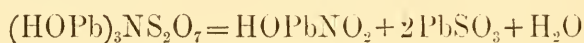


Such a mixture of salts must be very unstable, liable to hydrolysis and other changes, sufficient to account for that entire break-up of the salt which finally happens. Even before the change has proceeded far enough to affect the appearance of the crystals, the reversion can be detected by the presence of sulphite. For this detection direct acidification of the salt is hardly effective, because of secondary reactions which then consume the sulphurous acid. But by dissolving the salt, thus slightly decomposed, in water, adding barium chloride, and acidifying the washed precipitate, the sulphite can be readily found.

The disodium and dipotassium salts should, as neutral or nearly neutral salts, be liable to neither reversion nor hydrolysis, but their equilibrium is so unstable that the fact is that they are especially prone to rapid change. It is probable that they do really suffer no

reversion, but in practice, in consequence doubtless of the traces of alkaline salt purposely left in them as a guard against rapid hydrolysis, they do when kept dry develop a little sulphite before they hydrolyse. The disodium salt, which is an anhydrous salt, may however be kept for years in a desiccator without suffering sensible reversion. In comparison with the others, the five-sixths normal potassium salt and the disodium salt are the salts of their respective metals best fitted to keep in stock for any considerable time.

The dry lead salt reverts when heated. This salt is a basic or hydroxy-salt, and when moderately heated is decomposed, thus :—



The residue if further heated evolves red fumes, and if moistened with sulphuric or hydrochloric acid evolves much sulphur dioxide.

Other chemical properties of oximidosulphonates.

Hydrolysis.—The hydrolytic decomposition of oximidosulphonates has been repeatedly referred to in this paper, and is besides a fact long known. Its almost inevitable occurrence in every alkali oximidosulphonate is all that needs further notice. When by circumstances or by intention the salt is rendered acid, hydrolysis always quickly ensues. An alkaline salt when there is any water available, always becomes acid in time by suffering reversion, and thus its hydrolysis is brought about.

In dilute solutions of oximidosulphonates acidified, hydrolysis proceeds without complication, but in strong solutions, and when the change is carried through, by heating, to hydroxylamine sulphate, some gas is produced,—nitrous oxide and nitrogen. In hydrolysing for analytical purposes, this is manifested by pressure in the sealed tube, and by a deficiency in the quantity of hydroxylamine (*cf.* Raschig).

When moistened dipotassium oximidosulphonate hydrolyses spontaneously, it also gives out at the same time much nitrous oxide and a little nitrogen, while strong hot solutions of it or of the disodium salt effervesce a little when acidified.

Dipotassium salt which has been preserved as far as possible in a desiccator, also effervesces when dissolved in water, even in presence of an alkali. In these changes a little ammonia is also generated.

It thus appears that oximidosulphonates decompose in several ways combined; by reversion, by hydrolysis, and by secondary reactions with the sulphurous acid generated by the other changes. As products we therefore get in the first line, nitrite, sulphite, oxyamidodisulphonate, and sulphate. From the sulphite and oxyamidodisulphonate we may get imidosulphonate hydrolysing to amidodisulphonate, and from the sulphite and oximidosulphonate, nitrilosulphonate. By the hydrolysis of the oxyamidodisulphonate there will be got hydroxylamine, and by its reversion, hyponitrite. From the hydroxylamine or from the amidodisulphonate (slowly) may come ammonia, and from the former in alkaline solution also nitrous oxide and nitrogen. Nitrous oxide may also come from hyponitrite, and from hydroxylamine and nitrous acid, and nitrogen from ammonia and nitrous acid. But with all these possibilities before us, there are points which remain obscure, particularly that in the acid-decomposition of oximidosulphonates nitrous oxide and nitrogen should be evolved.

Behaviour when heated.—The dipotassium salt hydrolyses with its water of crystallisation quite suddenly when heated to 90° in a limited space, but by very gradual heating in a roomy air-bath it can be rendered anhydrous and raised to 100° and a little above without change (Claus). The disodium salt, which is anhydrous, can be safely heated to the same extent. In a current of dried air both salts can be heated much above

100° without being changed. The alkaline salts, though they contain water, may be heated also without change to 100° or a little above.

About 105° for the two-thirds normal salts and 110° for the alkaline salts, oximidosulphonates slowly take moisture from the air, increase consequently in weight, and become hydrolysed and acid. In this behaviour they are like imidosulphonates (*This Journ.*, **6**, 64). Claus observed that the dipotassium salt gained thus in weight, but set this down to absorption of oxygen. But any oxidation would be attended with a loss of nitrogen or oxides of nitrogen much more in weight than the oxygen absorbed. Besides, we find that, heated for hours at 120° in a current of well-dried air, oximidosulphonates, even the two-thirds normal potassium salt, gain nothing in weight.

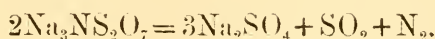
The effect of higher temperatures was investigated by Claus, who found that the anhydrous dipotassium salt gives off some acid-reacting vapour not sulphur dioxide, that on further heating a little ammonium sulphate sublimes (!), and that the residue then consists apparently of potassium pyrosulphate. His experience on heating the five-sixths normal potassium salt was, that above 200° the crystals swell up and spring asunder, give first nitric oxide and then ammonium sulphate, and leave a residue which does not fuse and is normal potassium sulphate. Fremy, Claus, and Raschig all state that this salt decomposes suddenly at about 200° (150° Raschig) with development of red vapours.

We have experimented more particularly with the sodium oximidosulphonates, the disodium salt being better fitted than the dipotassium salt for testing the effects of dry heat upon oximidosulphonates because it is anhydrous. With the object of collecting the resulting gases the heating has been conducted in a vacuum as well as in open vessels.

The dried normal sodium salt remains unchanged until the tem-

perature has risen to $182-3^{\circ}$ when it suddenly decomposes into a loose unfused residue, vapours depositing a very small sublimate, and gases. The residue is neutral or slightly alkaline and consists of sodium sulphate with a very little thiosulphate. The very small sublimate consists of sulphur and ammoniacal salt which when acidified yields a solution milky from sulphur and smelling faintly of sulphur dioxide. The gases are sulphur dioxide and nitrogen in nearly equal volumes.

Neglecting the very small quantities of sulphur, thiosulphate, and ammonia, all due to water retained by the salt, the decomposition may be written :—



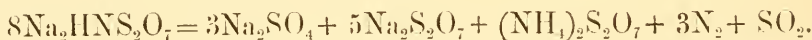
The anhydrous disodium salt begins to give off gas in a vacuum, but exceedingly slowly, at about 140° , and continues to do so nearly as slowly while the temperature is rising to about 170° , or while it is maintained steadily at any intermediate point. When the temperature is allowed to rise continuously though slowly the full decomposition of the salt occurs suddenly at 171° , whilst when it is kept steady for an hour or more at about 165° and then slightly raised, full decomposition occurs suddenly at $167-8^{\circ}$. At the moment of active decomposition the salt fuses and effervesces and, like the trisodium salt, yields a small sublimate as well as gases. The residue is exceedingly acid and consists of sodium acid-sulphate with a not inconsiderable quantity of ammonium acid-sulphate or more probably the corresponding pyrosulphates. The sublimate is a compound of ammonia and sulphur dioxide while the gases consist of nitrogen and sulphur dioxide, the former in somewhat greater volume than the latter. Nitrous oxide could not be detected by us ; but in one experiment the sulphur dioxide constituted more than half the total gas, a result only to be explained as due to some undetected error or to the presence of

a little nitrous oxide along with the nitrogen. The nitrogen obtained was about four-fifths of the total nitrogen of the salt, the rest remaining in the residue as ammonia. So far as was ascertained—and observation is difficult because of the explosive character of the salt—the first portions of gas are richer in sulphur dioxide than the last. If this be really so, it points to the not improbable intermediate formation of some imidosulphonate.

From the facts observed, the decomposition of the disodium salt seems to vary between that expressed by—



and that expressed by—



It will be seen that our observations do not agree with those made by Claus as to the nature of the products of the decomposition of oximidosulphonates by heat. Fremy, Claus, and Raschig all state that the five-sixths normal potassium salt gives red fumes when heated, and this also we cannot confirm; whether in crucible or in tube, heated slowly or quickly, it has never given us such fumes. Only where reversion to nitrite and sulphite has taken place, as with lead oximidosulphonate already noticed, does heat generate red fumes.

Oxidisability.—Characteristic of the oximidosulphonates are, firstly, that they do not reduce copper salts in presence of alkali, unless they have been previously hydrolysed with acid; secondly, that they yield by gentle oxidation Fremy's *sulphazilates*, unstable salts, blue-violet in solution, yellow and crystalline in the solid state. Traces of almost any oxidising agent capable of acting in alkaline solution effect this change in the alkaline salts, but oxidising agents of a basic character act also on the neutral salts. Silver oxide and lead peroxide are the best reagents for detecting any oximidosulphonate. These oxides

were used by Fremy. In hot acid solution oximidosulphonates are converted into sulphates and oxides of nitrogen by oxidising agents, such as nitric acid, bromine, and potassium chlorate, but the decomposition is seldom so complete as to be available for determining sulphur quantitatively.

Action of strong sulphuric acid.—That strong sulphuric acid disengages nitric oxide, as stated by both Fremy and Claus, we cannot admit. Heated with this acid, a sodium or potassium salt dissolves and evolves no gas until sulphuric-acid vapours are freely escaping, when nitrous oxide and sulphur dioxide are slowly generated. Indeed, addition of sulphuric acid should be made before igniting oximidosulphonates for analytical purposes, in order to avoid loss through explosive decomposition.

Behaviour with other salts in solution.—Disodium, dipotassium, or diammonium oximidosulphonate does not precipitate other salts and appears often not even to react with them, since on evaporation the salts mixed together crystallise out unchanged. This interesting fact was observed by Fremy in the case of the dipotassium salt mixed with those of zinc, manganese, copper, and silver. Claus denied its truth. We have fully verified it in the case of the disodium salt and copper sulphate. These salts in about equivalent quantities were dissolved in water and the solution made slightly turbid with copper hydroxide by adding a minute quantity of sodium hydroxide, in order to guard against hydrolysis supervening. By spontaneous evaporation copper sulphate first crystallised out and then the sodium oximidosulphonate, that is, in the order of their solubilities. This behaviour of the alkali oximidosulphonates serves to show that as sulphonates they have the stability of sulphates themselves.

A normal alkali oximidosulphonate reacts generally as alkali hydroxide towards salts the hydroxides of whose bases are insoluble ;

disodium, potassium, or ammonium oximidosulphonate remaining in solution. Silver oxide thus formed acts slowly as an oxidising agent. Triammonium oximidosulphonate added in excess to copper sulphate gives a chromium-green solution, which is turned blue by ammonia and which with excess of copper sulphate behaves as usual, yielding a precipitate of copper hydroxide and leaving diammonium oximidosulphonate in solution.

However, both the two-thirds normal and the normal salts of the alkalis enter into some reactions of complete double decomposition. Thus, either a two-thirds normal or a normal oximidosulphonate precipitates with hydroxy-lead acetate in excess or with barium hydroxide; a normal salt precipitates also with sufficient normal lead acetate or with barium chloride; but to secure precipitation of a lead salt the solutions must be dilute. The barium precipitates, too, are very slightly soluble in solutions of alkali oximidosulphonates. Calcium compounds yield no precipitates, but its hydroxide forms salts by decomposing the ammonium oximidosulphonates. Strontium compounds give with potassium oximidosulphonates clear solutions for a moment which then become filled suddenly with a voluminous silky precipitate. Fremy could get no precipitate with strontium salts, nor Claus either, strange to say. With sodium or ammonium oximidosulphonates, strontium salts do not precipitate, but in a few hours the solution deposits silky crystals in large hemispherical groups on the sides of the vessel.

The precipitates of barium, strontium, and lead salts are soluble in acids and in ammonium salts including ammonium oximidosulphonates. The lead precipitates are also soluble in sodium or potassium hydroxide.

Barium sulphate is distinctly soluble to a small extent in solutions of the normal alkali oximidosulphonates. Thus, barium chloride may

be added in very small quantity to a solution, not too dilute, of a normal alkali oximidosulphonate containing sulphate without occasioning any permanent precipitate even on standing. But when the alkalinity of the solution is about neutralised with hydrochloric acid, the solution becomes turbid from the separation of barium sulphate.

The purified barium or lead oximidosulphonates can be decomposed by sulphates and very completely by carbonates, and thus be used to furnish other oximidosulphonates. The ammonium salts which can be formed in this way serve in turn for the preparation of salts of metals which do not unite with ammonia by evaporating their solution with the oxide or hydroxide of the metal. However, the oximidosulphonates obtained in this way are the more unstable of these salts and we have not pursued their examination.

Oximidosulphonates combine readily with each other and with other salts, a fact which will be made fully evident in the brief descriptions which follow of the salts we have prepared.

Sodium oximidosulphonates.

Normal sodium salt, $\text{Na}_3\text{NS}_2\text{O}_7, 3\text{H}_2\text{O}$.—This salt forms rhombic prisms often a centimetre in length. It is strongly alkaline, soluble in 1·3 parts of water at 20° , and tends to form supersaturated solutions. It is only partly precipitated by alcohol but without decomposition, and is less soluble in sodium-hydroxide solution than in water. Its analysis gave—

	Calc.	Found.
Sodium	22·05	21·94
Sulphur.....	20·45	20·38
Oximide residue	9·58	9·32
Water	17·28	17·86

This and other oximidosulphonates were decomposed for the es-

timation of hydroxylamine and sulphuric acid by hydrolysing in sealed tubes with dilute hydrochloric acid kept for some time at 100° and only then heated to 130° . Some of the attempts to estimate hydroxylamine were however defective for the reason already given (p. 42). The sodium was directly estimated by cautious ignition with sulphuric acid, the presence of which prevents explosive decomposition of the salt by heat.

Disodium salt, $\text{Na}_2\text{HNS}_2\text{O}_7$.—This salt is anhydrous and is soluble in somewhat more than its own weight of water at 14° . It just reddens litmus. Its crystals are usually small, dense, brilliant prisms, united into hard masses and firmly adhering to the glass. On analysis it has given :—

	Calc.	Found.	
Sodium.....	19.41	19.21	19.53
Sulphur	27.00	27.06	26.98

Eight-ninths normal sodium salt, $\text{Na}_2\text{HNS}_2\text{O}_7$, $2\text{Na}_3\text{NS}_2\text{O}_7$, $3\text{H}_2\text{O}$.—The solutions intermediate in composition to those of the trisodium and disodium salts which yield this when evaporated may be prepared by adding either sodium hydroxide or trisodium salt to a solution of disodium salt in approximately the calculated quantity. Twice, when the solutions were left in the desiccator to evaporate and were of the composition of the unknown five-sixths normal salt, good sized prisms and plates of some salt filled the solution which when the attempt to remove them was made, rapidly melted away very remarkably, to give place to a precipitate of the eight-ninths sodium salt. Possibly, they were crystals of the five-sixths salt. The eight-ninths salt, redissolved in water or in its diluted mother-liquor, gives on evaporation small nodules of the salt. Only under the microscope can this precipitate and these nodules be made out to be composed of prismatic crystals. The crystals are efflorescent. Like the other

sodium salts this one is exceedingly soluble in water, requiring less than 1.5 parts of water at 14° to dissolve it, and can be repeatedly recrystallised unchanged. It tends greatly to form supersaturated solutions. We have made the following analyses of it prepared under different conditions :—

	Calc.	Found.			
		(a)	(b)	(c)	(d)
Sodium	22.77	22.51	22.64	22.63	22.77
Sulphur	23.73	23.76	24.04	23.73	23.49
Oximide residue.....	11.13	11.14	—	11.13	—
Water	6.68	3.59	3.53	—	—
Alkalinity, sodium...	5.69	—	5.45	—	5.83

The alkalinity of the preparations was determined with decinormal acid and methyl-orange. The water was estimated by drying at 110°, and it will be noticed that half of it was retained at this temperature. Similar retention of water will be seen in the case of the five-sixths potassium salt, of one of the sodium-potassium salts, and in that of the barium salts, and is not due to the fixation of the water by hydrolysis. It is remarkable that there should be such a difference in this respect between this and the normal salt.

Preparation (a) was deposited by evaporation in the cold of a solution of two molecules of the disodium salt to one of sodium hydroxide ; preparation (b) was the same salt recrystallised from water and separated as a precipitate from the supersaturated solution by stirring ; (c) was obtained by evaporating the mother-liquor of (b) ; and (d) was prepared by evaporating a solution of the calculated quantities of the disodium and trisodium oximidosulphonates.

Potassium oximidosulphonates.

Normal potassium salt, $K_3NS_2O_7, H_2O$. Also $K_3NS_2O_7, 2H_2O$.—We have several times prepared and analysed this salt with results

very nearly agreeing with those got by Raschig. But while he has given to the salt only one molecule of water to $K_6N_2S_4O_{14}$, we find that there is one molecule to $K_3NS_2O_7$. Neither he nor we happen to have succeeded in getting, the salt quite free from excess of alkali : we say 'happen,' because we are sure that we could prepare it so. Tabulating his and our analyses which show least excess of alkali with calculations for his formula and for ours—

<i>A</i>	<i>Raschig.</i>		<i>Divers and Haga.</i>	
	Calc.	Found.	Calc.	Found.
Potassium	37·09	36·67	36·06	36·29
Sulphur	20·25	19·33	19·67	19·42
Water	2·85	—	5·53	5·61

—we see that his analysis is less incompatible with our formula than with his, for which his sulphur is much, and inexplicably, too low, while agreeing with his other and all our determinations of sulphur. We estimated the water in our preparation, and it will be seen that potassium, sulphur, and water all agree well with our formula. Our analysis was made on a precipitate got by adding alcohol to a solution of dipotassium salt and potassium hydroxide in about the calculated proportions.

We now tabulate the other analyses by Raschig and by us along with a calculation for $K_3NS_2O_7, \frac{1}{8}KOH, \frac{5}{8}H_2O$ which matches Raschig's preparation :—

<i>B</i>	<i>Raschig.</i>		<i>Divers and Haga.</i>	
	Calc.		(a)	(b)
Potassium	37·53	37·55	37·48	37·37
Sulphur	19·66	19·56	19·23	19·54
Water.....	4·15	—	3·86	—

Our (a) preparation was got by using excess of concentrated

alkali, was drained on the tile, and then washed with alcohol ; (b) was got by recrystallising from water an unwashed salt like the last. This was the only time we succeeded in recrystallising the salt from water without adding fresh alkali. It will be seen that both Raschig's results and ours agree much better in composition with the formula of an impure salt than with that for a salt with half a molecule of water to one molecule of oximidosulphonate, (A) and from such results no deduction can be safely made as to the degree of hydration of the pure salt.

We once obtained the normal potassium salt crystallised with two molecules of water. The salt precipitated by adding excess of potassium hydroxide, and drained on the tile, was redissolved in water and precipitated by alcohol. In other trials in this way we got the salt with only one molecule of water as above. Our analysis included the estimation of the oximide radical as hydroxylamine :—

	Calc.	Found.
Potassium	34.17	34.10
Sulphur	18.64	18.81
Oximide residue	8.74	8.44

Dipotassium salt, $K_2HNS_2O_7 \cdot 2H_2O$.—Besides the two excellent methods of preparing this salt from potassium nitrite it can also be obtained, sometimes conveniently, from the disodium salt by precipitating this with potassium chloride, and is then more certainly free from nitrilosulphonate. The yield in this way can be made as much as 80 per cent. of the equivalent of the quantity of disodium salt taken. Fremy's method of preparing it pure from the alkaline five-sixths potassium salt by precipitating this with zinc or lead or barium salt thus leaving the dipotassium salt in solution, and which did him good service, is now only of interest as establishing the two potassium salts as salts of one and the same acid.

The dipotassium salt forms detached hard crystals remarkable in shape, being like somewhat flattened, very acute octahedrons. The crystals belong to the oblique rhombic system (Raschig and Fock). The analyses of Claus and Raschig established the composition of this salt. Fremy's recorded results are improbable and insufficiently concordant; but the identity of his *sulphazotate* with dipotassium oximidodisulphonate is beyond doubt.

According to Claus, the dipotassium salt is practically insoluble in cold water, and is very difficultly soluble according to Raschig. Its crystals can indeed be washed without much loss, but the salt in fine powder proves to be far from insoluble. It dissolves to the extent of about one part in thirty of water. When quite free from alkaline salt, it is faintly acid to litmus. It readily forms supersaturated solutions, as observed by Raschig, and in the preparation of the salt the mother-liquors retain much more of it than could simple water. We find that of a saturated solution of sodium chloride about 4 cub. cent. are able to dissolve 1 gram of the dipotassium salt, but the solution quickly deposits a compound of the two salts. Other salts also increase its solubility. Potassium or sodium hydroxide or normal oximidodisulphonate acts upon it and thus affects its solubility.

Five-sixths normal potassium salt, $K_2HNS_2O_7$, $K_3NS_2O_7$, H_2O .—There is really only one way of preparing this salt, namely, by dissolving up the dipotassium salt in hot solution of enough potassium hydroxide (or salts equivalent to it) and crystallising. The dipotassium salt should not be put into cold water or even potassium hydroxide, and then the vessel heated to dissolve it, unless the mixture is continuously stirred; for undissolved salt lying on the hot bottom of the vessel is liable to hydrolyse. Observing this precaution, no simpler way could be conceived to prepare the salt, all difficulties being imaginary. The belief that the salt can be formed from the nitrite direct is un-

founded in fact, the dipotassium salt always preceding it and then only yielding it by alkaline treatment.

The five-sixths potassium salt forms rhombic plates, crystallising in masses on the walls of the vessel. It is only sparingly soluble in water. It was analysed by Fremy and by Claus with results identical as to the sulphur and not greatly differing from each other as to the potassium. Raschig got results for potassium exactly intermediate to theirs, but for sulphur one per cent. less than they got; and he rightly gave to the composition of the salt a molecule of water, although Claus had emphatically stated it to be anhydrous, having found that it can be heated to 120° without any change or loss of weight. Raschig, however, so far from establishing the presence of this molecule of crystallisation-water, wrote of the salt that it 'remains wholly unchanged at 120° ,' a statement almost the same as that which Claus had made in proof that it contained no water. The facts, we find, are that it does lose in weight when heated, in fine powder, to 120° , and therefore contains water, but, as is usual in our experience with sulphazotised salts, it gives up this water very slowly. We further find that at 120° it slowly increases in weight again by re-taking water from the atmosphere and in doing so becomes hydrolysed and acid. When the hydrated salt is rapidly raised to, and maintained at 120° , such hydrolysis also occurs and thus fixes the water, causing the salt to suffer scarcely any change in weight.

The results of our determinations of sulphur and potassium agree with those got by Raschig, but with the potassium a little lower than the calculated number. As our methods of getting our preparations were somewhat exceptional, we submit the results of our analyses along with those got by the other investigators, also interesting from the peculiarities noticed of their agreements and variations :—

	Calc.	<i>Fremy.</i> Mean.	<i>Claus.</i> Mean.	<i>Raschig.</i> Mean.	<i>Divers</i> (a)	<i>and Haga.</i> (b)	(c)
Potassium ...	32·89	32·0	33·6	32·8	32·38	32·41	32·04
Sulphur	21·53	22·2	22·4	21·4	21·43	21·23	21·19
Nitrogen.....	4·71	4·4	4·9	—	—	—	—
Oximide res.	10·27	—	—	—	9·05	—	10·23
Water.....	2·97	—	—	—	3·00	3·29	—

Sample (a) was prepared by adding to a cold supersaturated solution of dipotassium oximidosulphonate the calculated quantity of potassium hydroxide in solution, and leaving to crystallise. Sample (b) was prepared by dissolving the tripotassium salt in warm water and crystallising out. Sample (c) was obtained on adding potassium chloride to dipotassium-oximidosulphonate solution and then excess of ammonia.

Dipotassium oximidosulphonate and potassium nitrate.

Dipotassium oximidosulphonate unites with potassium nitrate, but not with sodium nitrate or with sulphates. Again, it unites with sodium chloride but not with potassium chloride.

The potassium-nitrate compound is obtained when a cold saturated solution of potassium nitrate is mixed with a warm concentrated solution of dipotassium oximidosulphonate. The compound begins almost at once to crystallise out in long silky needles, which by their abundance and by interlacing often make the mixed solutions set. Drained on a tile, the compound proved to be the simplest double salt of the two radicals, $K_2HNS_2O_7$, KNO_3 , H_2O . Two preparations were analysed:—

	Calc.	(a)	(b)
Potassium	30·15	30·86	—
Sulphur.....	41·24	41·52	41·83
Nitrogen	7·22	6·83	—

Heated, it explodes giving off much red fume. Water decomposes it into its component salts. Its reaction to litmus is neutral.

Dipotassium oximidosulphonate and sodium chloride.

The compound of the dipotassium salt with sodium chloride is obtained by dissolving the finely powdered potassium salt in cold saturated solution of sodium chloride. Operating sufficiently quickly, 35–40 ccs. of the salt solution may be made to dissolve nine grams of the potassium salt before the compound begins to crystallise out. The compound salt forms small striated good crystals belonging to the orthorhombic system, and is neutral to litmus. Two preparations were analysed and gave results agreeing with the formula— $5K_2HNS_2O_7$, $8NaCl$, $3OH_2$, but as water decomposes the compound into indefinite potassium-sodium oximidosulphonates and chlorides of both metals, this formula can only claim to be the simplest expression of the composition (see further p. 63).

	Calc.		(b)
Potassium	20·89	20·83	—
Sodium	9·86	9·73	—
Sulphur	17·14	17·32	—
Chlorine	15·21	14·96	14·90
—			
Mixed sulphates ...	76·87	76·50	76·60

Potassium sodium oximidosulphonates.

It is perhaps not impossible, but it is certainly difficult to obtain potassium-sodium oximidosulphonates of very simple composition. Nor is it generally easy to get a salt of the same composition again and again. The salts now to be described must be regarded as examples only of an apparently indefinite number possible to prepare.

Most of them occur only in very small crystals, sometimes microscopic, generally aggregated in hard crusts and nodules. They were all however obtained in transparent though minute prismatic crystals, and under the microscope appeared to be homogeneous.

Normal sodium potassium salt.—Our attempts to produce normal mixed salts by adding the hydroxide of one metal to a hydrogen oximidosulphonate of the other metal have been unsuccessful. With any great excess of alkali, evaporation leads to the destruction of the oximide and crystallisation of potassium sulphite. With moderate excess of alkali and evaporation *in vacuo*, uncrystallisable or nearly uncrystallisable solutions are obtained. Such mixtures also fail for the most part to give a solid precipitate with alcohol ; at most, liquid droplets form which then sometimes slowly solidify to granules of microscopic crystals. Besides this, sodium oximidosulphonate in presence of alkali is largely soluble in strong spirit. When the alkali is used in only slight excess, salts are formed on evaporation, but these generally fall short of normal salts in composition. However, from the two single normal salts we did succeed in getting a mixed normal salt nearly pure, which we now describe.

This *mixed normal salt*, $2\text{Na}_3\text{NS}_2\text{O}_7$, $3\text{K}_3\text{NS}_2\text{O}_7$, $2\text{H}_2\text{O}$, but with one-fourth of the water replaced by its equivalent of potassium hydroxide, was obtained by evaporation in the desiccator of a solution of normal sodium and normal potassium oximidosulphonates in molecular proportions together with a little sodium and potassium hydroxides also in molecular proportions to each other. The salt appeared as a powdery deposit of microscopic crystals, and was drained dry on a tile, out of free contact with air. Reference to the description of the normal potassium salt, as obtained both by Raschig and by ourselves, will show that the small excess of potassium in the mixed salt is not peculiar to this preparation. Potassium and sodium

were estimated by the Finkener-Dittmar method. The calculation A is for the true normal salt; B for the salt with the small excess of potash mentioned above :—

	Calculated.		Found.
	A	B	
Potassium	23·85	24·85	24·89
Sodium.....	9·35	9·23	9·40
Sulphur	21·68	21·40	21·35

A seven-eighths-normal potassium sodium salt, $K_{18}Na_3H_3(NS_2O_7)_8$, $20H_2O$, in nodular masses of minute transparent prisms was obtained on evaporating in a vacuum-desiccator a solution of the five-sixths-normal potassium salt with sodium hydroxide in quantity calculated to form normal salt. The composition of this salt is not very far removed from that of a much simpler mixed eight-ninths-normal salt, $K_7NaH(NS_2O_7)_3$, $8H_2O$, corresponding to the eight-ninths sodium salt, but the deviation is a little beyond the probable errors of analysis. In the table, the calculated numbers marked A are for the 'eight-ninths' formula, and those marked B for the 'seven-eighths' formula.

	Calculated.		Found
	A	B	
Potassium.....	27·05	26·51	26·58
Sodium	2·27	2·59	2·49
Sulphur	18·98	19·28	19·22
Nitroxy-radical, NO.	8·90	9·03	9·01

A six-sevenths-normal potassium sodium salt, $K_2Na_{16}H_3(NS_2O_7)_7$, $5H_2O$, was obtained in some quantity by the evaporation of a solution of disodium oximidosulphonate to which a small proportion of potassium hydroxide and, unintentionally, some sodium hydroxide also had been added. It formed as a hard crust of small prisms on the bottom of the vessel. Attempts to prepare this salt again were unsuccessful;

the products then obtained will be noticed a few paragraphs later on. Preserved for many months, the salt was again examined, to confirm the results first obtained.

This salt scarcely loses weight in the vacuum-desiccator, although it contains water; is not so freely soluble in water as are other potassium-sodium oximidosulphonates rich in sodium; and may be recrystallised from solution almost unaltered. The analysis marked (*a*) is of the original preparation, and that marked (*b*) of the recrystallised salt:—

	Calc.	(<i>a</i>)	(<i>b</i>)
Potassium.....	4.17	4.10	4.27
Sodium.....	19.69	19.43	19.79
Sulphur	23.97	23.75	23.84
Nitroxy-radical, NO.	11.24	—	10.81

A five-sixths-normal potassium sodium salt, $K_{15}Na_5H_4(NS_2O_7)_8, 9H_2O$, or not very far from the five-sixths normal monosodium salt, $K_4NaH(NS_2O_7)_2, 2H_2O$, may from its mode of preparation possibly not be a single salt, yet is interesting on account of the way in which it was obtained, and for its relation in composition to the salt next described.

When the dipotassium salt was mixed with the calculated quantity of sodium hydroxide and the solution evaporated to a small bulk over sulphuric acid, no salt separated, and when alcohol was added, there occurred only a formation of droplets on the bottom of the vessel. But gradually these solidified to masses of minute crystalline particles having the composition formulated above instead of that of a normal salt or mixture of normal salts as it would have had if compounds of either metal had alone been present.

The calculation A is for the simpler and B for the more complex formula, given above:—

	Calculated.		Found.
	A	B	
Potassium.....	26·23	24·57	24·66
Sodium.....	3·86	4·82	4·93
Sulphur	21·46	21·44	21·37

The *fire-sixths-normal monopotassium sodium salt*, $\text{KNa}_4\text{H}(\text{NS}_2\text{O}_7)_2$, H_2O , is nearly represented by two salts obtained in different ways. Both salts were however slightly more basic than calculated, while in the first to be noticed the potassium was slightly replaced by sodium, and in the second salt the sodium was slightly replaced by potassium, as will be seen from the numbers given. The first salt (*a*) was in small well-formed prismatic crystals, and was got accidentally by the spontaneous evaporation of an ammoniacal solution of disodium oximidosulphonate containing some potassium chloride. It was readily soluble in water but could not be recrystallised unchanged. It was free from chlorine. The second salt (*b*) was obtained in one of the attempts to get again the six-sevenths mixed salt already described. To seven molecules of disodium oximidosulphonate in solution were added two of sodium hydroxide and two of potassium hydroxide. The solution was then evaporated *in vacuo* until crystallisation occurred :—

	Calc.	Found.	
		(<i>a</i>)	(<i>b</i>)
Potassium.....	7·36	6·89	7·61
Sodium.....	17·36	17·69	17·25
Sulphur	24·15	23·93	23·96

A *five-sixths-normal potassium sodium salt*, $\text{K}_{1·4}\text{Na}_{3·6}\text{H}(\text{NS}_2\text{O}_7)_2$, H_2O , was obtained as a crust of microscopic orthorhombic prisms when a mixture in unknown proportions of disodium oximidosulphonate in solution and potassium hydroxide was evaporated in the desiccator.

A salt differing only a little in composition from this was

obtained in another attempt to form the 'six-sevenths' mixed salt. In this case one molecule of dipotassium, two of disodium, and four of trisodium oximidosulphonates were brought together in solution and evaporated in the desiccator. The solution became somewhat viscid and only yielded a crystalline deposit when stirred with a glass rod. The deposit or precipitate was thoroughly drained on a porous tile, dissolved in a little water, and the solution evaporated to crystallisation. A salt now formed as a crust on the bottom of the vessel and resembled the salt sought for, from which however it differed materially in composition. The results of analysis of this preparation are given under (b), and those of the other preparation under (a). This is closely five-sixths normal, while (b) is a little less basic, and in other points deviates a little from the calculation :—

	Calc.	Found.	
		(a)	(b)
Potassium	10.20	10.16	9.71
Sodium	15.43	15.46	15.83
Sulphur	23.86	23.82	24.27
Alkalinity as sodium	4.29	4.29	4.30

Raschig's *potassium sodium sulphazotate* is a salt a little less basic than that just described as (b). He represents it as being—in the nomenclature of this paper, that is,—the *five-sixths normal monosodium oximidosulphonate*, $K_4NaH(NS_2O_7)_2, 2H_2O$, and was justified in doing so by his analysis. But from his results, the salt would nevertheless appear to have had a composition approaching more nearly that of a *four-fifths normal* salt than of a *five-sixths normal* one, and hence in the order of basicity of these potassium sodium salts stands between that last described and that to follow.

Raschig describes the salt as forming opaque spherical masses of the size of millet seeds, and therefore resembling some of our preparations.

A *four-fifths normal potassium sodium salt*, $K_{2.1}Na_{0.3}H_{0.6}NS_2O_7 \cdot 0.72H_2O$, was obtained by dissolving dipotassium oximidosulphonate in sodium-chloride solution and then adding ammonia in excess (p. 35). It separated out as a precipitate of exceedingly minute prisms slightly opaque when viewed under the microscope. It was free from chloride and ammonia. The atomic ratio of the metals, it will be seen, is $Na : K_7$, and of the metals to the hydrogen as $(KNa)_4 : H$.

	Calc.	Found.
Potassium	28.06	28.08
Sodium	2.33	2.40
Sulphur	21.87	21.86

A *four-fifths normal potassium sodium salt*, but a little less basic, so as to be *nineteen-twenty-fourths normal*, $K_{3.75}NaH_{1.25}(NS_2O_7)_2 \cdot 1.8H_2O$, was obtained almost in the same way as the last. Dipotassium oximidosulphonate in fine powder was dissolved in such quantity in a saturated solution of sodium chloride that crystallisation of an oximidosulphonate-chloride (p. 57) took place. This salt was redissolved by warming in its mother-liquor to which some ammonia water had been added, and the solution set aside. On cooling, the above salt crystallised out in minute prisms, which in a good vacuum over sulphuric acid lost nothing, although they contained water. The salt was free from chlorine and ammonia.

	Calc.	Found.
Potassium	25.13	25.11
Sodium	3.94	3.92
Sulphur	21.94	21.95

A *seven-ninths normal potassium sodium salt*, $K_6NaH_2(NS_2O_7)_3 \cdot 2H_2O$, forms hard crusts of small rhombic prisms, only moderately soluble in water, and recrystallisable unchanged. It is obtained from the

dipotassium oximidosulphonate, which exchanges one-third of its hydrogen for sodium on treatment with either sodium hydroxide, or sodium carbonate, or sodium chloride with ammonia.

In working with sodium hydroxide some small excess of this may be used without affecting the composition of the crystals. Analysis (*a*) was made upon crystals prepared by adding NaOH to $2K_2HNS_2O_7$, that is, in the proportion calculated for the five-sixths normal salt.

The dipotassium salt dissolved in an excess of warm solution of sodium carbonate, gave on cooling the crystals of which (*b*) is the analysis.

Analysis (*c*) is of crystals obtained by dissolving together in warm water, $NaCl + 2K_2HNS_2O_7$, and then adding concentrated ammonia-water in moderate quantity, and leaving to cool. The sodium chloride was, it will be seen, taken in small excess, namely, in the quantity calculated to produce the five-sixths normal salt. When the effect of a great excess of sodium chloride was tried, the result was less satisfactory, the crystals being then somewhat opaque and less definite in composition. As best prepared, this salt made by the use of sodium chloride, is somewhat less basic than a seven-ninths normal salt, being half-way, in composition, between this and a *three-fourths* normal salt.

	Calc.		Found.	
		(<i>a</i>)	(<i>b</i>)	(<i>c</i>)
Potassium	27.10	27.01	27.22	26.73
Sodium	2.66	2.53	2.50	2.62
Sulphur	22.18	22.00	22.08	22.27

Two-thirds normal potassium-sodium salt, $KNaHNS_2O_7, 3H_2O$.—This salt, but slightly more basic from the presence of about $\frac{1}{16}$ of an atom of potassium in excess of that in the formula and being therefore

a $\frac{2\frac{2}{3}}{3}$ normal instead of $\frac{2}{3}$ normal salt, was obtained in long rhombic prisms of small size by dissolving up 5.2 grams of dipotassium salt (slightly alkaline) in 30ccs. of a saturated solution of sodium chloride and 5ccs. of water, by the aid of a gentle heat, and leaving the solution to cool :—

	Calc.	Found.
Potassium.....	12.70	14.62
Sodium.....	7.49	7.60
Sulphur.....	20.85	20.87

Reaction between the dipotassium salt and sodium chloride gives also, as already described, p. 57, a double oximidosulphonate-chloride, besides other compounds allied to this and the salt just described. It is a reaction which requires fuller investigation than we have found time to make of it, and what follows is all we can add concerning it.

While from sodium oximidosulphonate and potassium chloride, potassium oximidosulphonate crystallises out nearly free from sodium, there is obtained from potassium oximidosulphonate dissolved in sufficient quantity in a hot concentrated but not saturated solution of sodium chloride, a crystallisation of oximidosulphonate, half potassium, half sodium. If the sodium-chloride solution is saturated and the potassium oximidosulphonate is dissolved in it in the cold, what appears to be a compound of these salts, already described, quickly separates. But if by the aid of heat more potassium oximidosulphonate is dissolved, the salt which crystallises out is what may be represented as sodium chloride in combination with potassium-sodium oximidosulphonate. And if in the mother-liquor of this salt, now containing sodium oximidosulphonate in place of some of its sodium chloride, potassium oximidosulphonate is again dissolved by heat, the crystals which form on cooling consist of an oximidosulphonate, more of sodium than of potassium, and with only very little sodium

chloride along with it. We give an example of the composition of such a salt, which was obtained in good transparent small rhombic prisms :—

Potassium	7.45
Sodium	12.98
Sulphur	23.65*
Chlorine	0.63

The potassium is to the sodium as $K : Na_2$, plus a very little for the chlorine.

By dissolving dipotassium oximidosulphonate in a warm concentrated solution of disodium oximidosulphonate, it is practically certain from what precedes that a potassium-sodium salt would crystallise out. We regret we have no experience to record on this point.

Ammonium oximidosulphonates.

Hydroxy-lead oximidosulphonate (p. 80) yields at once, when shaken with enough solution of ammonium acid-carbonate to convert its lead to carbonate, a solution of normal ammonium oximidosulphonate almost pure. Barium oximidosulphonate may be used in place of the lead salt, but not being basic it requires the normal instead of the acid ammonium carbonate to be used with it. The solution smells mildly of ammonia, but in a closed vessel may be preserved unchanged. Heated with exposure to air, it becomes a solution of the diammonium salt, by loss of ammonia. Guarding against hydrolysis, to which this salt is very liable, by adding a drop of strong solution of ammonia occasionally so as to maintain its alkalinity, the solution of the diammonium salt may be evaporated on the water-bath to an exceedingly small volume without decomposing. If, now,

* This number is calculated, as the sulphur was lost.

the very concentrated solution be quickly evaporated cold over sulphuric acid, it is possible to get the *diammonium salt* in good prismatic crystals, very soluble in water. If instead, the solution is mixed with concentrated ammonia-water and evaporated in an ammoniacal atmosphere over solid potassium hydroxide, crystals of another very soluble salt are obtained, having an unconquerable tendency to climb the sides of the vessel and there exfoliate. These crystals slowly effloresce in the desiccator, and occur in masses of overlapping flat prisms. They are believed to be the *five-sixths normal ammonium oximidosulphonate*, corresponding to Fremy's basic potassium sulphazotate.

We have no analyses to bring forward. The diammonium salt has hydrolysed before it could be freed from its mother-liquor. The alkaline salt has also not been obtained in quantity in such definite form as to promise results of value from its quantitative analysis. Heated, this salt decomposes like other oximidosulphonates suddenly, and leaves a residue of ammonium acid sulphate.

On evaporating a solution of normal ammonium oximidosulphonate with ammonium acetate over a water-bath, ammonia escaped as usual and the concentrated solution on cooling yielded a magma of lustrous needles. This magma slowly drained itself dry on the tile and for days evolved strong acetic-acid vapour without the oximidosulphonate hydrolysing or the compact dry mass losing its silky lustre. In the course of weeks, however, hydrolysis occurred and the mass became loose opaque crystals of ammonium hydrogen sulphate. It thus seems that diammonium oximidosulphonate like the potassium salt combines with other salts such as ammonium acetate.

Barium oximidosulphonates.

Barium chloride does not precipitate a solution of dipotassium or disodium oximidosulphonate, but the mixed solutions are very

unstable, soon hydrolysing into sulphate and oxyamidodisulphonate (*cf.* Claus). From the normal potassium or sodium salt it precipitates barium oximidodisulphonate, in combination always with some potassium or sodium oximidodisulphonate, and with very much when the alkali salt is in concentrated solution and kept in excess. The alkali salts intermediate to the normal and two-thirds normal salts behave towards barium chloride as mixtures, the two-thirds normal alkali salt remaining unprecipitated except in concentrated solutions when much of it, if the potassium salt, becomes insoluble in combination with the barium salt. The precipitates dissolve to some extent in warm concentrated solutions of the normal and two-thirds normal alkali oximidodisulphonates but, for the most part, separate on cooling, sometimes richer in alkali salt.

Barium hydroxide precipitates solutions of the two-thirds normal as well as normal salts, and all the precipitates are combinations of barium with potassium or sodium salts. When the hydroxide is used in excess, and when barium chloride is used in excess with the normal or the intermediate salts, the precipitates are generally basic, that is, have some of the barium as hydroxide.

Ammonium oximidodisulphonates behave much like the potassium and sodium salts, but have a much larger solvent action upon the barium precipitates. We have made no analyses of the ammonium barium salts.

There are apparently three barium salts $\text{Ba}_3(\text{NS}_2\text{O}_7)_2$, BaHNS_2O_7 , and $(\text{HOBa})_2\text{HNS}_2\text{O}_7$ or $(\text{HOBa})\text{BaNS}_2\text{O}_7$; of which, however, the second is so unstable as to be known only in solution, while the third is known only in combination.

Normal barium oximidodisulphonate, $\text{Ba}_3(\text{NS}_2\text{O}_7)_2 \cdot 4\text{H}_2\text{O}$ and $8\text{H}_2\text{O}$.—The precipitate obtained by using barium chloride or hydroxide in excess with a potassium or sodium oximidodisulphonate is washed and

then nearly all dissolved by adding dilute hydrochloric acid with continual stirring until a solution is obtained neutral or only slightly alkaline to litmus. The turbid solution is as quickly as possible filtered, by vacuum-pump, into excess of warm baryta-water. The precipitate thus obtained when washed with boiled-out water, hot or cold, is the normal barium salt free from alkali and from carbonate. It is a voluminous curdy precipitate at first, but generally changes to a denser powder, crystalline under the microscope but to the naked eye chalk-like when dry. It is practically insoluble in water, but soluble in ammonium chloride. It is fully decomposed in the cold by ammonium or sodium carbonate solution. Heated dry, it suddenly decomposes into barium sulphate and gases. It loses water at common temperatures in dry air, and nearly all at 110° . Its composition, calculated and found, is shown by the following table, in which A refers to one preparation with 4 Aq., and B to another with 8 Aq. :—

	A		B	
	Calc.	Found.	Calc.	Found.
Barium	47.62	47.50	43.96	43.74
Sulphur	14.83	14.80	13.69	13.46

Two-thirds normal barium oximidosulphonates, BaHNS_2O_7 , can be obtained in solution by adding just enough acid to the normal barium salt. When the normal salt is free from alkali and the acid is sulphuric the solution filtered is pure. When the normal barium salt is combined with any potassium or sodium salt the solution obtained by means of an acid is treated as described for getting the pure normal salt, which is then again converted to the two-thirds normal salt by sulphuric acid. The solution of this salt is distinctly acid to litmus and hydrolyses too readily to admit apparently of crystallising out the salt from it.

Barium sodium oximidosulphonates.—A normal one-fifth sodium salt, $\text{Ba}_6\text{Na}_3(\text{NS}_2\text{O}_7)_5, 7\text{H}_2\text{O}$, is obtained when to normal sodium oximidosulphonate in strong solution, one mol., there is added barium chloride, one mol. It was also obtained by adding baryta-water to a strong solution of the normal sodium salt kept in excess. The precipitate unwashed or slightly washed is dried on a porous tile. It loses only one-third of its water at 120° . (*a*) was obtained from barium chloride and (*b*) from barium hydroxide :—

	Calc.	Found.	
		(<i>a</i>)	(<i>b</i>)
Barium.....	41.79	42.02	42.24
Sodium	3.51	3.60	3.44
Sulphur	16.27	15.69	15.97

Another normal, only one-seventh sodium, salt, $\text{Ba}_9\text{Na}_3(\text{NS}_2\text{O}_7)_7, 7\text{H}_2\text{O}$, can be obtained by adding baryta-water to a mixed solution of one mol. of disodium salt and one mol. of barium chloride, and in other ways. (*a*) and (*b*) were separate preparations :—

	Calc.	Found.	
		(<i>a</i>)	(<i>b</i>)
Barium.....	44.71	44.70	44.18
Sodium	2.50	2.40	2.47
Sulphur	16.24	15.83	—

A normal salt richer in sodium was got from 3 mols. of trisodium and 3 mols. of disodium salt ground together with hardly enough water to dissolve all, by adding to the mixture very gradually and with trituration a dilute solution of one mol. of barium chloride, half of which was added before it caused any precipitation in the now dissolved salts. The mixture quickly thickened to a nearly opaque firm jelly which after a time broke up into a thoroughly liquid

mixture of a powdery precipitate and an abundant mother-liquor. The precipitate was drained on a tile, and then had changed from a powder to a horny hard mass.* This was ground fine in a mortar and dried in a good desiccator, the grinding being once or twice repeated. Analysed it proved to be $\text{BaNaNS}_2\text{O}_7, \text{H}_2\text{O}$ with a little $\text{Ba}_3(\text{NS}_2\text{O}_7)_2, 9\text{H}_2\text{O}$:—

	$\text{Ba}_{18}\text{Na}_{15}(\text{NS}_2\text{O}_7)_{17}, 24\text{H}_2\text{O}.$	Found.	
Barium.....	38·10	38·36	38·01
Sodium	5·33	—	5·25
Sulphur	16·81	—	16·72

Barium potassium oximidosulphonates.—The normal potassium salt decomposes so quickly in aqueous solution into the five-sixths normal salt and potassium hydroxide that constant and simple results would probably not be obtained with it as a precipitating agent. We have used, as Fremy did, the K_5 salt, or with baryta the K_2 salt.

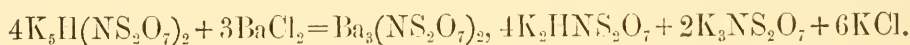
The seven-ninths normal salt, $\text{Ba}_3\text{K}_8\text{H}_4(\text{NS}_2\text{O}_7)_6, 9\text{H}_2\text{O}$, is obtained as a crystalline powdery precipitate when a concentrated solution of barium chloride, two mols., is added to three or more mols. of the five-sixths normal potassium salt in warm concentrated solution, this salt, it should be remembered, being very little soluble in cold water. The precipitate first formed redissolves in its mother-liquor up to the point when only half the barium chloride has been added, but is all again precipitated on adding the rest of the barium chloride and letting the mixture cool. The mother-liquor must be decanted soon and the precipitate drained on a tile, because the former in about an hour after cooling begins to deposit potassium salt. The results of the analysis of the precipitate were—

* The behaviour of the mixture in gelatinising and in breaking up then into a thin liquor and a precipitate which became horny on drying, is much like that of one of Fremy's potassium salts, his *metasulphazate*.

	Calc.	Found.
Barium	20·25	20·40
Potassium	15·41	15·66
Sulphur.....	18·92	18·25

The sulphur, it will be seen, is a little low for the formula, a deviation attributable perhaps to the presence of a little K_5 salt from the mother-liquor.

In this salt the oximidosulphonate exists one-third as normal barium salt and two-thirds as dipotassium salt. In its formation the K_5 salt behaves, as usual, as mixed normal and dipotassium salts, of which one part of the former suffers decomposition with the barium chloride and the other remains in solution, while the dipotassium salt precipitates in combination with the barium salt:—



On adding to a warm concentrated solution of three mols. of the K_5 salt, one mol. of barium chloride, agitating to redissolve the precipitate and then letting the solution cool, a relatively very large quantity of crystalline powdery precipitate is obtained, which from its quantity must consist largely of potassium salt. Being uncertain concerning its freedom from separate potassium salt we have not analysed it. It can be redissolved in its warmed mother-liquor and recovered by cooling, again and again. But if the mother-liquor decanted is allowed to stand some hours to deposit most of its potassium salt and then re-decanted on to the barium precipitate it fails to dissolve this when warmed with it, though it effects great change in its composition, dissolving out potassium salt. Thus treated, the precipitate was found to differ only a little from a salt having the formula $BaKNS_2O_7, H_2O$ and could be regarded as this salt retaining unchanged a little of the salt from which it had been

prepared. It had the composition expressed by the formula, $\text{Ba}_{12}\text{K}_{14}\text{H}(\text{NS}_2\text{O}_7)_{13}, 10\text{H}_2\text{O}$:—

	Calc.	Found.
Barium	33·95	33·93
Potassium	11·35	11·57
Sulphur.....	17·18	17·03

The attempt to get the salt BaKNS_2O_7 by direct reaction thus:—



was only partly successful. The precipitate obtained on adding to a warm concentrated solution of one mol. of the K_5 salt one of barium chloride had the composition shown by the formula $(\text{HO})_2\text{Ba}_6\text{K}_4\text{H}(\text{NS}_2\text{O}_7)_5, \text{H}_2\text{O}$, or quite probably, $\text{HOBa}_6\text{K}_4(\text{NS}_2\text{O}_7)_5, 2\text{H}_2\text{O}$. This composition would agree with that of a barium sodium salt already described, $\text{Ba}_6\text{Na}_3(\text{NS}_2\text{O}_7)_5$, if an atom of barium were not half displaced by one of potassium.

	Calc.	Found.
Barium	41·49	41·68
Potassium	7·89	8·22
Sulphur.....	16·15	16·06

This precipitate may be represented as a mixture or compound of $4\text{BaKNS}_2\text{O}_7$ with $(\text{HOBa})_2\text{HNS}_2\text{O}_7, \text{H}_2\text{O}$ (see the lead salts 3, p. 81, and 7, p. 85) and it is probable that the precipitate first formed consists largely of the latter salt, and that when, by the accumulation of dipotassium salt the mother-liquor lessens in basicity, the BaK salt alone precipitates. The reaction representing the formation of the hydroxy-barium salt—



—in which $(\text{HOBa})_2\text{HNS}_2\text{O}_7$ when doubled is equivalent to $\text{Ba}(\text{OH})_2 +$

$\text{Ba}_3(\text{NS}_2\text{O}_7)_2 + \text{H}_2\text{O}$, shows the normal sodium salt behaving with a salt of barium somewhat as it does with zinc, manganese, copper, and other salts, that is, as dipotassium salt and potassium hydroxide (p. 47).

Fremy, by treating the K_8 salt with barium chloride, got a precipitate, gelatinous at first, afterwards crystalline, the composition of which, as given by him, may be expressed (1) pretty closely by $(\text{HO})_3\text{Ba}_3\text{K}_8(\text{NS}_2\text{O}_7)_7$, or (2) by BaKNS_2O_7 as an approximation sufficiently near to Fremy's analytical results, considering the liability to impurity of his preparations, and his imperfect methods of analysis :—

	Calculated.		Found. (Fremy)
	(1)	(2)	
Barium.....	39.29	37.43	39.90
Potassium.....	11.19	10.65	11.55
Sulphur	16.07	17.49	15.80

He must have kept the potassium salt in excess when precipitating. He found that baryta-water gave the same salt. Although much reliance cannot, we admit, be placed on Fremy's analytical results as closely accurate, we are not inclined to adopt the simpler formula rather than the other of the two we have calculated ; for our own results, here and in the case of strontium and lead salts, where we have been sure of the comparative purity of our preparations and accuracy of our methods of analysis, have taught us that compound oximidosulphonates are generally of complex composition. Claus, after experimenting for himself, pronounced Fremy's results to be worthless. He found that after ten minutes or so, the barium precipitates always began to decompose and to contain sulphate. We have not met with this difficulty, the barium salts (except the two-thirds normal salt, as already noticed) proving to be stable, and free from sulphate when analysed. As the barium salts when acidified are

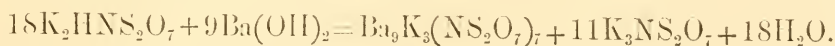
extremely rapidly hydrolysed, the best way to test their freedom from sulphate is to digest them with sodium carbonate, filter off and wash the barium carbonate, and observe whether this dissolves perfectly in nitric or hydrochloric acid.

Claus's experience may be accounted for by the presence of much dipotassium oximidosulphonate in the mother-liquor, which with barium chloride very rapidly yields sulphate by hydrolysis. He refused credence to Fremy's statement as to the formation of the dipotassium salt when the K_5 salt is precipitated by barium chloride and other salts, and consequently took no steps to protect himself against this accident.

On adding to a supersaturated solution of the dipotassium salt warm baryta-water not in excess, a precipitate was obtained by us somewhat poor in potassium; but then neither a supersaturated solution of the dipotassium salt nor a warm one of barium hydroxide is a solution of much concentration, which is the condition for the precipitate to contain much potassium. The composition of the precipitate agrees with that for the formula $Ba_3K_3(NS_2O_7)_7, 14H_2O$, matching which, except as to water, there is a barium sodium salt already described on a previous page:—

	Calc.	Found.
Barium	42.05	42.14
Potassium	4.01	4.19
Sulphur	15.28	15.32

Two mols. of dipotassium salt react with one mol. of barium hydroxide to yield $BaKNS_2O_7$, which is resolved by water into normal potassium salt and normal barium salt retaining a twelfth of the former, thus:—



To show the effect of adding the barium hydroxide in greater

quantity upon the normal potassium salt otherwise found in the mother-liquor, we may give the result of adding it in some excess to a warm solution of the K_5 salt. The precipitate obtained had a composition which may be regarded as resulting from the addition of barium hydroxide to the preceding salt, or as that of a hydroxy-derivative of that salt, in which half the barium is as half-hydroxide. The composition of the salt was $(HO)_3Ba_8K_2(NS_2O_7)_5, 14H_2O$, which is nearly equivalent to $(HOBa)_6Ba_6K_3(NS_2O_7)_7, 19\frac{1}{2}H_2O$, or $3Ba(OH)_2 + Ba_9K_3(NS_2O_7)_7$. It is also related to Fremy's salt, being that salt with K_5 replaced by $(HOBa)_3Ba$:—

	Calc.	Found.
Barium	45.17	45.29
Potassium	3.21	3.21
Sulphur.....	13.18	13.11

The reaction has been one in which potassium hydroxide is largely formed :—



The mother-liquor was tested and found to contain much potassium hydroxide and some barium hydroxide and nothing else.

When barium chloride in slight excess and in moderately dilute solution is added to the K_5 salt, it yields a similar compound to that obtained by excess of baryta-water, but with some chloride in place of hydroxide of barium. The precipitate had the composition $Cl(HO)_2Ba_{12}K_3(NS_2O_7)_6, 7H_2O$ or $BaCl_2 + 2Ba(OH)_2 + Ba_3(NS_2O_7)_2 + 2Ba_9K_3(NS_2O_7)_7 + 14H_2O$ (see lead salt 4, p. 82).

	Calc.	Found.
Barium	47.29	47.73
Potassium	3.37	3.24
Sulphur.....	14.73	14.67
Chlorine	1.02	0.96

Strontium oximidosulphonates.

We have not made a general examination of the strontium salts. What we have observed qualitatively and at variance with Fremy we have recorded on p. 48. On adding warm concentrated strontia-water to the five-sixths normal potassium salt the mixed solutions remain clear for a few moments, but then become suddenly filled with fine needles of very silky lustre. Yet this precipitate, which retains its lustre when dry, has a composition as complex as that of any barium salt we have examined, the formula for it being $(\text{HO})_3\text{Sr}_{11}\text{K}_8(\text{NS}_2\text{O}_7)_9, 16\text{H}_2\text{O}$, which may be expressed more simply as $(\text{HOSr})_3\text{NS}_2\text{O}_7, 8(\text{SrKNS}_2\text{O}_7, 2\text{H}_2\text{O})$, and in other ways :—

	Calc.	Found.
Strontium	28.96	28.69
Potassium	9.39	9.39
Sulphur	17.33	17.41
Nitroxy-rad., NO....	8.12	8.16
Water	8.44	8.67
Hydroxyl.....	1.53	—

A strontium sodium salt crystallises out in hemispherical tufts of brilliant silky needles, some hours after mixing either the disodium salt with warm concentrated strontia-water, or the normal sodium salt with strontium chloride. No analysis has been made of the salt.

Calcium oximidosulphonates.

Calcium salts, including calcium hydroxide, are not precipitated by alkali oximidosulphonates. When solution of the normal or the two-thirds normal ammonium salt is mixed with pure soft calcium hydroxide, one mol. of the salt dissolves about one mol. of the hydroxide, this not having been added in excess, and ammonia is liberated. The

solution evaporated on the water-bath gives off ammonia and leaves a crystalline residue, which may be taken to be a compound or mixture of the salts $\text{CaAmNS}_2\text{O}_7$ and CaHNS_2O_7 . Treated with water some nearly insoluble calcium oximidosulphonate is left, while the greater part of the mass dissolves up as calcium ammonium salt.

If after dissolving, as described in the preceding paragraph, two mols. of calcium hydroxide in two mols. of the ammonium salt, a third mol. of the softest moist calcium hydroxide be stirred in, it may be seen to give place to a voluminous precipitate which most probably is normal calcium oximidosulphonate. The precipitate has very little solubility in water, and is so free from ammonia as to evolve none when mixed with calcium hydroxide. a test however which is not quite conclusive.

Lead oximidosulphonates.

Reactions of alkali oximidosulphonates with lead acetates.—The reactions of oximidosulphonates with lead salts are complex. The disodium and dipotassium salts give no precipitate with normal lead acetate, but precipitate with the basic lead acetates. With highly basic acetates in good excess the precipitate is mainly or wholly the normal hydroxy-lead salt (numbered 2 in the description of these salts which follows). With a basic acetate not in excess and in solution not concentrated, the precipitate is the salt numbered 3; while with medium quantities of basic acetate and concentrated solutions the precipitate is approximately the salt 4. The precipitates usually contain, especially when strong solutions are worked with, acetate and alkali salt, only partially removable by washing with hot water.

Normal sodium oximidosulphonate and normal lead acetate show no immediate precipitation unless the solutions are dilute. With enough of the sodium salt present, two mols. or more, to one mol. of

the acetate, and using concentrated solutions, the mixture in a few hours becomes filled more or less with a soft mass of minute crystals of a lead sodium salt (7). Using much less sodium salt the mixed solutions, if not dilute, also remain clear, but very slowly deposit spherical crystalline hard grains of the salt 4. The mother-liquor of this salt precipitates with water. If less concentrated the mixed solutions yield a precipitate at once, which however redissolves on heating, while concentrating the mixture by evaporation on a water-bath causes the precipitate in what were even dilute solutions, to gradually redissolve. Evaporation to dryness, yields a gum-like mass soluble in a little water but decomposed by much.

Normal potassium oximidosulphonate and also the K_5 salt give an immediate precipitate with normal lead acetate, soluble in excess of either mother-salt in strong solution, but the solution remains clear for only a few moments and then gives a crystalline precipitate unlike the flocculent and voluminous one first formed (*cf.* Fremy). When the oximide salt is in excess the precipitate is 6; with the acetate in excess it is 5. Normal ammonium oximidosulphonate is like the sodium salt in keeping clear for a long time after being mixed with normal lead acetate when in moderately concentrated solution, and in precipitating when largely diluted. Its behaviour has not been further observed.

The normal sodium salt and a basic lead acetate give no precipitate when the former is in the proportion of one mol. or more to one mol. of hemihydroxy-lead acetate, even after a long time or upon dilution with water. With the basic lead acetate in excess precipitation of the normal hydroxy-lead salt (2) is immediate.

The normal potassium salt in excess and in concentrated solution remains clear for a very short time after being mixed with basic acetate; it then deposits a crystalline flocculent precipitate redissolv-

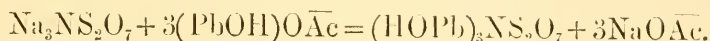
ing on heating and re-forming on cooling (6). The basic acetate being in excess, instant precipitation takes place. The K_5 salt in excess with basic lead acetate behaves as with the normal acetate (*cf.* Fremy). With the basic acetate in excess it reacts in the same way as the normal potassium salt. The normal ammonium salt with basic lead acetate behaves essentially as the sodium salt, but the solvent action of ammonium salts prevents complete precipitation. Except when a basic acetate in excess is used, the mother-liquors of the lead precipitates are rich in the two-thirds normal salt of the alkali metal used, a fact pointed out by Fremy, but emphatically denied by Claus.

Description of the salts.—Like other oximidosulphonates the lead salts combine readily and in varying proportions with other salts. Accordingly, several double salts of lead and sodium, of lead and potassium, and even of lead and hydrogen can be prepared. These are partly decomposed by water, but washing even with hot water never removes all alkali salt. In nearly all the salts the lead is present half as hydroxide or oxide. An exception is the unstable two-thirds normal lead salt. With admissible and only slight qualifications, all the salts we have analysed may be expressed as derivatives of one, two, three, or four molecules of the acid.

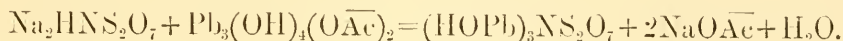
(1) *Two-thirds normal lead oximidosulphonate*, $PbHNS_2O_7$. This salt can be obtained from the normal hydroxy-lead salt (2) which is insoluble in water, by stirring it with water containing almost enough sulphuric acid to deprive it of two-thirds of its lead. The mother-liquor, which is slightly acid, when moderately evaporated in the desiccator gives a crust of minute crystals. These appear under the microscope as transparent prisms somewhat hollowed at their middle to a dumb-bell-shape. The salt is only sparingly soluble, and is very unstable. We have not analysed it.

(2) *Normal hydroxy-lead oximidosulphonate*, $(HO Pb)_3NS_2O_7$,

$3\text{H}_2\text{O}$.—Hemihydroxy or still more basic lead acetate in good excess of the calculated quantity yields this salt in the pure state when normal sodium oximidosulphonate is poured into it with stirring. When the lead solution is poured gradually into the sodium salt the precipitate contains generally a little lead hydroxide, free or combined. The salt is quite stable and may be freely washed with water hot or cold. It is a very voluminous flocculent precipitate, not in the least slimy or gelatinous, evidently crystalline under the microscope. The reaction by which it is formed is expressed by the equation—



The salt can also be prepared, when sufficient care is taken, by adding to good excess of tribasic lead acetate a solution of the disodium salt. The reaction then is—

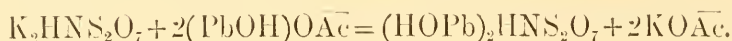


This lead salt decomposes when heated, as already described (p. 42) into sulphite and nitrite. It is soluble in acetic and other acids, in sodium hydroxide, in ammonium chloride and other ammonium salts, including ammonium oximidosulphonate. It is easily and fully decomposed in the cold by sodium, potassium, or ammonium hydrogen carbonate. The last named salt gives as an intermediate product a soluble lead ammonium salt (7). The results of analysis agree well with calculation :—

	Calc.	Found.
Lead	67.79	67.57
Sulphur.....	6.99	6.89

(3) *Two-thirds normal hydroxy-lead oximidosulphonate*, $(\text{HOPb})_2\text{HNS}_2\text{O}_7, \text{H}_2\text{O}$.—Hemihydroxy-lead acetate added to a solution of dipotassium oximidosulphonate, the latter in small excess, gives a

precipitate of this salt, not quite pure however, but containing small quantities of potassium, acetic acid, and lead in excess. The corresponding sodium salt could no doubt be used in place of the potassium salt, but has not been tried. The precipitate is voluminous and flocculent. Its formation is represented by the equation—



Analysis of a preparation freest from potassium gave the following results, which are compared with the results of calculation for the above formula with one-ninth of an atom of PbO additional:—

	Calc.	Found.
Lead	64.10	63.92
Potassium.....	—	0.23
Sulphur	9.39	9.36

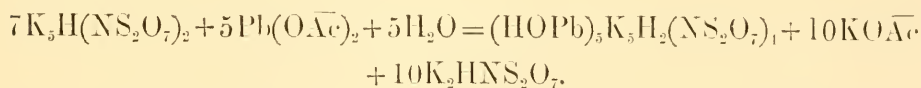
The slight approach in composition to the normal hydroxy-lead salt, which the small excess of lead indicates, is in agreement with what is observed in preparing the dipotassium salt, which is apt to crystallise with a little excess of potassium. Calculation for the pure salt gives lead 63.01 and sulphur 9.74 per cent.

(4) *Five-sixths normal aceto-hydroxy-lead oximidosulphonate*, $(\overline{\text{AcOPb}})(\text{HOPb})_4\text{H}(\text{NS}_2\text{O}_7)_2 \cdot 2\frac{1}{2}\text{H}_2\text{O}$.—When to two mols. of normal sodium oximidosulphonate in somewhat concentrated solution excess—say three to six mols.—of normal lead acetate is added, there form in some hours or days hard, almost opaque, spherical granules, of radiating crystalline structure, some at the surface of the solution, others adherent to the sides and bottom of the vessel. These granules are of the required salt not quite pure however and incapable of being purified because slowly decomposed by water. The preparation of which the analysis follows, contained, it will be seen, acetic acid in excess of that indicated by the formula. It would be possible to include this extra

eighth of acetic acid in a complex formula, but to do so would be almost vain, for while the acetic acid was in other preparations also in excess, the excess was variable though the total acetic acid was still about 4 per cent :—

	Calc.	Found.
Lead	65.18	65.10
Sulphur.....	8.06	8.14
Acetic acid	3.78	4.29

(5) *Five-sixths normal hydroxy-lead potassium oximidosulphonate*, $(\text{HOPb})_5\text{K}_5\text{H}_2(\text{NS}_2\text{O}_7)_4$.—To form this salt calculation points to the use of 7 mols. of the K_5 salt and 5 mols. of normal lead acetate. It was obtained by us by taking 8 mols. to 6 mols. of the respective salts, the former in warm concentrated solution and the latter also in concentrated solution. Almost immediately after mixing them together a dense granular but still somewhat curdy precipitate separated, which was drained on a tile. Air-dried, it lost weight in a desiccator equal to 5.3 per cent.; the loss of $2\text{H}_2\text{O}$ would be about that. It was the dehydrated salt which was analysed. The equation expressing its formation is—



This salt is somewhat soluble in its mother-liquor. On adding more of the potassium salt to the mother-liquor, a precipitate of apparently the salt next to be described formed.

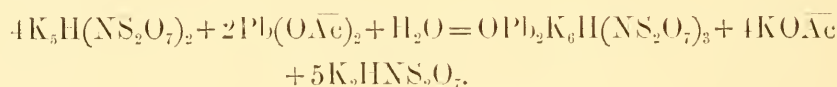
	Calc.	Found.
Lead	49.79	50.04
Potassium.....	9.42	9.15
Sulphur.....	12.34	12.46

(6) *Eight-ninths normal oxy-lead potassium oximidosulphonate*, $(\text{OPb}_2)\text{K}_6\text{H}(\text{NS}_2\text{O}_7)_3$, or the *normal salt* $(\text{HOPb})\text{PbK}_6(\text{NS}_2\text{O}_7)_3$.—Pro-

ceeding as for the last described salt, but taking to two mols. of the potassium salt only one mol. of the lead salt, the clear solution obtained after agitating to dissolve the precipitate first formed, yielded us when stirred with a glass rod a sandy crystalline precipitate of the composition expressed by either of the above formulæ, but with nearly $\frac{1}{16}$ of the potassium replaced by hydrogen, as the calculation shows :—

	Calc.	Found.
Lead	33·51	33·46
Potassium.....	18·98	18·33
Sulphur.....	15·54	15·82

The mother-liquor proved to be free from lead and practically neutral. The following equation expresses, therefore, the reaction which had taken place :—



Fremy's salt.—Almost in the same way, that is, by adding normal lead acetate drop by drop to warm concentrated solution of the K_5 salt, agitating to dissolve the immediate precipitate, and going on until suddenly a second crystalline precipitate appeared. Fremy got a salt having nearly the same content of sulphur as our salt just described, but with more lead and less potassium, which admits of no simple representation by a formula, although there are, closely enough, 4K to 1Pb. We give the calculation for $\text{PbK}_4(\text{NS}_2\text{O}_7)_2$, along with Fremy's finding :—

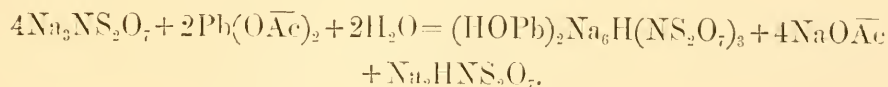
	Calc.	Found. (Fremy)
Lead	27·85	28·40
Potassium.....	21·04	21·44
Sulphur.....	17·22	15·65
Nitrogen	3·77	3·48

from which it will be seen that his sulphur is about one-tenth too low for this formula, one which is besides improbable from the fact of the lead in it being wholly combined with the oximide radical.

(7) *Eight-ninths normal hydroxy-lead sodium oximidosulphonate*, $(\text{HOPb})_2\text{Na}_6\text{H}(\text{NS}_2\text{O}_7)_3, 14\text{H}_2\text{O}$.—Concentrated solutions of two mols. of the normal sodium salt and one mol. of normal lead acetate are mixed together and left, protected from the air, till crystallisation occurs, when the solution becomes filled, or partly filled, with minute crystals forming with it a soft magma. The crystals are drained from their mother-liquor and pressed between porous tiles. They are efflorescent and are dissolved and decomposed by water. Two quantities were prepared and analysed, the one more effloresced than the other. Calculation A is for 14 Aq. and B for 10 Aq. :—

	A		B	
	Calc.	Found.	Calc.	Found.
Lead	29.38	29.31	30.97	31.32
Sodium	9.79	9.69	10.32	10.05
Sulphur	13.63	13.73	14.36	14.35

This salt, like the potassium salt, can be formulated as a normal salt, $(\text{HOPb})\text{PbNa}_6(\text{NS}_2\text{O}_7)_3, 15\text{H}_2\text{O}$, or as an oxy-lead salt, $(\text{OPb}_2)\text{Na}_6\text{H}(\text{NS}_2\text{O}_7)_3$, but with water of crystallisation. The mother-liquor of the crystals contains much disodium oximidosulphonate :—



(8) *Normal diammonium hydroxy-lead oximidosulphonate*, $\text{HOPbAm}_2\text{NS}_2\text{O}_7$.—A normal salt with one atom of the hydroxy-lead radical to two of ammonium has not been isolated in the pure state, but can be got in solution almost pure, and the solution can be evaporated in a desiccator to dryness with the loss of a small fraction only of its ammonia. Normal hydroxy-lead oximidosulphonate (2) is pre-

precipitated from a known quantity of normal sodium oximidosulphonate, washed by decantation, and its last washing-water closely decanted. The precipitate even on long standing still occupies a large volume of liquid, and if to it there is now added powdered ammonium hydrogen carbonate in quantity calculated to decompose completely barely two-thirds of the lead salt and the mixture is well agitated and then left to stand, lead carbonate, filling a very small space, settles down and a clear mother-liquor can be decanted having only a scarcely noticeable odour of ammonia and containing one atom of lead to two of ammonium, practically all the oximidosulphonic radical, and no carbonic acid. Much water added renders it milky. It can be evaporated to dryness in the cold and the residue redissolved in water.

The disodium and dipotassium hydroxy-lead salts cannot be prepared in a similar way, but a solution of either of these salts, or of the diammonium salt, along with acetate, is apparently obtained by mixing together in concentrated solution one mol. of hemihydroxy-lead acetate and one mol. of normal oximidosulphonate of sodium, potassium, or ammonium. The solution dries up to a vitreous mass with a little confused crystalline matter, and is precipitated on dilution with water.



Constitution of Glycocoll and its Derivatives.

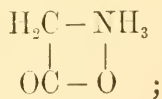
(Appendix: General theory and Nomenclature of Amido-acids.)

By

Jōji Sakurai, F. C. S., *Rigakuhakushi*,

Professor of Chemistry, Imperial University.

Glycocoll, the prototype of that large and important class of compounds generally called amido-acids, is usually represented as amido-acetic acid, $\text{H}_2\text{N}.\text{CH}_2.\text{COOH}$, in spite of an abundance of facts which show that it must be considered as an internal ammonium salt of the constitution,

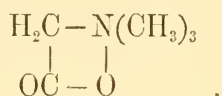


and the object of this paper is to strengthen the evidence in support of the latter view and, at the same time, to deprecate the almost universal employment of the open formula in describing the reactions of this compound.

The view that glycocoll is constituted as an internal ammonium salt was first suggested by Erlenmeyer and Sigel (*Lieb. Ann.*, **176**, 349 [1875]), in order to account for the perfect neutrality of this compound towards litmus ; it may also be advanced to explain the high melting

point of glycocoll and its insolubility in alcohol and ether. A similar view of the constitution of taurin had already been put forward by Erlenmeyer.

The great analogy in properties between glycocoll and anhydrous betain (trimethyl-glycocoll) on the one hand, and the close relationship between the latter and choline on the other, a relationship which establishes the constitution,



for anhydrous betain, give a further support to the correctness of the closed formula.

Again, the observation made by Marekwald, Nenmark, and Stelzner (*Ber.*, **24**, 3279 [1891]), that glycocoll does not readily react with mustard oils to form derivatives of thio-urea, contrary to the behaviour of all primary amines, speaks against the commonly adopted open formula for glycocoll.

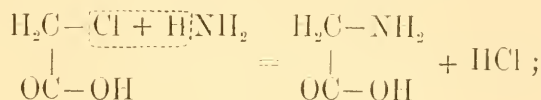
All other properties of this compound are in perfect accordance with the constitution of the internal ammonium salt, and there is not a single reaction which needs to be expressed by the open formula ; on the contrary, there are several, as will be shown later on, which can not be explained by it.

In spite of the evident claim of the internal ammonium theory to be exclusively adopted, it is astonishing to find that there are most eminent authors who do not recognise it frankly and make common use of it, and others who even disregard it altogether.⁽¹⁾ While insist-

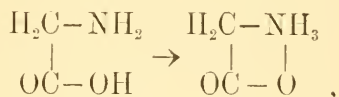
(1) Beilstein (*Handbuch* : 2 Aufl., I, 1182-1183 [1893]) does not even mention the closed formula. Roscoe and Schorlenner (*Treatise* : 2 Ed., III, Pt. 2 [1890]) say in one place (p. 20) that glycocoll must be considered as an ammonium salt, but in describing its reactions they expressly use the open formula (pp. 20 and 100) ; alkyl glycocolls are, on the other hand, represented by the closed formula. V. Meyer and Jacobson (*Lehrbuch* : I, 828 [1893]) seem to be greatly in favour of the internal ammonium theory, but do not use it consistently ; in fact, like

ing upon the analogy between glycocoll and its trimethyl derivative, they represent the one by the open, and the other by the closed, formula. This inconsistency on their part is, I think, to be attributed partly, at any rate, to the erroneous manner in which the modes of formation of glycocoll have been hitherto represented.

Thus, the production of glycocoll by the action of ammonia upon chloracetic acid is always regarded as if taking place by the direct replacement of chlorine by the amidogen group—



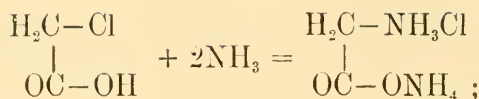
it, therefore, requires a further strain of mind to represent glycocoll as an internal ammonium salt,



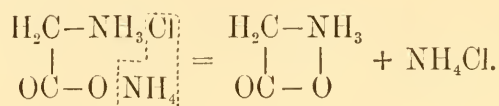
and it is this awkwardness, no doubt, which has had much to do in making authors hesitate in adopting the closed formula.

The above universally employed representation of the mode of formation of glycocoll is, however, erroneous, inasmuch as it does not take into account the evident fact that ammonium chloracetate must first be produced. In order to obtain glycocoll by this method, an excess of ammonia must be employed, which not only goes to neutralise chloracetic acid but also to form an ammonium chloride.

Roscoe and Schorlemmer, they only employ the open formula in describing the modes of formation of glycocoll. Mono- and dimethyl glycocolls are represented by the open, and trimethyl glycocoll by the closed, formula. Strecker and Wislicenus (*Text Book*: English Translation, 416 [1885]) are more decidedly inclined to this theory; but still they are not quite consistent, in one or two places using the open formula and in several others the closed double formula. The only work I have seen where the internal ammonium theory is exclusively adopted, though on an insufficient ground, is the article on Glycocoll, contributed by Hell to the *Neues Handwörterbuch der Chemie*, III, 446 [1878].

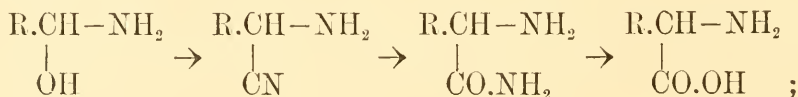


the latter part of this change being analogous to that which occurs between ammonia and alkyl halides. This ammonium compound must, then, be regarded as decomposing into glycocoll and ammonium chloride by the action of heat :

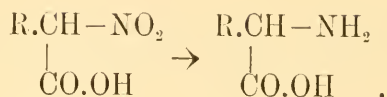


The conception of the mode of formation of glycocoll here advanced is not a matter of speculation, but is only an expression of actual facts, and it *necessarily* leads to the internal ammonium theory of the constitution of this compound ; for, if we regarded glycocoll as $\text{H}_2\text{N} \cdot \text{CH}_2 \cdot \text{COOH}$, we should have to assume that the highly acid group, COOH , remains unneutralised by ammonia even in the presence of an excess of the latter !

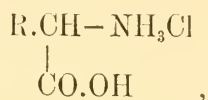
Again, it may be observed that in describing various other modes of formation of glycocoll and of the “amido-acids” generally, the fact is almost always either concealed or forgotten that it is the hydrochlorides or other analogous compounds which are first obtained. Thus, it is stated that “amido-acids” are formed by heating a mixture of aldehyde-ammonias and hydrogen cyanide with hydrochloric acid, the changes which occur being indicated by some such scheme as follows :



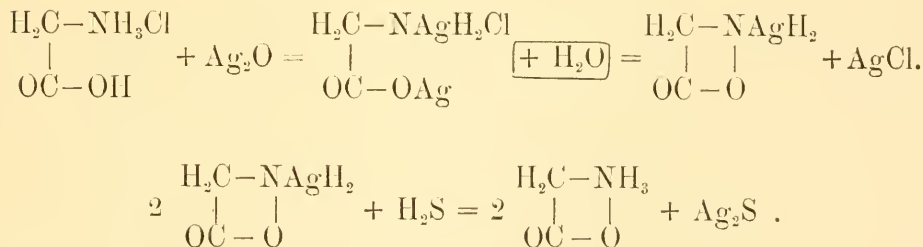
or, that they are produced by reducing nitro-acids with tin and hydrochloric acid :



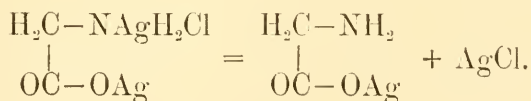
“Amido-acids” are, however, *not* produced in these reactions ; it is their hydrochlorides,



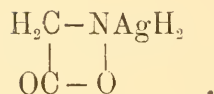
which are actually obtained. In order to prepare the “amido-acid” itself, glycocoll for example, the hydrochloride is digested with silver oxide (or litharge), and the silver-glycocoll decomposed by sulphuretted hydrogen. The changes which occur in these operations are in accordance with the following scheme :



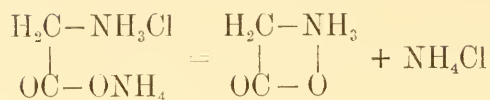
It may be thought, however, that the separation of silver chloride occurs, not in the manner indicated above, but as follows :



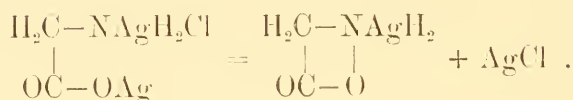
Against this objection, it may be pointed out that the properties of silver-glycocoll cannot, as will be shown later on, be accounted for by regarding it as an amido-acetate, but that all its reactions are most satisfactorily explained by the formula,



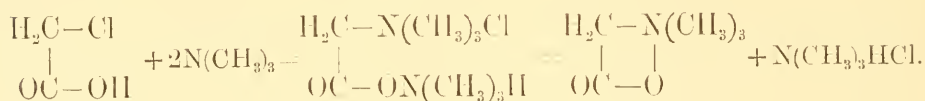
Moreover, there is a complete parallelism between the two equations :



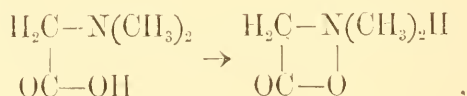
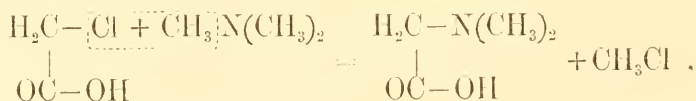
and



The synthetic formation of anhydrous betain from trimethylamine and chloracetic acid (Liebreich : *Ber.*, **2**, 167 [1869]) can be expressed in a manner perfectly analogous to that indicated for glycocoll :

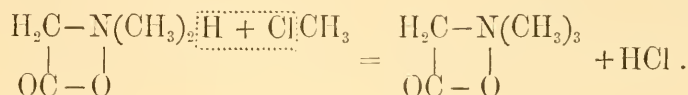


If, however, we attempted to represent the above change in a manner analogous to that by which the formation of glycocoll is usually represented—



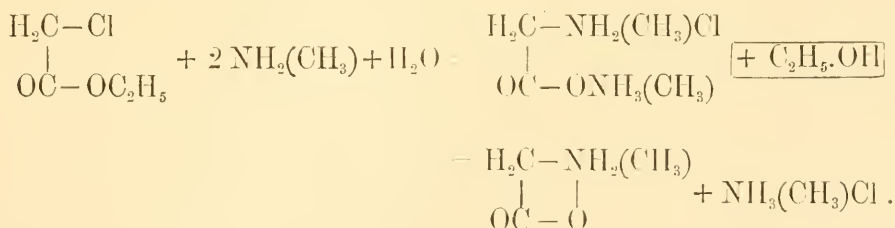
we should misrepresent the actual fact, inasmuch as dimethyl-glycocoll is thus made to be the product instead of betain, unless, indeed, we

assume that methyl chloride, supposed to be liberated during the first stage of the reaction, goes to interact with dimethyl-glycocoll, producing betain and hydrogen chloride :



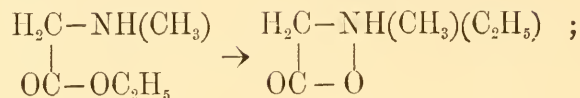
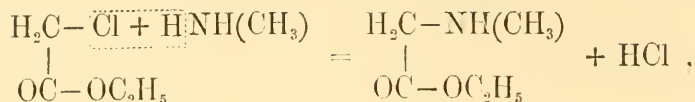
This assumption, which has no merit as an explanation over that already given, is, moreover, hardly warranted by facts. For, even if we suppose that dimethyl-glycocoll is first formed and then changed into trimethyl-glycocoll by interaction with methyl chloride, it is difficult to imagine that this interaction would be so complete that none of the interacting bodies should be left unaltered. The fact, then, that by the action of trimethylamine upon chloracetic acid, betain is produced unmixed with dimethyl-glycocoll goes to deprive the ground of the assumption necessitated above of any probability and, therefore, to corroborate the view here advocated as to the mode of formation of glycocoll and its trimethyl derivative.

The consideration of the mode of formation of sarkosine (monomethyl-glycocoll), a body which Volhard synthetically obtained by the action of methylamine upon ethyl chloracetate (*Lieb. Ann.*, **123**, 261 [1862]), leads to the same conclusion. The reaction is expressed as follows, according to the view here urged :



If, however, we attempted as before to represent the above change in

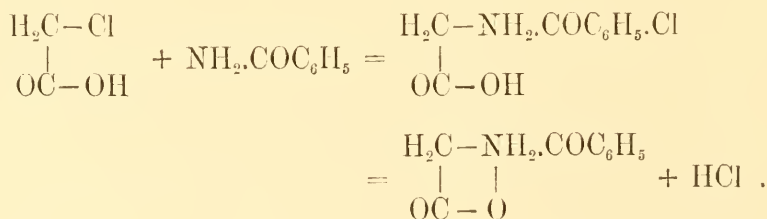
the way commonly adopted in the case of glycocoll, we should have



and in order to avoid the misrepresentation here involved that methyl-ethyl-glycocoll is produced instead of sarkosine, we should have to make an assumption similar to that made in the case of betain, an assumption which is not supported by facts.

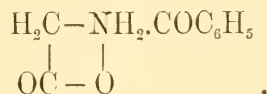
It is, then, that property, characteristic of the class of compounds under consideration, of producing internal ammonium salts, which alone can satisfactorily explain the mode of formation of sarkosine and betain. This consideration gives a fresh confirmation of the correctness of the closed formula for glycocoll, unless we disregard the evident analogy which exists between it and its methyl derivatives.

The mode of formation of hippuric acid by the action of benzamide upon chloracetic acid has also to be represented in a manner analogous to that above advanced for glycocoll and its methyl derivatives, thus :



This way of regarding the formation of hippuric acid makes it also a ring compound, a conclusion which is not contradicted by facts ; on

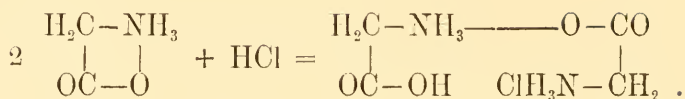
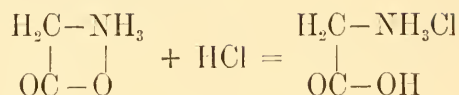
the contrary, very slight solubility of this compound in cold alcohol and ether, its feeble acid character, and its almost neutral taste rather go to support this view of its constitution. That it does possess an acid character, feeble as it is, is easily accounted for by the presence of the benzoyl group, which imparts to the amidic hydrogen the property of being more easily replaced by metals generally than in the case of glycocoll; and it is scarcely necessary to mention that there are many well known non-carboxylic compounds, uric acid for example, which possess acid characters. Again, the fact that all attempts hitherto made to obtain what may be called hippuryl chloride, $\text{COCl.CH}_2\text{.NH.COC}_6\text{H}_5$, have been attended with failure, goes towards upsetting the accepted view that it is a carboxylic compound, an argument which, indeed, may be used for denying carboxylic constitution to all the so-called amido-acids. In one word, the name benzoyl-glycocoll, already in common use, expresses its constitution perfectly—



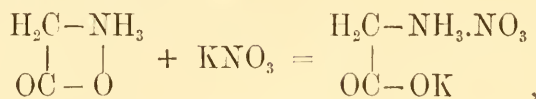
Passing, now, from the modes of formation of glycocoll and its derivatives, let us consider some of its transformations and discuss the constitution of the compounds thereby produced.

A. *Addition compounds.* That, although glycocoll and its derivatives must be regarded as closed bodies, its addition compounds should possess an open constitution is easy to admit, remembering that a nitrogen atom is not yet known to be capable of combining with more than five monovalent radicals. The ordinary or hydrated betain, $\text{COOH.CH}_2.\text{N}(\text{CH}_3)_3.\text{OH}$, and the corresponding chloride, $\text{COOH.CH}_2.\text{N}(\text{CH}_3)_3.\text{Cl}$, are bodies of this kind. Their formation presents, therefore, no difficulties with the closed formula for glycocoll.

(a) *Combination with acids :*

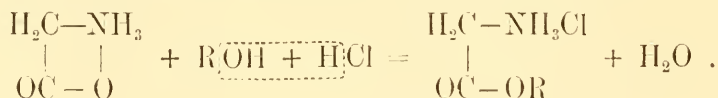


(b) *Combination with metallic salts :*



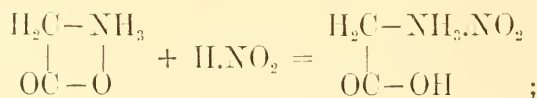
whereas with the open formula for glycocoll, we must assume, in this case, that a double decomposition first takes place with formation of potassium amido-acetate and nitric acid and that, then, the latter unites to the amidogen group, unless we give the irrational formula, $\text{COOH}.\text{CH}_2.\text{NH}_2.\text{K}.\text{NO}_3$, to this compound.

(c) *Conversion into the hydrochloride of an amido-acetic ester :*

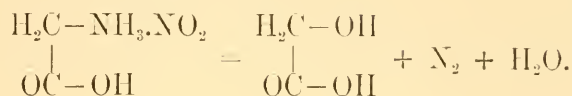


(d) *Conversion into glycollic acid :*

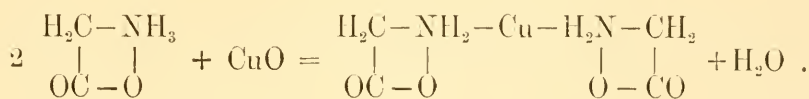
Although this change is usually regarded as consisting in the direct replacement of the amidogen group by hydroxyl and, therefore, may appear as offering a difficulty to the closed formula, the first action of nitrous acid must be admitted to be analogous to that which occurs between glycocoll and hydrochloric acid, namely the formation of an addition compound—



the latter, then, decomposes, under the conditions of the experiment, into glycollic acid, nitrogen, and water :



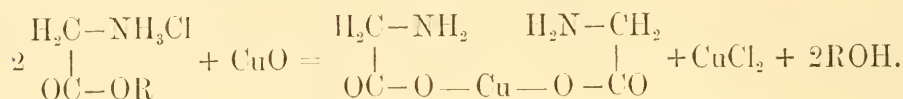
B. *Metallic derivatives.* Among the derivatives of glycocoll one, which is best known and most characteristic, is the *copper* compound. The deep blue colour of this body and its solubility in alkalis distinguish it from ordinary carboxylic salts of copper, and lead us to the conclusion that it is most probably a cuprammonium compound, a conception which can be readily expressed by the following scheme :



If we attempted to represent the copper compound as an ammoniated derivative by help of the open formula and gave it the constitution $\text{Cu}(\text{NH}.\text{CH}_2.\text{COOH})_2$, we should have to make the baseless assumption that the hydrogen of the carboxyl group remains unreplaced by copper even in the presence of an excess of cupric oxide. If, on the other hand, we regarded the copper derivative as $(\text{H}_2\text{N}.\text{CH}_2.\text{COO})_2\text{Cu}$, looking upon glycocoll as a carboxylic compound (an acid) because it dissolves oxide of copper and some other metallic oxides, we might argue that ammonium chloride or even ammonia itself is also an acid !

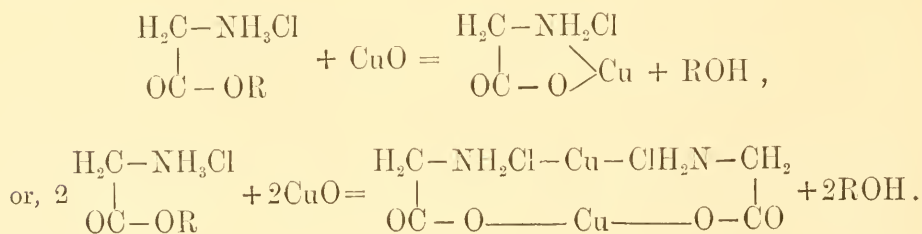
Reference must here be made to two important papers bearing upon the question : one by Curtius and Goebel (*J. prakt. Chem.*, **37**, 150 [1888]), the other by Kraut (*Lieb. Ann.*, **266**, 292 [1891]). By digestion of the hydrochloride of an amido-acetic ester with an excess of freshly precipitated cupric oxide, Curtius and Goebel obtained a

copper compound which they represent by the formula, $\text{COOR} \cdot \text{CH}_2 \cdot \text{NH} - \text{Cu} - \text{NH} \cdot \text{CH}_2 \cdot \text{COOR}$; whilst Kraut denies the existence of such a compound and shows, by a careful experimental study, that the interaction between the hydrochloride of an amido-acetic ester and cupric oxide occurs according to the equation :



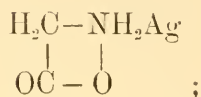
Kraut concludes from this that it is the hydrogen of the hydroxyl, not that of the amidogen, which is replaced by copper.

It must be observed, however, that what Kraut obtained is a double copper compound, having the composition, $\text{CuCl}_2 \cdot (\text{H}_2\text{N} \cdot \text{CH}_2 \cdot \text{COO})_2\text{Cu}$, or more simply, as he himself puts it, $\text{H}_2\text{N} \cdot \text{CH}_2 \cdot \text{COO} \cdot \text{CuCl}$; but he does not give any explanation of the formation of this *double* salt in support of his views. I think a satisfactory explanation of the formation and existence of this double salt can be readily obtained by supposing that cupric oxide displaces an equivalent of an alcohol from the hydrochloride of an amido-acetic ester, according to the equation,

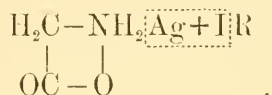


The view here advanced as to the constitution of the copper compound obtained by Kraut is in perfect harmony with his experimental data; at the same time, it shows that his conclusion that copper must be wholly carboxylated is groundless.

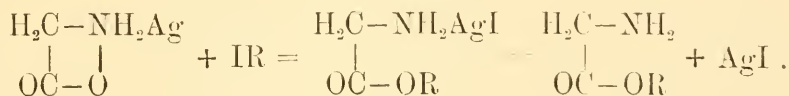
Turning now to the *silver* derivative, the metallammonium theory gives it the constitution,



and although the formation of amido-acetic esters by the action of alkyl iodides upon it is brought forward against this view (Kraut : *op. cit.*, 310). I see no ground whatever for this argument. Representing, as is usually done, the action of alkyl iodides upon silver-glycocoll, as if direct replacement of silver by alkyl radicals took place,



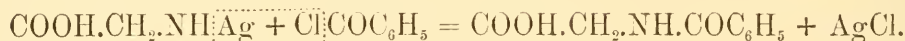
the formation of amido-acetic esters, $\text{H}_2\text{N}.\text{CH}_2.\text{COOR}$, would, indeed, offer a difficulty to the metallammonium theory ; but the action in this case, as in other analogous cases, must be regarded as primarily of an additive nature, the unstable addition compound, then, decomposing into silver iodide and the ester, under the conditions of the experiment :



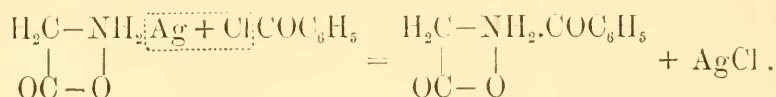
The formation of amido-acetic esters, and not alkyl-glycocolls, by the action of alkyl iodides upon silver-glycocoll is a strong argument against the *metallamido-acetic acid theory* ($\text{MHN}.\text{CH}_2.\text{COOH}$), but it does not touch the metallammonium theory.

The formation of hippuric acid by the action of benzoyl chloride upon silver (or zinc) glycocoll is, on the other hand, a death-blow to

the *metallic amido-acetate theory* ($\text{H}_2\text{N}.\text{CH}_2.\text{COOM}$) ; it is this reaction which has led many eminent chemists to regard metallic derivatives of glycocoll as $\text{MHN}.\text{CH}_2.\text{COOH}$, and to represent the formation of hippuric acid accordingly :



As already pointed out, however, there are reasons to believe that hippuric acid is benzoyl-glycocoll, and its formation is, therefore, no more easily expressed by the scheme above given than by



It must be observed that an opening of the ring, as in the case of the action of alkyl iodides upon silver-glycocoll, could not occur here, because both the radicals, Cl and COC_6H_5 , are negative.

The metallic amido-acetate theory can furnish no satisfactory explanation of the formation of hippuric acid by the action of benzoyl chloride upon silver-glycocoll, nor can the metallamido-acetic acid theory that of the production of amido-acetic esters by the action of alkyl iodides upon silver-glycocoll. Both theories are imperfect, inasmuch as each considers only one set of facts ; the *metallammonium theory*, on the contrary, takes complete account of the reactions of the derivatives under consideration and, at the same time, meets the objections and requirements of the other two theories.

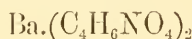
With regard to other metallic derivatives, such as those containing *mercury*, *cadium*, *zinc*, *magnesium*, and *lead*, they are no doubt constituted like the copper or the silver derivative. Those containing the metals of the alkaline earths—*barium*, *strontium*, and *calcium*—have only been recently obtained in definite and crystalline states (Kraut:

op. cit., 299). Horsford (*Lieb. Ann.*, **60**, 33 [1846]) and, more recently, Curtius (*J. prakt. Chem.*, **26**, 159 [1882]) attempted in vain to prepare these bodies in the pure state; it is only by mixing concentrated aqueous solutions of glycoll and an alkaline earth, pouring the mixture into alcohol, and leaving the precipitated oil for some days in contact with the mother liquor that Kraut was able to obtain them in a pure and crystalline condition. The evident difficulties, under which these bodies are formed, mark them off from ordinary carboxylic salts, a circumstance which shows that they are probably also metallammonium compounds. The comparative ease with which derivatives containing copper, silver, mercury, &c. are formed and the difficulties, which attend the preparation of those containing the metals of the alkaline earths, and which increase still more in the case of those containing alkali metals (these derivatives, in fact, do not seem to have been isolated as yet), speak again in favour of the metallammonium theory; inasmuch as metals like copper, silver, and mercury are eminently characterised by the ease with which they form ammoniated derivatives. In opposition to Kraut's words, "Ich halte daher sämtliche Metallverbindungen des Glycolls für wahre amido-essigsäure Salz," I would rather say that *all metallic derivatives of glycoll are metallammonium compounds.*

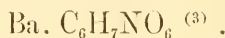
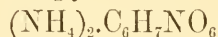
The above examination of the modes of formation and transformation of glycoll irresistably leads to the internal ammonium theory of its constitution; there is, however, one other point which must be considered before going further. The point in question is the relation between glycoll and the so-called diglycolamidic and triglycolamidic acids. The latter compounds are usually represented by $\text{HN}(\text{CH}_2\text{COOH})_2$ and $\text{N}(\text{CH}_2\text{COOH})_3$ respectively, and glycoll and these two bodies are compared to mono-, di-, and triethylamine; this relation is even regarded as an argument for the open formula for

glycocoll (Kraut: *op. cit.*, 309). It is to be observed, however, that di- and triglycolamidic acids behave respectively as mono- and dibasic acids towards alkalies and alkaline earths, the composition of their anhydrous salts containing alkali metals and metals of the alkaline earths⁽²⁾ being

Diglycolamidates.

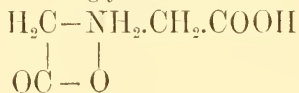


Triglycolamidates.

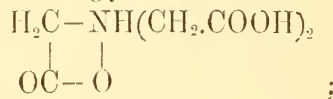


It is only those of their derivatives containing copper, silver, zinc, or lead⁽⁴⁾-metals more or less characterised by the ease with which they form ammoniated compounds—which may be regarded as dibasic ($\text{M}'_2 \cdot \text{C}_4\text{H}_5\text{NO}_4$) and tribasic ($\text{M}'_3 \cdot \text{C}_6\text{H}_6\text{NO}_6$) salts respectively. These facts are unexplicable by the accepted constitution of di- and triglycolamidic acids, but receive an ample and ready explanation from the following formulae:

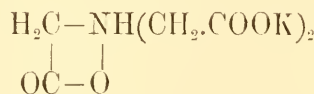
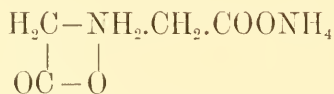
Diglycolamidic acid.



Triglycolamidic acid.



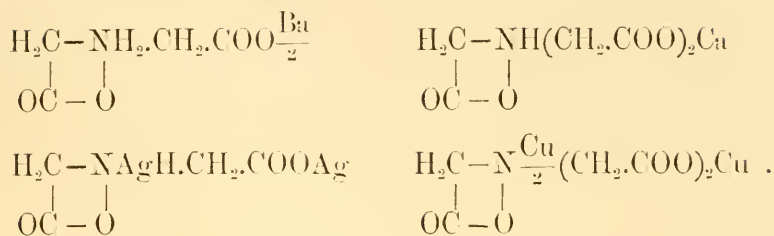
the constitution of their salts or metallic derivatives being



(2) For the composition of these and other salts, see Heintz: *Lieb. Ann.*, **122**, 269; **124**, 297; **156**, 51; Lüddecke: *Ibid.*, **147**, 272; Beilstein: *Handbuch*, 3 Aufl., I, 1191-1192.

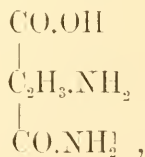
(3) Barium forms another salt of the composition $\text{Ba}_3(\text{C}_6\text{H}_6\text{NO}_6)_2$, but it readily changes into the above dibasic salt on addition of acetic acid.

(4) A dibasic salt of lead of the composition $\text{Pb} \cdot \text{C}_6\text{H}_7\text{NO}_6$ is also known.



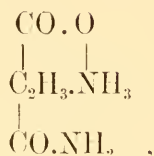
A word must now be said with regard to the term "amido-acids."⁽⁵⁾ This term which is so generally employed should, in accordance with the internal ammonium theory, be dropped from chemical nomenclature and replaced by the word *glycocolls*, at least in the case of the so-called amido-carboxylic acids. This designation is to be recommended, because it is not new, and is to be preferred to other difficult names which might be suggested as expressing constitution. Moreover, a particular glycocoll may be easily specified by prefixing *acetic*, *propionic*, &c.; alanine would thus be *propionic glycocoll*. These expressions have to be distinguished from those, which are already in use to designate derivatives of a particular glycocoll, such as acetyl-glycocoll (aceturic acid) and benzoyl-glycocoll (hippuric acid). We may also use such expressions as "glycocollic constitution," "glycocollyl group—CO₂NH₃—," &c. without confusion or other inconvenience.

Asparagin and aspartic acid, as well as all other similar bodies, such as glutamin, glutamic acid, leucine, leucic acid, &c. must be looked upon as glycocollyl compounds. Thus, asparagin which is usually regarded as "amido-succinamic acid,"

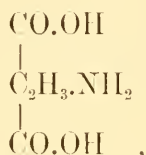


(5) In the new edition of his *Handbuch*, Beilstein changes it into "amino-acids," but very little is gained by the alteration.

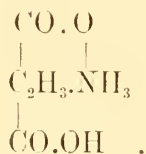
is exceedingly like glycocoll both in its physical and chemical properties, and does not possess any acid character. It ought, consequently, to be represented by the formula,



and called *succinamic glycocoll*. Aspartic acid, generally expressed in name and formula as "amido-succinic acid,"



must in reality be considered as monobasic, inasmuch as its so-called normal salts containing alkali metals are easily decomposed by carbonic acid, and the only salts known in a definite state are the so-called acid salts. This body must, therefore, be regarded as *succinic glycocoll*, and represented by the formula,



The fact that a mono-ethyl ester is obtained by direct etherification of aspartic acid also goes to support this view of its constitution.

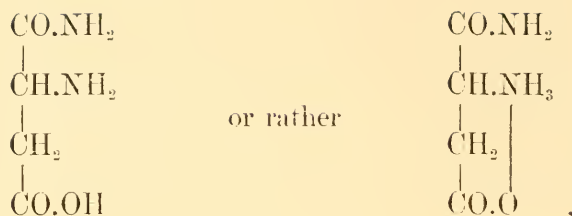
The above consideration leads to the prediction not only of the existence of two chemically isomeric asparagins, as is predicted by the current view,



but also of aspartic acids—

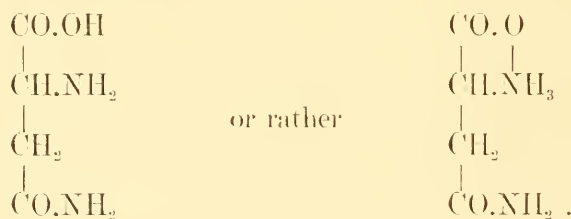


each of which is further capable of existing as dextro-rotatory, laevo-rotatory, and racemoid modifications. It must be frankly acknowledged, however, that all the aspartic acids known at present appear to possess one and the same constitution, the two active acids obtained from the ordinary and the sweet asparagins being optical isomers, and all the inactive aspartic acids obtained in various ways being identical among themselves and also with that produced on mixing the two active acids in equal quantities (Piutti : *Ber.*, **19**, 1684 [1886]; Engel: *Bull.*, **50**, 150 [1888]). There are, however, certain points in the chemistry of asparagin and aspartic acid, which require further investigation. The singular fact that, of the two asparagins which are regarded to be optical isomers, one is tasteless and the other exceedingly sweet, the fact also that these two active asparagins, mixed in equal quantities, do not produce an inactive racemoid compound, are some of the points above referred to. Moreover, the inactive asparagin obtained from monoethyl α -aspartate (m. p. 165°) and which, from its mode of formation, has to be represented by the formula,



has not yet been resolved into active components; but since van't Hoff's theory does not admit of the existence of an inactive amesotomic compound in the case of bodies containing only one asymmetric carbon atom, the mesotomism of the above inactive asparagin has yet to be achieved. It is not unlikely that the new active asparagins thus obtained would yield active aspartic acids different from those known at present.

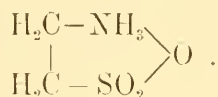
It is to be observed that the inactive asparagin is here assumed to be *chemically* isomeric with the two known active asparagins (*Cf.* Victor Meyer and Jacobson, *loc. cit.*), and since the former possesses the constitution above given, the latter should be



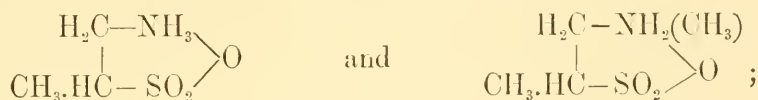
It is probable, however, that the inactive asparagin will prove to be the racemoid compound of the two known active asparagins, in which case an asparagin possessing the latter constitution has yet to be discovered. Our non-acquaintance of a second aspartic acid presents, therefore, no greater difficulties to the acceptance of the view here advocated as to the constitution of aspartic acids than our non-acquaintance of a second asparagin presents to the current theory. From this point of view, the existence of new aspartic acids is again, at

least, as probable as that of new asparagins. Time will show whether these predictions are verified or not by actual facts.

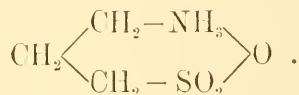
Returning to the question of nomenclature, the generic name *taurins* should be used in the case of the so-called amido-sulphonic acids, and different taurins distinguished by prefixing the names of the divalent hydrocarbon radicals. Thus, “ β -amido-ethyl sulphonic acid,” or “amido-isethionic acid,” terms which are intended to express the constitution of common taurin, should be replaced by *ethylene-taurin* (or simply, *taurin*) in accordance with the formula,



The “ β -methyl taurin” and the “ β - ν -dimethyl taurin,” described by Gabriel (*Ber.*, **22**, 2988 [1889]), would be respectively *propylene-taurin* and *methyl propylene-taurin*, and expressed by the formulae,



whilst the “ γ -amido-propyl sulphonic acid” (Gabriel and Lauer: *Ber.*, **23**, 92 [1890]) should be named *trimethylene-taurin*, and expressed by the formula,



The above examples would, I hope, suffice to show that the nomenclature of the “amido-acids” here suggested is capable of general application.

I have much pleasure, in conclusion, in tendering my best thanks to Dr. E. Divers, F.R.S., for kindly looking over this paper.



Addendum to the preceding paper.

June 1894.

In the *Proceedings* of the Chemical Society, London, issued May 2, 1894, there appeared an abstract of the preceding paper and, immediately following it, a note on the same subject but from a physical stand-point, by Dr. James Walker. Dr. Walker compared the electrical conductivities of glycoll, phenyl-glycoll, hippuric acid, and aceturic acid with that of acetic acid, and found that, whilst glycoll itself is an extremely feeble conductor, its derivatives are far better conductors than acetic acid. "Consequently", he says, "if acetic acid contains a carboxyl group, phenylglycocine (etc.) must, *a fortiori*, contain one." He then concludes with the words, "If we are to trust to analogy, therefore, the evidence afforded by the electrical conductivity goes to show that glycocine has not the ring constitution, but the ordinarily accepted constitution represented by the formula $\text{NH}_2 \cdot \text{CH}_2 \cdot \text{COOH}$."

In coming to this conclusion I cannot follow him. That glycoll derivatives, with their high dissociation constants, are to be regarded as open-chain compounds when dissolved in water is a statement from which I do not, indeed, dissent. But, then, by the very same reasoning glycoll itself must be regarded as a *closed-chain* compound, even when dissolved in water. Dr. Walker has not a word to say in explanation of the remarkable difference, which he has observed, between the non-conductivity of glycoll and the conductivity of its derivatives. Before attempting myself to find some explanation for it, I wish to emphasise the point that, if the electrolytic behaviour of glycoll proves anything as to its constitution, it is that

this body is not an open-chain compound,—is not a carboxylic compound, or acid.

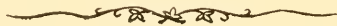
A probable explanation of the facts, and one which gives every support to the view of the cyclic constitution of the glycocolls seems obvious to me. It is that while glycoll itself appears not to form a hydrate when dissolved, its derivatives do form such hydrates, and these *addition* compounds will be, as pointed out in my paper, open-chain compounds. Ordinary betain, for example, is, by universal admission, $N(CH_3)_3OH.CH_2.COOH$, and anhydrous betain

$$\begin{array}{c} N(CH_3)_3-CH_2 \\ | \qquad \qquad | \\ O \text{-----} CO. \end{array}$$

I am not aware whether the electrical conductivity of betain has been determined or not, but it will doubtless prove to possess a rather high dissociation constant. Glycoll hydrochloride, an undeniable open-chain compound, was, some years ago, found by Dr. Walker himself to be an electrolyte.

Electrical conductivity of glycoll derivatives must, then, be regarded as most probably due to their forming addition compounds with water and their subsequent dissociation, but there is nothing to show that these derivatives are themselves open-chain compounds. On the contrary, the non-conductivity of glycoll rather points to its being close-chained (even in solution), and by analogy we may regard its derivatives as being also ring compounds.

I must, therefore, maintain with added force that all facts, *physical* as well as chemical, so far as known, point exclusively to the cyclic constitution of the glycocolls as the one in every way to be preferred.



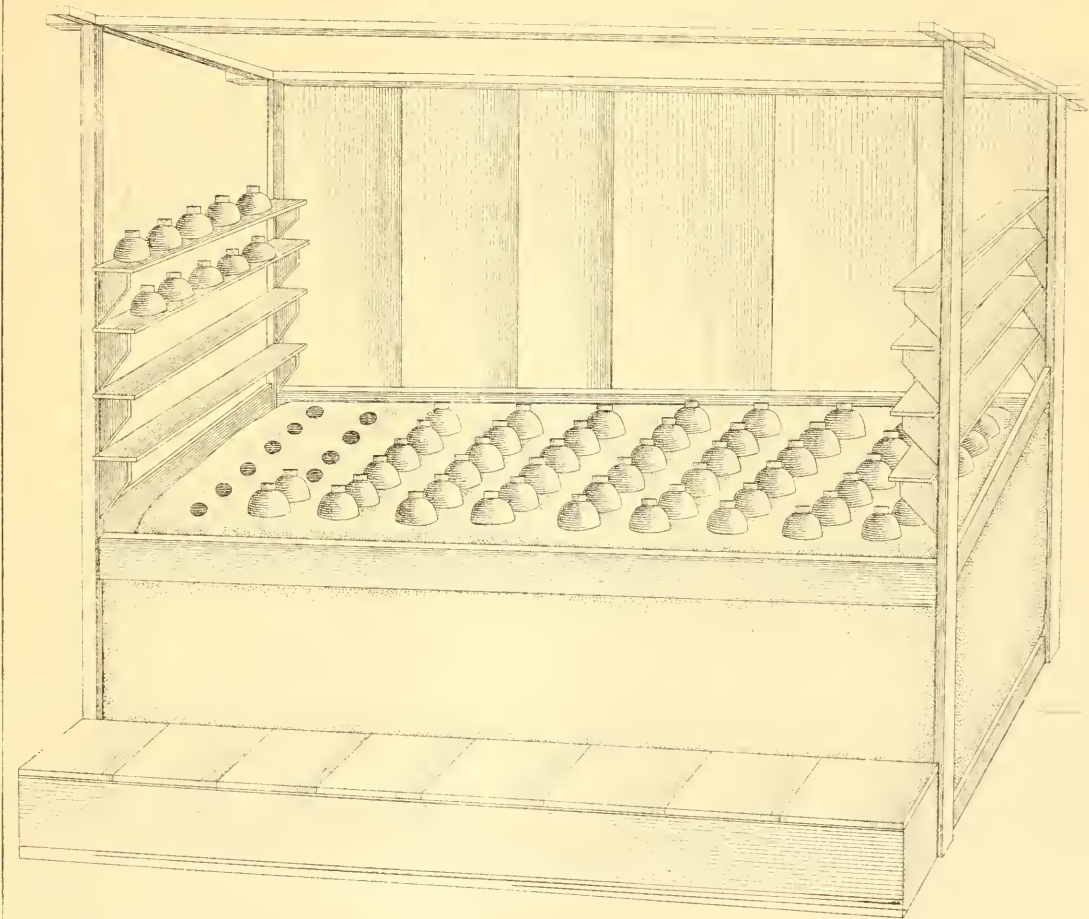


Fig. 1.

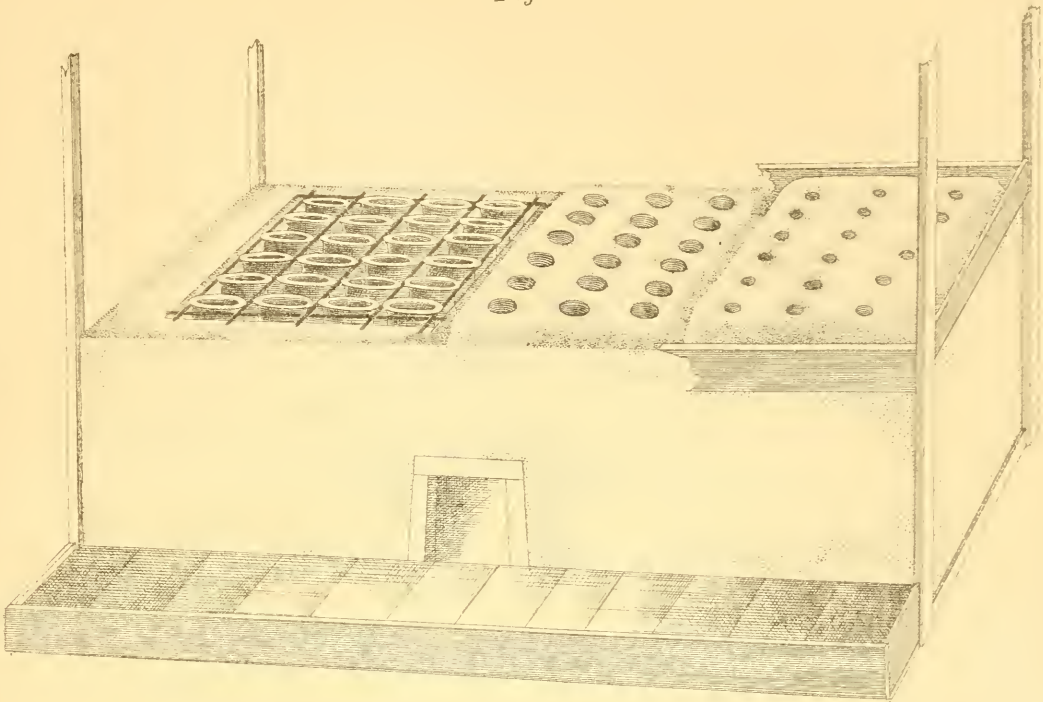
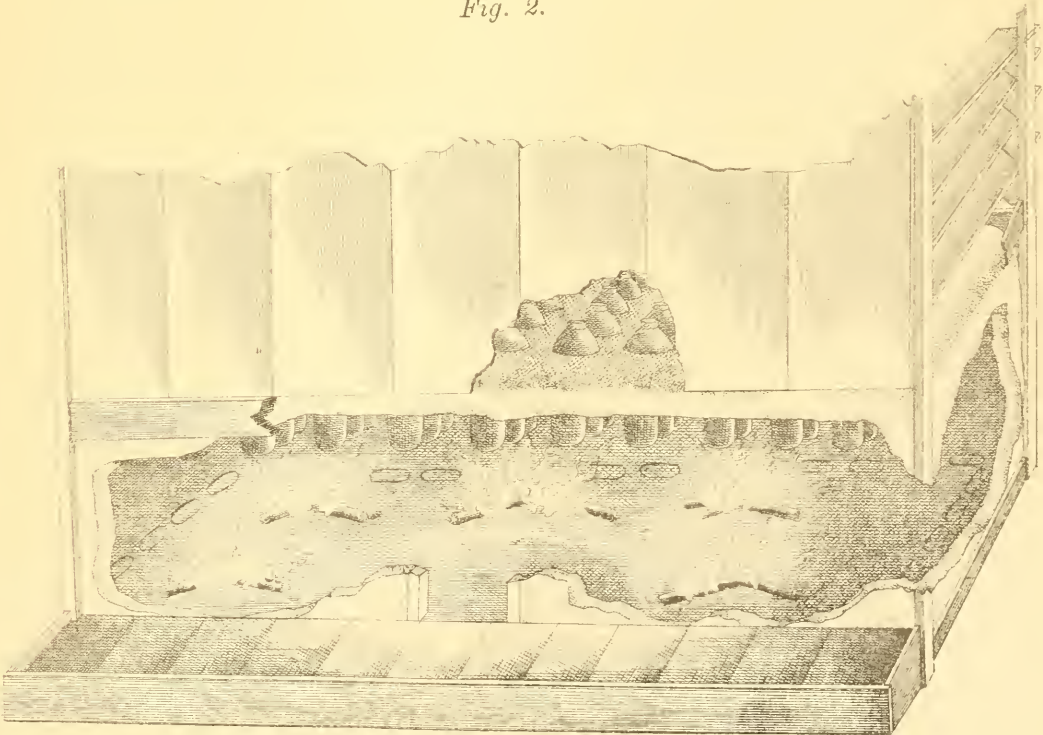


Fig. 2.



1.



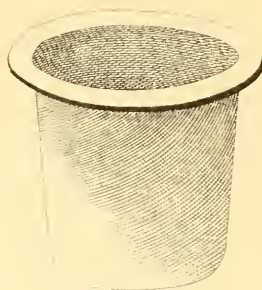
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3.



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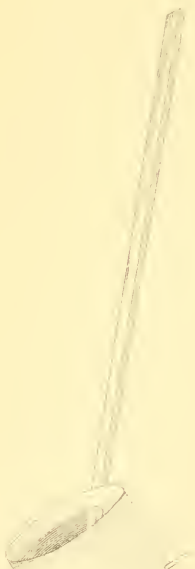
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On the After-shocks of Earthquakes.

by

F. Ōmori, *Rigakushi.*

I. General Considerations.

§ 1. A strong earthquake is almost invariably followed by weaker ones and when it is violent and destructive the number of minor shocks following it may amount to hundreds or even thousands. When after-shocks are not reported to have happened it is probably because they were deemed unimportant to record. Or it may be that the seat of origin of the earthquake being very deep or far out under the ocean-bed, the after-shocks did not reach the observer.

Complete records of after-shocks were obtained, I believe, for the first time in the cases of the three recent great earthquakes in Japan; namely, those of Kumamoto in 1889, of Mino and Owari in 1891, and of Kagoshima in 1893. The discussion of these records forms the subject of the present paper.

§ 2. The numbers, daily, monthly, etc., of the after-shocks of these three earthquakes, together with other matters relating thereto are contained in Tables I—XXII at the end of the paper; the shocks being distinguished as “violent,” “strong,” and “weak” (or “feeble”) according to their intensity; the total or aggregate intensity of a number of shocks is obtained by multiplying each shock by one of the coefficients 3, 2, 1, according to its intensity, and taking the sum of the numbers so obtained. The after-shocks of the Mino-Owari earth-

quake were recorded at the Meteorological Stations in Gifu, Nagoya, Tsu, Kyōto, and Ōsaka; those of the Kumamoto earthquake, first at the Prefectural Office, and subsequently at the Meteorological Station in Kumamoto; and those of the Kagoshima earthquake at the Police Station and the District Office in Chiran in Kagoshima Prefecture.

§ 3. The number of after-shocks are found to diminish rapidly with distance from the origin of the initial earthquake,¹ as may be seen from the following list:—

AFTER-SHOCKS OF THE MINO-OWARI EARTHQUAKE.						
Stations.	Gifu.	Nagoya.	Tsu.	Kyōto.	Ōsaka.	Tōkyō.
Number of shocks during the first two years.	3365 ⁴	1298 ⁴	314	125	70	30
Distance in ri^2 from the central part of the Neo-Valley. ³	7	15	25	25	36	68

Of the six cities mentioned in the above table, Gifu, Nagoya, and Tsu are situated nearly along the axis of the depression which extends from the Sea of Isé to the plain of Mino and Owari, while the remaining three, Kyōto, Ōsaka, and Tōkyō are situated in a line roughly perpendicular to this axis. These two sets of places must be regarded as distinct. The decrease with distance in the number of after-shocks is, in the case of the second group, seen to be more rapid than in the inverse ratio.

In the case of the Kumamoto and Kagoshima earthquakes, the

¹ The origin of the initial earthquake is not necessarily the same with those of the after shocks, so that in the immediate neighbourhood of the origins the statement in the text is by no means true.

² $1\ ri = 2.44$ miles or 3.93 kilometres.

³ The earthquake of October 28th, 1891 was most violent in the Neo-Valley. See Dr. Kotō's excellent paper, "On the Cause of the Great Earthquake in Central Japan, 1891." (Jour. Sci. Coll., vol. V. pt. IV); also § 35 of the present paper.

⁴ These numbers do not include the shocks within a few hours immediately after the earthquake, which were not recorded. See Tables XI and XII, and also § 17.

observing stations, Kumamoto and Chiran, may practically be considered as being on the respective epi-centres.

§ 4. Of the 3365 after-shocks recorded at Gifu during the first two years, 10 were "violent," 97 "strong," 1808 "weak," and 1041 "feeble," while in the remaining 409 only sounds were heard without shocks. From Tables IV and V, it will be seen that, in the days immediately succeeding the initial great earthquake, "violent" and "strong" shocks occurred very often, but that these became rarer as time advanced. Of the 10 "violent" shocks, 9 occurred within the first four months and the remaining 1 in September, 1892, twelve months after. The "strong" shocks occurred all within the first thirteen months, and the "weak" shocks all within the first twenty months. It thus seems that at the earthquake-origin great instabilities are removed quicker than small ones. Again, the numbers of "weak" and "strong" shocks at Gifu during the first two years are greater respectively than the total numbers recorded during the same time interval at Nagoya and Ōsaka. Assuming that the after-shocks all originated somewhere near Gifu,¹ it may be concluded that nearly two-thirds of these had radii of propagation less than 10 *ri* and that only one-fiftieth had radii greater than 35 *ri*.

§ 5. The number of after-shocks of the Kumamoto earthquake recorded at Kumamoto up to the end of 1893 is 922, including 1 "violent" shock which happened on August 3rd, 1889, five days after the initial earthquake, 76 "strong" shocks, and 845 "weak" and "feeble" shocks and sounds. These shocks all extended over small areas, only the "violent" one reaching a distance greater than 20 *ri* from the origin.

§ 6. In the cases of the Mino-Owari and Kumamoto earthquakes, all the after-shocks were very much smaller in extent than

¹ See § 33.

the initial shocks. In the case of the Kagoshima earthquake, there was a second great shock about four months after the first, but the two had quite different origins.¹

The frequency or activity² of after-shocks at a given distance from the origin is different for different earthquakes, and depends mainly on the magnitude of the initial shock, and at the epicentre, of course on the depth of the focus also. I may here state that the focal depths of the three great earthquakes we are discussing were all comparatively very small, being probably between 2 and 4 *ri*.

§ 7. The table following will serve for making a comparison of the frequency and activity of the after-shocks of the three earthquakes as observed at or near the epi-foci of the latter in the days immediately succeeding the initial shocks :—

COMPARISON OF FREQUENCY AND ACTIVITY OF AFTER-SHOCKS.												
(I) MINO-OWARI EQ.			(II) KUMAMOTO EQ.			(III) KAGOSHIMA EQ.			(IV) RATIO $\frac{(I)}{(II)}$		(V) RATIO $\frac{(I)}{(III)}$	
Date.	Freq.	Act.	Date.	Freq.	Act.	Date.	Freq.	Act.	Freq.	Act.	Freq.	Act.
Oct. 29th, 1891	318	333	July 20th, 1889	27	32	Sep. 8th, 1893	45	49	12	10	7.1	7.0
„ 30th, „	173	193	„ 31st, „	15	16	„ 9th, „	41	49	12	12	4.0	4.0
„ 31st, „	126	129	Aug. 1st, „	11	12	„ 10th, „	28	29	12	11	4.5	4.5
Nov. 1st, „	99	104	„ 2nd, „	8	9	„ 11th, „	23	23	12	12	4.3	4.5
Average.....									12	11	5.	5.

The Kumamoto earthquake took place on July 28th, 1889, at 11h. 49m., p.m.; the Mino-Owari earthquake, on October 28th, 1891, at 6h. 37m., a.m.; and the Kagoshima earthquake on September 7th, 1893, at 2h. 46m., a.m. Though the frequencies and activities of the first three groups, (I), (II), and (III), in the above table, which

¹ See § 32.

² “Frequency” means the number, and “activity” the total *intensity* defined as in § 2, of shocks during any given interval of time.

are put in the same horizontal rows, may not be strictly comparable with each other, yet the ratios in the columns (IV) and (V) are seen to be in each nearly constant. This will show that the law of decrease with time in the frequency or activity of the after-shocks was nearly the same, at least in the days immediately succeeding the initial earthquake, in each of the three cases.

The dependence of the frequency or activity on the magnitude of the initial disturbance is more clearly shown by the following table of total numbers and activities of after-shocks during the first 30 days and the first 2 years :—

Time Interval.	(I) MINO-OWARI EQ.		(II) KUMAMOTO EQ.		(III) KAGOSHIMA EQ.		(IV) RATIO $\frac{(I)}{(II)}$		(V) RATIO $\frac{(I)}{(III)}$	
	Number	Activity	Number	Activity	Number	Activity	Number	Activity	Number	Activity
First 30 days.	1746	1821	340	385	278	292	5.1	4.8	6.2	6.2
.. 2 years.	3365	3482	833	905			4.0	3.8		

Now the total (land and sea) areas of disturbance of the Mino-Owari, Kumamoto, and Kagoshima earthquakes were respectively about 54000, 6500, and 5000 square *ri*. The magnitudes of these earthquakes, therefore, if represented by their areas, are in the ratios of 11 : 1.3 : 1 approximately; i.e., the Mino-Owari earthquake was greater by eight or nine times than the Kumamoto earthquake, and by eleven times than the Kagoshima earthquake. The frequency or activity for the first earthquake is seen from the above table to be greater by four or five times than that for the second, and greater by six times than that for the third. If we compare together the two earthquakes of Kumamoto and Kagoshima, we find that the ratio of the frequencies or activities of their after-shocks for the first 30 days is 1.3, which is equal to the ratio of their areas of disturbance.

Thus the proportionality of the frequency or activity to the area

of disturbance is not strictly verified between these two earthquakes and the Mino-Owari earthquake, and the number of after-shocks for the last as recorded at Gifu seems to be comparatively too small. It may be that the town of Gifu is not sufficiently near to the district which is to be regarded as the proper centre of activity of after-shocks of the Mino-Owari earthquake. The effect that may be due to difference of focal depths must also be kept in mind.

Anyhow the frequency or activity of after-shocks increases with the magnitude of the initial earthquake and therefore can to a certain extent be regarded as a measure of the latter. Thus, for instance, the Kumamoto earthquake of July 28th, 1889, was followed, on August 3rd, five days later, by a second strong shock. According to the above criterion, the first shock was double or treble as great as the second, the numbers of their after-shocks being in the ratio of 90 : 30 approximately.

§ 8. To show the time relation of after-shocks, curves have been drawn, (Figs. 1,5,6,...), whose abscissae are equal intervals of time, and ordinates the numbers or activities of after-shocks during these intervals. The form of the curves drawn, in red, through the ultimate mean positions will be seen to be very like that of a rectangular hyperbola.

§ 9. The time rate of decrease of the frequency or activity of after-shocks is at first very rapid, but afterwards becomes slow and asymptotic. As far as the rate of variation is concerned, the after-shock curve may be regarded as consisting of two nearly straight portions, during the first of which the rate is great, and during the second of which it is small, the turning point occurring sooner with smaller earthquakes. In the case of the after-shocks of the Mino-Owari earthquake this transition took place ten or twelve days after the initial shock.

§ 10. To deduce theoretically the time-relation of the activity (or frequency) of after-shocks, it will be assumed, firstly, that the activity at any moment is proportional to the magnitude of disturbance in the geotechnic condition then existing at or near the origin of the initial earthquake; and, secondly, that the reduction of this magnitude depends on the corresponding activity of after-shocks; that is, it is assumed that the after-shocks remove so many points of instability or weakness at the origin. It must also be supposed that each of the after-shocks has its own series of secondary after-shocks depending on its magnitude.

Let y =the activity of after-shocks at any instant of time x ; m =the corresponding magnitude of disturbance at or near the origin of the initial earthquake, expressed in any arbitrary measure; and k, k' be constants. We then obtain the following two equations:—

$$y = k.m;$$

$$-dm = k'.y.dx - k''.y.dx,$$

whence, by integration, supposing k'' to be constant,

$$y = a.b^{-x}, \tag{a}$$

a and b being constants.

The logarithmic time-decrement of the activity of after-shocks seems to be a likely one, when considered from the analogy of certain physical phenomena, but the result obtained by applying equation (a) to the records of after-shocks of the three recent great earthquakes is not very satisfactory.

A formula which gives nearly satisfactory results is the following,—

$$y = \frac{k}{h+x}, \tag{b}$$

in which y is the frequency (or activity) at time x , k and h being

constants. This equation which represents a rectangular hyperbola may be deduced by taking terms only as far as the first power of x from either of the equations—

$$y = k(h+x)^{-1} + k'(h+x)^{-2} + \dots,$$

and

$$y = \frac{1}{p+qx+rx^2+\dots}$$

where h, k, k', \dots and p, q, r, \dots are constants.

Equation (b) implies that the frequency varies nearly in an inverse ratio to the time.

§ 11. *The Mino-Owari Earthquake.*

Let us apply equation (b) to the record of after-shocks taken at the Gifu Meteorological Station. As may be seen from Fig. 5, the curve of actual frequency presents a series of maxima and minima, and in deducing the mean values of the constants k and h from observations, the latter must either cover a number of such fluctuations, or be taken to be those at an early epoch when the rate of variation in the frequency is very great and little affected by periods of long duration.

Taking the half-daily numbers of earthquakes during the first five days, from October 29th to November 2nd, 1891,¹ and applying the method of Least Squares, we obtain the equation

$$y = \frac{440.7}{x+2.31}, \quad (c)$$

in which x denotes successive intervals of 12 hours, beginning with the first half of October 29th, 1891, and y the corresponding numbers of earthquakes. Actual and calculated values of y are tabulated below.

¹ See Fig. 6.

x	Date.	y , actual.	y , calculated.
0	1891, 10, 29, 0—12 a.m.	171	190
1	0—12 p.m.	147	133
2	30, 0—12 a.m.	109	102
3	0—12 p.m.	64	76
4	31, 0—12 a.m.	71	
5	0—12 p.m.	55	56
6	11, 1, 0—12 a.m.	58	
7	0—12 p.m.	41	42
8	2, 0—12 a.m.	46	
9	0—12 p.m.	46	

Now $x=493$ represents the half-day interval which is equally distant from the beginning and end of the year 1892, and $x=1225$ that of the year 1893. The values of y corresponding to these x as calculated from equation (c) are respectively 0.9 and 0.36. In the actual case, there were 867 and 308 shocks at Gifu during the above two years, giving the average half-daily numbers of 1.2 and 0.42.¹

It is interesting that equation (c), deduced from the observations of only a few days immediately succeeding the initial earthquake, thus represents with tolerable accuracy the frequency of earthquakes a year or two later on.

In the curve represented by equation (b), the point of the maximum curvature occurs at $x=\sqrt{k}-h$. Substituting the values of the constants k and h adopted in equation (c), we find $x=19$, denoting an epoch of time about ten days after the great earthquake (see § 9).

Again, by taking the monthly activities for the successive eighteen

¹ Assuming equation (a) and using the same data as in deducing equation (c), we obtain

$$\log y = 2.25 - x \times 0.106,$$

which gives satisfactory results when x is small, (thus, when $x=0$, $y=175$; when $x=1$, $y=133$; when $x=2$, $y=103$, etc.), but fails when x becomes great.

months, from November, 1891, to April, 1893, we get the following equation,—

$$y = \frac{16.9}{x + 0.397}, \quad (d)$$

in which x denotes time in months, the origin being November, 1891, and y the mean daily activity during the month x . Now $x=7.5$ represents the middle of the year 1892, and $x=19.5$ that of the year 1893. The corresponding values of y calculated from the above equation are respectively 2.1 and 0.84, the actual mean daily activities of after-shocks in the two years being respectively 2.4 and 0.84.

If we put $y=\frac{1}{36}$, equation (d) gives $x=510$ months or 42 years; and if we put $y=\frac{1}{60}$, equation (e) gives $x=27000 \times \frac{1}{2}$ days or 37 years. The meaning is that the seismic activity or frequency at Gifu due to the residual effect of the great earthquake after about forty years from the initial date may be such that one “weak” or “feeble” shock occurs in each month. Making great allowance for error of calculation and quartering the above figures, we may conclude that at least some ten years will elapse before the disturbed tract about Gifu can practically regain its stability, that is before the activity or frequency of after-shocks at that place reduces to the state of having one small shock per month.

The above conclusion, though only the result of rough approximations, seems a very likely one, when considered in reference to the Kumamoto earthquake of July 28th, 1889, which was far smaller than the Mino-Owari earthquake, and whose after-shocks are still occurring at the present day, about $4\frac{1}{2}$ years after.

§ 12. The variation with time of the frequency or activity of after-shocks of the Mino-Owari earthquake is comparatively simple, and none of these shocks was of a magnitude comparable with that of the initial earthquake itself. A few of the after-shocks, such as those of

January 3rd and September 7th, 1891, were pretty severe and followed by their own (secondary) after-shocks, fifty or more in number. Their residual effects were, however, of very short durations, being sensible only for a month or two. Besides, in deducing equation (*c*), all the after-shocks during the first eighteen months, some of which were due to the severe ones above named, were taken account of, and, therefore, the conclusion stated at the end of the last paragraph will not be materially affected by the occurrence at future times of similar severe after-shocks in the Mino-Owari district.

§ 13. *The Kumamoto Earthquake.*

The district about Kumamoto is steadily settling down to equilibrium, and there has thus far been no new great earthquake.¹ The rate of decrease of the frequency of after-shocks seems in this case to tend finally to be a little quicker than according to equation (*b*). The mean annual frequency at Kumamoto is very well represented by the following equation,—

$$y = \frac{1}{0.0048 + x \times 0.0021 + x^2 \times 0.0043}, \quad (c)$$

in which x denotes time, in years, 1890 being the origin, and y the corresponding yearly number of earthquakes. The values of y calculated from this equation for the years 1890, 1891, 1892, and 1893 are respectively 208, 89, 38, and 20, agreeing exactly with the actual numbers for these years.

According to equation (*c*), the numbers of earthquakes for the years 1894, 1895, and 1896 would be respectively 12, 8, and 5. Now, before the earthquake of July 28th, 1889, the average yearly number of shocks at Kumamoto had been 3 or 4. We may, therefore, conclude that it would be about seven or eight years from the date of

¹ See Fig. 4.

the initial earthquake before the number of shocks in the disturbed tract about Kumamoto can attain its original yearly average, if at all.

§ 14. We have seen (§ 7) that the number of after-shocks of the Mino-Owari earthquake during the first two years is about four times greater than that of the Kumamoto earthquake during the corresponding interval of time. If we now assume that the seismic frequency at Gifu after a given interval of time from the Mino-Owari earthquake is always greater by this ratio than that at Kumamoto after an equal interval from the Kumamoto earthquake, the result inferred from the analogy of the latter is that some nine or ten years will elapse before the yearly number of earthquakes at Gifu is reduced to ten or twelve (compare § 11).

The great earthquake of October 28th, 1891, must have removed the principal geotechnic instability which had existed beneath the Mino-Owari district, and in this part of the country the present epoch of seismic activity will be followed probably by one of rest. The average yearly number of earthquakes at Gifu before 1891 was about 15.

§ 15. I shall here remark that the space distribution of seismic energy as represented by the relative number of earthquakes during a given interval of time may vary from time to time, and particularly with the occurrence of great earthquakes. In general there is a coincidence between the distribution of destructive shocks and that of ordinary minor ones, and the latter may be the consequences of the former. In Japan the seismic activity, as far as small earthquakes are concerned, is much greater on the Pacific than on the Japan Sea side, and so is it with destructive shocks, of which 57 % took place on the Pacific side, 28 % on the Japan Sea side, and only the remaining 15 % remote from either. More especially the great shock of the 1st year of Ansei (1854), which affected severely

almost the whole of Japan, might have had some share in making the distribution of seismic energy in this country such as it is at the present day.

§ 16. *Digression on the Seismic Frequency in Tōkyō.*

In connection with this subject, it may be interesting to examine whether there has been in recent years a secular variation in the seismic frequency in Tōkyō.

The systematic instrumental observation of earthquakes at Tōkyō dates from the 8th year of Meiji (1875) and has now been continued for more than eighteen complete years. At first the record was taken by means of Palmieri's seismograph, but since 1887, it has been taken by means of the Gray-Milne seismograph. The numbers of earthquakes during eighteen years (1876-1893) are given in the following table:—

(1) RECORDED BY PALMIERI'S SEISMOGRAPH.				
Year.	Total Number of Eqs.	Tremors. ¹	Difference.	
1876	56	38	18	Average of "Difference." = 34.7 ²
1877	71	40	31	
1878	50	21	29	
1879	70	30	40	
1880	77	32	45	
1881	66	32	34	
1882	46	19	27	
1883	32	12	20	
1884	68	25	43	
1885	68	38	30	
1886	54	4	50	

¹ "Tremors" mean those shocks whose direction of motion was not distinctly shewn by Palmieri's Seismograph.

² The Central Meteorological Observatory, where the record has been taken, was moved in 1883 from its old position in the southern part of Tōkyō to its present one in the Castle grounds. The observation during that year might be imperfect and is rejected in taking the average number of earthquakes.

(II) RECORDED BY THE GRAY-MILNE SEISMOGRAPH.				
Year.	Total Number of Eqs.	Tremors. ¹	Difference.	
1887	80	44	36	Average of "Difference." = 33.3
1888	101	64	37	
1889	113	75	38	
1890	93	60	33	
1891	97 ²	49 ²	48	
1892	73	52	21	
1893	59	39	20	

As far as the total number of recorded earthquakes is concerned, the frequency appears to be greater in the later epoch, (II), than in the earlier, (I). But, as the record of the number of tremors or very small shocks may have been modified by the change in sensibility with the change of instrument we can safely take only those shocks whose motion was distinctly registered, and then we see that their average yearly number is nearly identical in the two groups (I) and (II).³

The Yedo (Tōkyō) earthquake of the 2nd year of Ansei (1855) took place 21 years before 1876, and 38 years before 1893. The above table shews that the residual effect of this earthquake had ceased to be sensible before 1876, and that the *mean* seismic frequency in Tōkyō has remained ever since practically constant. It may here be remarked that the intensity of motion in Tōkyō on the occasion of the earthquake of the 2nd year of Ansei was far less than that in the Neo-Valley and neighbouring tracts on the occasion of the recent Mino-Owari earthquake, the former earthquake being also smaller in extent than the latter.

§ 17. *On the Estimation of the Probable Total Number of After-*

¹ "Tremors" here mean those shocks whose motion was too small to be distinctly measured by the Gray-Milne seismograph.

² See Note to Table XVII.

³ See § 29.

shocks of a Given Earthquake.—If the time relation of the frequency of after-shocks of an earthquake be represented by an equation, we can readily calculate from it their approximate total number. Thus, if $y_0, y_1, y_2, \dots, y_n$ be values of y corresponding to $x=0, 1, 2, \dots, n$, we see that

$$\sum_0^{n-1} y_m > \int_0^n y \, dx > \sum_1^n y_m,$$

from which inequality we can approximately estimate the total number (or activity) of after-shocks, namely, $\sum_0^n y_m$, n being made suitably great. To take an example, we have, from equation (c), for the Mino-Owari earthquake,—

$$\sum_0^{1458} y_m > 440.7 \times \log_{10} \left(\frac{1459 + 2.31}{2.31} \right) \times \log_e 10 > \sum_1^{1459} y_m$$

or

$$\sum_0^{1458} y_m > 2840 > \sum_1^{1459} y_m,$$

in which y_0 denotes the number of shocks at Gifu during twelve hours, from 0 to 12 a.m., on October 29th, 1891, and is equal to 190, the total number of shocks during two complete years, from October 29th, 1891, to October 27th, 1893, being denoted by the sum $\sum_0^{1459} y_m$. The calculated value of the latter thus comes out to be 2900 or 2950. The corresponding actual number is 3257.

Similarly the calculated total number of shocks during ten years, or the sum $\sum_0^{7307} y_m$, is found to be some 3600.

Now the great earthquake of October 28th, 1891, took place at 6.37 a.m., and the record of after-shocks at Gifu was not taken till about 2 p.m. of the same day, the number recorded during the remaining 10 hours, from 2 to 12 p.m., being 101. The total number of shocks during the above initial day was probably not less than 300.

From these considerations I conclude that the entire number

of after-shocks of the Mino-Owari earthquake disturbing the *vicinity of Gifu* is about 4000.¹

Some of the great earthquakes in the world had areas of disturbance many times bigger than the Mino-Owari earthquake, and we may assume that 10000 is probably the highest possible total number of after-shocks of an earthquake.

The after-shocks of the Kumamoto earthquake are now approaching the end and their total number is 950 or 1000.

§ 18. *The Kagoshima Earthquake.*

The Kagoshima earthquake took place on September 7th, 1893, and is yet only a few months old. Making an estimate from the record already obtained of its after-shocks (see Figs. 15 and 16), we find that, at Chiran, the epi-focal tract, they may continue for three or four years, and that the total number may be some six hundred.

The earthquake took place at about 2.46 a.m. and the record of its after-shocks was not taken till about 9 p.m. of the same day, there being during this interval probably some 100 shocks. Making this addition, the total number of shocks at Chiran during about five months, up to the end of January, 1894, is nearly 480, being less than the number of shocks at Kumamoto during the corresponding interval of time after the earthquake of July 28th, 1889, which may be taken in round numbers as 600.

II. *On the Periodicity of the Frequency of After-shocks, etc.*

§ 19. What has been said so far about after-shocks relates to the ultimate mean time-relation of their frequency or activity. When

¹ It must be remarked that the conclusions regarding the after-shocks of the Mino-Owari earthquake thus far stated are supposed to hold for Gifu, and may not hold necessarily for other places which are not sufficiently near to the principal epi-focal tract.

examined particularly, however, there are to be seen in the variations of the latter various sets of periodic fluctuations (see Figs. 1, 2, 3, 5, etc.).

Earthquakes being isolated or discontinuous events from the nature of their causes, we can from analogy readily conceive why the frequency of those after-shocks which happen in close succession, should present a series of well-marked maxima and minima.

The *fluctuations in the decrease* of the frequency of after-shocks of an earthquake may be of two kinds; namely, those which are proper to the earthquake under consideration, and those whose maxima and minima occur at fixed epochs. The former are of the nature of forced oscillations and may disappear after a time; while the latter are of the nature of free oscillations and may become finally predominant.

The amplitude of the fluctuations would evidently increase with the magnitude of the earthquake.

As far as I can ascertain from the records of after-shocks there are, besides the diurnal and annual fluctuations, six different series of periods in the variation of the frequency, whose lengths range from a few hours to several months.

These various periods have been obtained by drawing curves through the mean positions of points whose abscissae are equal time-intervals of 1 hour, 2 hours, 6 hours, 1 day, 2 days, 5 days, 10 days, or 1 month, and whose ordinates are the numbers of earthquakes during the corresponding intervals. The results given below were obtained by a direct measurement from Figs. 1, 2, 3, 5, etc.

§ 20. *The Kumamoto Earthquake.*

The curves of daily and 2-daily earthquake frequencies (Figs. 2 and 3) respectively indicate periods whose average lengths are 4.6 and 12 days.

The curves of 5-daily and 10-daily earthquake frequencies seem to indicate periods whose average lengths are about 33 days and 3 months respectively.

The curve of monthly earthquake frequency (Fig. 1) indicates distinct fluctuations, of which there are seven between August, 1889, and December, 1893. The dates of maximum and minimum frequencies as given by the curve are as follows:—

Maximum.	Minimum.
October, 1889,	September, 1889, (?)
May, 1890,	February, 1890,
February, 1891,	November, „,
October, „,	August, 1891,
May, 1892,	February, 1892,
October, „,	August, „,
„, 1893.	between March and August, 1893.

The successive intervals between the earlier well-defined maxima or minima are from 7 to 9 months.

§ 21. *The Mino-Owari Earthquake.*

The curve of hourly earthquake frequency for Gifu (Fig. 10) indicates a period of 8 or 9 hours, and also a shorter one of about 4 hours. The former is clearly shewn in the curve of 2-hourly earthquake frequency (Fig. 11), which gives an average length of 9 hours.

The curve of hourly earthquake frequency for Nagoya (Fig. 12) indicates regular fluctuations, whose average length is about $4\frac{1}{2}$ hours, and amongst which prominent maxima, marked *a, b, c, d, e, f*, occur at successive intervals of nearly twenty-four hours, shewing an evident diurnal variation. The curve of 2-hourly earthquake frequency indicates a period of mean length of about 9 hours.

The curves of daily, 2-daily, and 5-daily earthquake frequencies

for Gifu (Figs. 7, 8, and 9) indicate respectively fluctuations, whose average length are $4\frac{1}{2}$, 12.3, and 33 days. Longer periods are not evident.

The curve of monthly earthquake frequency for Gifu (Fig. 5) indicates four maxima and minima between November, 1891 and December, 1893, whose dates are as follows.—

Maximum.	Minimum.
April, 1892,	June, 1892,
September, „	February, 1893,
April, 1893,	June, „
September, „	December, „

The intervals between successive maxima or minima are from 4 to 8 months.

§ 22. *The Kagoshima Earthquake.*

The curves of daily and 2-daily earthquake frequencies for Chiran (Figs. 16 and 17) indicate respectively periods whose average lengths are 4.4 and 12 days. The curve of 5-daily earthquake frequency (Fig. 18) seems to indicate a period whose average length is about 33 days.

The Kagoshima earthquake is not yet sufficiently old to give indications of longer periods. The after-shocks of this and of the Kumamoto earthquake were not numerous enough to enable us to draw curves of hourly frequencies.

§ 23. We have before noted that a few severe after-shocks are likely to be followed by their own after-shocks. It might be supposed that the maxima, which occur in the curves of monthly earthquake frequencies (Figs. 1 and 5) represent merely the effects of such shocks and not the real fluctuations in the residual effect of the initial earthquake itself. But the fact is, on the contrary, that strong shocks

occurred when the frequency was going to reach a maximum, as with the earthquakes of January 10th, 1894, and of September 7th, 1892 (see Figs. 5 and 26). In the case of the latter earthquake, a maximum frequency took place indeed at Gifu, which was quite near the origin, in the same month (September); but, at Mitake, a town about 7 *ri* from the origin, greater numbers of shocks were recorded in the following two months. The three other maxima of frequency for Gifu which took place on April, 1891, and April and September, 1893, were accompanied by no particularly severe shocks. Similarly with the maxima in the monthly earthquake frequency for Kumamoto.

In the case of the Mino-Owari earthquake, some of the after-shocks are doubtless to be regarded as “fore-shocks” of the stronger ones which followed.

§ 24. *The Diurnal Fluctuation of the After-shock Frequency.*

The diurnal and annual fluctuations of the earthquake frequency have been discussed by various European investigators. The results obtained by Perry, Mallet, and others, however, are more or less doubtful, as they were chiefly based on statistics of vulgar records. In the present instance, after-shocks have been recorded at meteorological stations provided with seismographs, and the results here deduced should have therefore far greater weight than those hitherto obtained.

The curves of six-hourly earthquake frequencies for Gifu and Nagoya (Figs. 13 and 14) indicate the daily fluctuation very clearly.

The Mino-Owari earthquake took place on the morning of October 28th, 1891, and during the next thirteen complete days, from October 29th to November 10th, there were 1258 shocks at Gifu and 572 shocks at Nagoya. The distribution of these shocks in the twenty-four hours of the day are shewn in Fig. 19, (1) and (2) (see Tables XIII and XIV).

The curve of the diurnal earthquake fluctuation for Gifu (Fig. 19. (1)) indicates distinctly *three maxima*, which occur respectively between 4 and 5 a.m., between 11 a.m. and noon, and between 6 and 7 p.m., and *three minima* which occur respectively between 9 and 10 a.m., between 3 and 4 p.m., and at 11 p.m. The intervals between successive maxima are 7, 8, and 9 hours, and those between successive minima 6, 8, and 10 hours.

The corresponding curve for Nagoya (Fig. 19. (2)) indicates six more or less distinct oscillations, giving a mean period of 4 hours.

It is evident (see § 21) that both the 4-hour and the 8-hour periods existed together, but that the longer period predominated in the diurnal earthquake frequency for Gifu and the shorter one in that for Nagoya.

The curves drawn in red (Fig. 19. (1) and (2)) seem to indicate a very slight diurnal variation of the *mean* frequency, the minimum occurring between 1 and 2 p.m., and the maximum at about 1 a.m.

The hourly distributions of 148 shocks at Kumamoto during fourteen days, from July 31st to August 13th, 1889, and of 233 shocks at Chiran during an equal time interval, from 8th to 21st, September, 1893, (see Tables III and XVI), are, as shewn in Fig. 20, very similar respectively to those for Gifu and Nagoya.

§ 25. *The Annual Fluctuation of the After-shock Frequency.*

Curves (1), (2), and (3), Fig. 21, shew the annual fluctuation of after-shock frequency at Kumamoto averaged respectively for four years (1890-1893), three years (1891-1893), and two years (1892-1893). The first curve indicates *three maxima* which occur respectively in March, May, and October; and *three minima* which occur respectively in April, between August and September, and in December. The curves drawn in red indicate the annual variation of the *mean* frequency.

The Mino-Owari and the Kagoshima earthquakes are not yet sufficiently old to give positive results respecting the annual fluctuation.

§ 26. We have, in §§ 20, 21, and 22, found various periods of the after-shock frequency whose lengths vary from a few hours to several months. The five periods of about 4 hours, 8 or 9 hours, $4\frac{1}{2}$ days, 12 days, and 33 days, seem to occur constantly in after-shocks of different earthquakes. Besides these, there may exist also a constant period of some three months, (see § 20). But the longest period is different in the cases of the Mino-Owari and the Kumamoto earthquakes, the average length being about 6 months in the former and 8 months in the latter.

To see whether these periods and the diurnal and annual fluctuations stated in §§ 24 and 25 can be identified in the frequency of *ordinary* earthquakes, we shall next consider the seismometrical observations in Tōkyō and over the whole of Japan.

§ 27. (a) *Seismometrical Observations in Tōkyō.*

The hourly distribution of 1168 earthquakes recorded instrumentally at Tōkyō during sixteen years, from 1876 to 1891, (Table XVIII),¹ is shewn in Fig. 19. (4). The diurnal fluctuation presents *three maxima*, which occur respectively between 9 and 10 a.m., between 3 and 4 p.m., and between 8 and 9 p.m.; and the *three minima*, which occur respectively between 2 and 3 a.m., between 11 a.m. and noon, and between 6 and 7 p.m. The intervals between successive maxima are 6, 5, and 13 hours, and those between successive minima 9, 7, and 8 hours. The mean curve, drawn in red, seems to indicate a very slight diurnal variation, of which the maximum occurs in the evening and the minimum in the early morning.

The monthly distribution of 1300² earthquakes instrumentally

¹ Tables XVIII and XIX are taken, by permission, from the Earthquake Report of the Central Meteorological Observatory.

² See the note to Table XVII.

recorded at Tōkyō during eighteen years, from 1876 to 1893, (see Table XVII) is shewn in Fig. 21, (4). The curve presents three maxima which occur respectively in March, May, and December; and three minima which occur respectively in January, April, and August or September.¹

(b) *Seismometrical Observations over All Japan.*

The hourly distribution of 3842 earthquakes in Japan during six years, from 1885 to 1890, (see Table XIX), is shewn in Fig. 19, (3). The diurnal fluctuation presents again *three maxima*, which occur respectively between 2 and 3 a.m., between 2 and 3 p.m., and between 10 and 11 p.m., the successive intervals being 12, 8, and 4 hours; and *three minima*, which occur respectively between midnight and 1 a.m., between 8 and 9 a.m., and between 5 and 6 p.m., the successive intervals being 8, 9, and 7 hours. The mean curve, drawn in red, seems to indicate a slight variation, having a maximum in the early morning and a minimum in the evening.

Fig. 19, (5) shews the hourly distribution of 5333 earthquakes in Japan, including the after-shocks of the Mino-Owari and the Kagoshima earthquakes, (see Table XX). The character of the curve is very similar to that in Fig. 19, (3) above described.

The monthly distribution of earthquakes during the same six years² is shewn in Fig. 21, (5). The annual fluctuation indicates *three maxima*, occurring respectively in February, May, and November; and *three minima*, occurring respectively in April, August, and December.

§ 28. *Conclusions.*

Three distinct maxima and minima occur in the diurnal fluctuation of the frequency of after-shocks as well as in that of ordinary

¹ The maximum in March and the minimum in April, which are here very slight, are markedly shewn in the Kumamoto curve, Fig. 21, (1).

² See Table XIX.

earthquakes. The hours at which these occur seem to be different for different localities and therefore these may not each be shewn, when we mix up earthquake records from distant places of the world together. Whether there are more earthquakes during the night than during the day is not certain, as may be inferred from the mean curves drawn in red.

The 4-hourly and 8-(or 9) hourly periods indicated in the curves of hourly earthquake frequency (Figs. 10, 11, and 12) are evidently due to the above diurnal fluctuation. We have not at present sufficient data to determine whether other three constant periods of $4\frac{1}{2}$, 12, and 33 days occur likewise in the frequency of ordinary earthquakes.

With regard to the annual fluctuation, it is to be noted that all the maxima and minima of the monthly after-shock frequency for Kumamoto (§ 20), with the two exceptions next mentioned, occurred in exactly, or nearly, the same months as the maxima and minima of frequency for all Japan (§ 27, (*b*)). Only the second and third minima at Kumamoto (§ 20) took place respectively in February and November, (1890), which are generally months of maximum earthquake frequency. These may denote fluctuations due to the "proper" period of the Kumamoto after-shocks.

Again, the four maxima of the monthly frequency for Gifu (§ 21) occurred in April and September, which are generally months of minimum earthquake frequency. These may denote the fluctuations due to the "proper" period of the Mino-Owari after-shocks. Of the four corresponding minima, three occurred in months of minimum frequency, and one in a month of maximum.

As already remarked, strong shocks seem to have a tendency to happen when the frequency is going to reach a maximum, and it is interesting to find that of the four severest *after-shocks* (three of the

Mino-Owari, and one of the Kagoshima earthquake) three took place in January and one in September. I believe that periodicity plays a very important part in the frequency of earthquakes, and its attentive study may be of help in the prediction of changes in seismic activity and other events.

I shall here confine myself to merely stating the facts. Theoretical speculations on this interesting branch of the earth's physics are reserved for a future occasion, when more materials respecting after-shocks shall have been obtained.

§ 29. *Again of the Seismic Frequency in Tōkyō, etc.*

As seen above, after-shocks indicate many periods in the variation of the seismic frequency whose lengths range between a few hours and one year. It is of course possible that there should exist periods of still longer duration. We shall here again consider the seismic record taken by instruments in Tōkyō during eighteen years from 1876 to 1893.¹ The curve of monthly earthquake numbers, a portion of which is shewn in Fig. 22, indicates very clearly the existence of the annual period. There seem to exist also fluctuations of a few months' duration, of which, for instance, I can count more or less distinctly—

10 during 39 months between January, 1876, and April, 1879;
 15 „ 50 „ „ September, 1882, and November, 1885;
 and 15 „ 50 „ „ November, 1885, and January, 1891;
 The average length of the period is 3.5 months.

The curve of yearly earthquake numbers at Tōkyō (Fig. 23) shews fluctuations, the maxima of which occurred in the years 1880, 1884-5, and 1889, and the minima in the years 1878, 1883, and 1886, there being besides these a slight maximum in 1891. The average length of the period is 4 years.

¹ See Table XVII.

The mean curve (drawn in red), Fig. 23, seems to indicate the existence of a period¹ whose length is some 12 years.

One interesting fact is that the three severest of all the earthquakes recorded at Tōkyō during the eighteen years, namely, those of February 2nd, 1880, of October 15th, 1884, and of February 18th, 1889, happened respectively in the years which mark, exactly or nearly, the epochs of successive maximum frequency. It may likewise be the case that the maxima of the longer (12 years) period may often be marked by great earthquakes.

It may hereby be remarked that the greatest monthly earthquake number, namely 45, was recorded in October, 1891. Of these, 28 happened within four days immediately after the great Mino-Owari earthquake (which took place on the 28th of the same month), and the remaining 17 before the latter, so that the proper number of earthquakes in Tōkyō may be assumed to have been $17 \times \frac{3}{2}$, or 19.² Of the other 225 monthly numbers the next greatest namely 18, occurred in April, 1889.

A good example of the occurrence of destructive earthquakes at rather regular intervals is found in the seismic frequency for Kansu, a Department in the north-western corner of China, where the intervals between successive paroxysmal epochs range from 15 to 64 years, giving the mean length of 33 years.³

To discover periodicity in seismic frequency we must treat records of earthquakes for different localities separately. By way of example, in Fig. 23, (*A*), (*B*), (*C*) and (*D*), are shewn the curves of yearly seismic activity during 19 years, 1865 to 1883, in Switzerland, the Vesuvian

1 If the sensibilities of the seismographs by which the record of shocks has been made have been different in the earlier and later years, the chief consequence will have been to affect slightly the amplitude.

2 At Gifu only one earthquake had been recorded in October, before the 28th.

3 See the present author's paper, "On Chinese Earthquakes," *Seis. Jour.* Vol. I.

district, Sicily, and in the Balkan Peninsula and neighbouring Islands.¹ The curves for the Vesuvian district and Sicily shew each a series of periods of about 5 years; and that for Switzerland shews one well-defined of 12 years besides some ill-defined fluctuations of shorter average length. It will be observed that the maxima and minima of seismic activity for the two Italian districts occurred simultaneously, but at quite different epochs from those for Switzerland.

§ 30. To investigate the relations, if any, between earthquakes and the phases of the moon, sun-spots, temperature of the atmosphere, etc., seems not likely to lead to valuable results and would be, as Mallet remarked, a waste of scientific time and labour. With atmospheric changes of pressure it may be different and I shall here, therefore, treat shortly of the possible connection of the barometric height with the frequency of after-shocks.

In Table IV are given the mean barometric heights and the fluctuations during successive days from October 28th, 1891, to April 30th, 1892. It seems that earthquakes happen equally often with low as with high pressures. Thus from an examination of the record of the daily seismic frequency at Gifu, I can count, between the above dates, 55 maxima and 55 minima in the frequency, and the means of the barometric heights corresponding to these two sets are respectively 762.64 and 763.34 mm., which are practically identical.

Again, big barometric falls of 10 or 20 mm. or rises of 5 or 10 mm. were not accompanied by any marked change in the seismic frequency.

A single abrupt change in the atmospheric pressure is not likely to be accompanied by any fluctuation in the frequency of earthquakes. If, however, barometric changes, whether small or great, occur at regular intervals, then the earth's crust may finally assume certain corresponding oscillations. Thus the daily and annual fluctuations in

¹ The data are taken from Fuchs' "Statistik der Erdbeben von 1865-1885"; see Table XXI.

the seismic frequency may partly be due to those in the atmospheric pressure. Especially are the curves of the annual barometric and seismic fluctuations very similar to each other.

III. On the Distribution of After-shocks, etc.

§ 31. We shall lastly consider more particularly the magnitude of after-shocks and their distribution.

Earthquakes are produced when strains in the earth's crust reach a certain limit, and, as a very great shock would remove a correspondingly great underground instability, it is probable that such a shock would not, for a long time, be followed by another of a magnitude comparable to its own, in the same or a neighbouring district. When, however, the initial shock is not very great, it may be followed by another like it, and, even in this case, the position of the origin of the second shock would usually be quite distinct from that of the first.

The above statements can well be illustrated by the four recent destructive earthquakes in Japan, namely, those of Mino-Owari, of Noto, of Kumamoto, and of Kagoshima; the three last ones were much smaller than the first, which was indeed very great and violent.

§ 32. The Kumamoto earthquake of July 28th, 1889, was followed five days later, on August 3rd, by a second shock, which was, however, as we have already seen (§ 7) only one-half or one-third as great as the first. All the other after-shocks were much weaker.

The Noto earthquake of December 9th, 1891, was followed, two days later, on the 11th of the same month, by a second severe earthquake. These two had nearly an equal area of disturbance, and the intensity of motion near the epicentres was almost the same. Their origins were, however, different, the epi-centre of the first shock being at a point, latitude $37^{\circ}4' N$, longitude $136^{\circ}40' E$, or in the sea at about

$1\frac{1}{2}$ or 2 *ri* to the SSW of the Togi-mura, Hagui District (Noto province); and that of the second shock at a point about 2 *ri* to the SSE of that of the first, also in the sea and near to the town of Takahama in the same district. All the shocks in Noto which followed these two were small.¹

The Kagoshima earthquake of September 7th, 1893, was followed, on January 4th, 1894, about four months later, by a second shock whose area of disturbance was a little greater than that of the first. These two shocks again originated from different centres. The epicentre of the first shock was inland and near Chiran-mura in the Kiire District, at about 7 *ri* to the SSW of Kagoshima; while that of the second shock was in the sea at about 2 *ri* to the west of Nomatsuki, the distance between the two epi-centres being about 10 *ri*. The damage caused by the first shock was much greater than that by the second. The effect of the second shock on the frequency of earthquakes at Chiran was merely to increase slightly the amplitude of the period of seismic activity during which it occurred (at the epoch marked *a*, Fig. 15).

In the case of the Mino-Owari earthquake, all the subsequent shocks, more than 3000 in number, were far smaller than the initial one itself. It is to be remarked that the three severest of these numerous after-shocks, namely, the earthquakes of January 3rd and September 7th, 1892, and of January 10th, 1894, all originated in the Mino-Owari Plain, and not in the Neo-Valley, wherein indeed no very strong shock has ever occurred since the date of the first great one.

§ 33. *The Mino-Owari Earthquake.*

It thus seems that the Neo-Valley, or the principal epi-focal tract, is steadily settling down to equilibrium, while the Mino-Owari Plain, having probably lines of weakness under it not completely

¹ Unfortunately the after-shocks of the Noto earthquake were not carefully recorded.

removed by the earthquake of October 28th, 1891, has been affected by the severe shocks mentioned above. It may be that the latter tract is still to be disturbed in future by a few such shocks, which, however, will then be of only secondary magnitude and not so violent as to destroy houses.

Soon after the great earthquake of October 28th, 1891, temporary seismological observatories were established at Ōgaki and Midori to cooperate with the Meteorological Stations of Gifu and Nagoya. I have myself passed, in the latter part of November and the early part of December, 1891, several days at Midori and Ōkawara, the latter of which is a small village about 5 *ri* to the NNW of the former. Again the town of Gifu is about 7 *ri* to the SSE of Midori, which is in the central part of the Neo-Valley, where remarkable faults have taken place. In the following table are compared the daily numbers of earthquakes I have recorded at Midori and Ōkawara with those observed at Gifu :—

Date.	Place.		
	Gifu.	Midori.	Ōkawara.
November 22nd, 1891	12	7	...
23rd	23	9	...
...
25th	9	...	5
26th	15	...	15
27th	11	...	10
28th	16	...	7
29th	19	...	6
30th	14	3	...
December, 1st, „	7	2	...
2nd	16	7	...
3rd	17	4	...

Date.	Place.	Gifu.	Midori.	Ōkawara
December, 4th, 1891		18	11	...
5th		32	3	...
6th		7	6	...
7th		8	5	...
8th		19	4	...
9th		22	6	...
10th		22	8	...
11th		14	5	...
12th		23	12	...
13th		14	2	...
14th		23	11	...

The results contained in the above table are graphically represented in Figs. 24 and 25. Fig. 24 shews that the number of earthquakes at Ōkawara was a little less than that at Gifu, and that the seismic frequencies at the two places did not synchronize in the occurrence of maxima and minima. Fig. 25 shews that the seismic frequency at Gifu synchronized in the occurrence of maxima and minima with that at Midori, but that at the latter place it was generally less than half what it was at the former.

From the above we may conclude that the after-shocks which were felt at Midori were in the main the same as those felt at Gifu, but that the latter place was much nearer to the principal centre of these shocks than the former; also that the shocks which were felt at Ōkawara had mostly proceeded from a centre different from and less active than that of the shocks at Gifu.

Thus it seems that the activity of after-shocks at the principal epi-focal tract was distinctly less than at the regions to its north and south. Again, from what I can infer from information or from my

own experience while travelling in the valleys of the Ibi-kawa and the Mugi-gawa, which are respectively to the west and east of the Neo-Valley, the activity of after-shocks in these two valleys was less than in the latter.

It is difficult to determine the exact position of the most active centre of after-shocks, but it was situated somewhere to the south of the Neo-Valley for some time immediately succeeding the great earthquake of October 28th, 1891, quite near to Gifu and probably in the tract adjoining the town on the west.

Below are tabulated the monthly numbers of earthquakes recorded, from November, 1891, to February, 1894, at various places in the three provinces of Mino, Owari, and Mikawa;—

Year, Month.		1891			1892													
Place, District.		XI	XII	Sum	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Sum	
(MINO.)																		
Hachiman, Gijō		139	36	175	18	9	9	3	2	2	3	1	5	3	2	4	61	
Kōzuchi, Mugi		304	91	395	21	10	11	7	3	3	5	8	9	10	13	6	106	
Takatomi, Yamagata		318	98	411	22	11	18	13	2	3	3	3	7	14	17	7	120	
Kitagata, Motosu		154	40	194	17	10	8	4	2	2	2	1	2	4	2	4	60	
Ibi, Ono		152	40	192	13	10	8	4	2	2	2	1	1	3	2	4	52	
Tarui, Fuwa		137	48	185	12	10	8	5	2	2	2	1	1	4	2	4	53	
Takata, Tagi		232	60	293	16	13	10	7	2	2	2	2	2	5	1	5	70	
Takasu, Shimo-Ishizu		260	66	332	16	13	10	7	1	2	2	2	2	5	4	6	70	
Ogaki, Ampachi		110	128	238	17	17	12	7	2	2	2	2	2	5	4	6	78	
Gifu		1087	416	1503	164	114	87	90	54	30	35	52	107	47	48	39	867	
Ota, Kamo		428	143	571	62	36	22	21	10	8	9	13	17	21	22	13	254	
Mitake, Kani		486	166	652	82	55	32	26	17	14	9	17	18	25	24	14	333	
Takayama, Toki		310	78	388	38	24	19	11	6	6	3	10	11	11	12	10	161	
Nakatsugawa, Ena... ..		182	31	213	20	11	11	5	1	4	2	5	7	9	4	2	81	
(OWARI.)																		
Ōtajima, Haguri		533	206	739	80	45	25	28	22	9	13	28	33	33	43	30	389	
Inazawa, Nakajima		575	204	779	74	45	28	14	9	5	10	15	17	24	21	18	273	
Tsushima, Kaito		462	126	588	31	27	16	10	5	4	4	9	11	13	17	11	158	
Koori, Niwa		468	217	685	150	52	36	55	53	50	52	46	64	57	69	43	727	
Biwajima, Nishi-Kasugai		591	215	806	141	51	36	48	44	44	49	42	54	54	55	40	658	
Katsukawa, Higashi-Kasugai.		456	178	634	139	49	35	47	44	44	49	42	54	53	55	37	648	
Nagoya		416	113	529	43	29	16	11	11	12	4	15	13	8	12	14	188	
Atsuta, Aichi		294	72	366	31	29	14	9	8	10	4	15	11	13	18	14	176	

Place, District.	1891			1892													
	XI	XII	Sum	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Sum	
Handa, Chita	177	42	219	15	15	7	6	4	4	2	5	1	4	5	6	77	
(MUKAWA.)																	
Chiryū, Aomi	194	47	241	17	13	6	6	3	3	3	5	5	6	6	6	79	
Okazaki, Nukada	175	13	218	19	12	8	6	3	4	3	5	5	1	5	5	79	
Nishio, Hazu	142	36	178	14	11	7	5	2	4	1	6	5	5	5	4	69	
Gou, Hoi	98	26	124	8	6	2	3	...	4	1	2	4	3	2	4	40	
Toyohashi, Atsumi...	72	19	91	6	5	1	1	...	3	1	1	1	2	1	3	25	
Koromo, Nis' i-Kamo ...	288	75	363	29	24	16	9	4	6	3	11	11	10	15	11	149	
Asuke, Higashi-Kamo ...	287	70	357	37	27	23	11	5	6	3	10	10	8	11	12	163	
Shinshiro, Minami-Shidara ...	138	33	171	14	8	3	3	...	4	1	2	5	3	3	4	50	
Tomicka, Yana	58	17	75	5	5	1	3	2	2	1	3	22	
Taguchi, Kita-Shidara...	135	32	167	13	9	2	4	2	3	1	2	5	3	3	4	51	

Place, District.	1893													1894	
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Sum	I	II
(Mino.)															
Hachiman, Gujō	3	2	1	...	1	2	1	1	11	4	1
Kozuchi, Mugi... ..	5	6	1	...	3	1	4	...	1	2	1	...	24	3	...
Takatomi, Yamagata	5	3	5	2	4	2	5	2	7	8	5	10	58	12	6
Kitagata, Motosu	12	...
Ibi, Ōno	2
Tarui, Fuwa	1	1	1	1	1	...	5	6	...
Takata, Tagi	1	1	2	...
Takasu, Shimo-Ishizu	1	1	1	3	11	1
Ōgaki, Ampachi	1	2	1	4	5	1
Gifu	31	20	52	59	32	12	18	13	20	19	16	16	308	62	14
Ōta, Kamo	28	...
Mitake, Kani	14	11	9	10	13	9	8	7	8	3	8	7	107	57	14
Takayama, Toki	3	3	3	2	3	2	1	...	3	1	21	8	1
Nakatsugawa, Ena	2	3	5	3	...	1	...	1	2	3	21	13	...
(OWARI)															
Otajima, Haguri	28	12	19	17	19	4	5	13	4	11	5	13	150	341	49
Inazawa, Nakajima... ..	11	5	12	4	5	1	1	5	4	3	2	3	56	64	7
Tsushima, Kaito	5	3	1	1	2	1	1	1	1	2	18	17	3
Koori, Niwa	12	40	48	33	24	20	15	13	23	35	25	32	350	366	95
Biwajima, Nishi-Kasugai	9	6	12	6	7	2	2	6	2	6	3	4	65	39	10
Katsukawa, Higashi-Kasugai.	6	5	8	6	3	2	1	2	...	4	3	5	45	62	6
Nagoya	5	5	10	11	14	9	14	14	8	9	4	7	110	89	29
Atsuta, Aichi	4	6	4	3	2	1	...	3	1	2	2	2	30	25	7
Handa, Chita	1	1	2	1	2	1	1	...	1	...	2	1	13	4	3
Maegasu, Kaisai	2	5	2	2	1	1	1	1	15	14	2
Toyohama, Chita	1	1	...	1	1	4	7	...

Place, District.	1893														1894	
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Sum	I	II	
(MIKAWA).																
Chiryū, Aomi	1	2	2	3	1	1	1	...	1	...	2	1	15	13	1	
Okazaki, Nukada	3	2	1	1	2	2	1	...	1	2	15	8	1	
Sakushima, Hazu	1	2	1	1	5	5	...	
Nishio, Hazu	1	1	1	...	1	1	...	2	2	9	8	2	
Horikiri, Atsumi	1	1	2	4	...	
Gou, Itoi	1	1	1	1	1	2	1	...	1	...	9	6	...	
Toyohashi, Atsumi	1	1	1	...	1	4	3	...	
Koromo, Nishi-Kamo	6	2	3	5	4	2	...	2	2	1	1	5	32	20	9	
Asuke, Higashi-Kamo	8	2	3	2	5	2	1	2	1	3	29	40	8	
Shinshiro, Minami-Shidara...	4	1	1	1	4	2	1	2	16	13	1	
Tonioka, Yana	1	1	1	1	1	1	1	...	7	5	...	
Taguchi, Kita-Shidara...	...	2	1	1	1	1	2	1	9	10	...	
Shimoda, Kita-Shidara...	...	1	1	1	1	1	1	...	1	2	9	16	...	

Earthquake reports from District Offices and other stations in the Aichi Prefecture, *i.e.*, in the two provinces of Owari and Mikawa, have been sent in satisfactorily, and the numbers of shocks for these as tabulated above are practically correct. The numbers as recorded at the Gifu Meteorological Station and three District Offices in eastern Mino, namely, those at Mitake, Takayama, and Nakatsugawa, are also accurate; the records at other stations in the province, however, are imperfect and give only the numbers of stronger shocks.

During 1891 and 1892 the greatest number of earthquakes was recorded at Gifu. It is likely indeed that shocks may have been more accurately recorded at a meteorological station than at District Offices, but I believe the number of shocks was actually greatest in the vicinity of Gifu for some time after the great earthquake.

The records during 1892 for the six places in the western part of Mino, namely, Kitagata, Ibi, Tarui, Takata, Takasu, and Ōgaki, were month by month nearly alike, the highest seismic frequency having occurred at Ōgaki and the lowest at Ibi and Tarui. Of the two places in the central part of the province, namely, Takatomi and

Kozuchi, a greater number of shocks was recorded at the former than at the latter. Of the remaining six places, Nakatsugawa, Takayama, Mitake, Ōta, and Hachiman, the greatest number was recorded at Mitake and the least at Hachiman.

The records taken at several stations in Owari shew an evident change with time of localisation of seismic frequency. In 1891, the greatest number of earthquakes was recorded at Biwajima, Ōtajima, and Inazawa, these places being situated in a zone where, it should be remarked, the motion had been very strong on the occasion of the great earthquake of October, 1891. In 1892, the greatest number was recorded at Koori, and the next greatest at Biwajima and Katsukawa. Again, in 1893 the greatest number was recorded still at Koori, but the next greatest at Ōtajima and Nagoya. The least numbers always occurred at Atsuta and the stations in the Chita Peninsula.

The numbers of earthquakes at Nagoya, Atsuta, Handa, and Toyohama indicate an evident decrease of seismic activity with distance as we go southwards.

In Mikawa, the greatest numbers of earthquakes occurred at Koromo and Asuke in the north-western part of the province, the activity there being nearly the same as in the vicinity of Atsuta in Owari.

The seismic activity in the Atsumi Peninsula was less than that in the Chita Peninsula.

§ 34. The distribution of seismic activity in Mino, Owari, and Mikawa during 1892, 1893, and January, 1894, will be clearly seen from Figs. 27, 28, and 29, respectively, in which the curves are loci of places where equal numbers of shocks have been recorded during each of these intervals. In drawing Fig. 28, the numbers of earthquakes during January and September have been omitted as a severe shock

occurred in the beginning of each of these months in the district under consideration, and where consequently the seismic activity in some particular places was greatly increased. The residual effect of these severe secondary-earthquakes, however, as before remarked, soon died away, so that Fig. 28 will fairly represent the distribution of after-shocks in 1892 due wholly to the great earthquake of 1891. On the other hand, Fig. 29 will shew principally the effect of the strong shock which took place on January 10th, 1894.

Fig. 27 shews more or less distinctly four axial lines, which radiate from the vicinity of Koori and along which the seismic activity was greater than in the neighbouring tracts. In Figs. 28 and 29, two of these axial lines, one of which proceeded towards Ibi in the north-western corner of the Mino-Owari Plain, and the other of which proceeded to the basin of the Isé Gulf, became insignificant and we have only two distinct axial lines which extend from the same centre towards ENE and ESE, that is, respectively along the upper valley of the Kiso-gawa and into the mountain districts of the northern Mikawa.

The origin of the severe earthquake of January 10th, 1894, was just at the point of intersection of the above two axial lines, and those of the severe earthquakes of January 3rd and September 7th, 1892, were respectively in the vicinities of Gifu (near to the Kiso-gawa) and of Katsukawa (near to Nagoya), so that all these three shocks had their origins approximately on the same axial line running from WNW towards ESE.

§ 35. *On the Cause of the Great Earthquake of October 28th, 1891.*

The fact that the centre of the greatest activity of after-shocks is, not in the Neo-Valley where the shock had been strongest on the occasion of the earthquake of October, 1891, but in a locality near to it, suggests the idea that there had existed a very extensive

instability under the Echizen, Mino, Owari, and Mikawa provinces, and that the great earthquake was caused by some big fractures produced in this underground strained portion of the earth's crust.

The district adjoining the south-eastern extremity of the Neo-Valley is not yet on the way of steadily settling into equilibrium, and the four axial lines in Fig. 27 probably indicate the positions of four weaker or deeper fractures, the peculiar violence of motion on the occasion of the great earthquake in the zonal tract at the western part of the Owari Plain and in the regions to the south-east of Koori (see Fig. 26) being probably due to the existence of such lines of weakness beneath.¹ The underground fracture along the Neo-Valley was probably sufficiently great and must have removed the chief centre of weakness at that district.

The two axial lines in Fig. 28 seem to indicate the positions of fracture lines which became gradually prominent, and along which the activity of after-shocks is at present greatest.

The centre of activity of after-shocks may in future change its position, but it will very probably recede from, and not approach the Neo-Valley.

The distribution of seismic activity before 1891 is shewn in Fig. 30 (see Table XXII). It indicates no such peculiarity as that seen in Figs. 27, 28, and 29.

§ 36. *Earthquake Sounds*.—I take this opportunity of making some remarks upon earthquake sounds.

Many of the after-shocks of the Mino-Owari earthquake were attended with sounds, which were essentially of two types, being either rushing feeble noises like those caused by winds, or loud rumbling sounds like those caused by the falling of a heavy weight on ground, or by the discharge of a gun.

¹ See Fig. 31.

The sounds of the second type, which were sometimes like detonations of thunder, were most frequent and distinct in the Neo-Valley, where, as I believe, their main origin really was. It is remarkable that tremblings of the ground accompanying these sounds were invariably very feeble, and often not to be felt at all, while severe sharp shocks were usually not accompanied by distinctly audible sounds. This peculiar phenomenon was ascertained likewise to have been observed with the after-shocks of the Noto and Kagoshima earthquakes. The following may be one of the possible explanations.

Among numerous depressions of small pieces of ground produced by the great earthquake of 1891, there was one which took place on a high mountain flank near the village of Higashi-Yokoyama, in the Ōno District, Mino, and which was cylindrical in form, being about 9 feet in diameter and 10 feet in depth. The top of the cavity was just shewn on the surface slope as a circular aperture about 4 feet in diameter. From examples like this, we may suppose that some cavities have probably been formed underground which are not shewn on the surface. Especially in the focal region there may exist big vacant spaces, and the falling down into these of superincumbent rock masses would give rise to sounds accompanied by small movements of ground, as muffled sounds are heard when we throw stones into a deep well or hole. On the other hand, shocks may originate by fracturing of strata in districts near the focus which would cause sharp tremblings of the ground, but not be accompanied by loud sounds.

The slight rushing sounds often accompanying earthquakes may, as usually supposed, be caused by quick tremors preceding the principal earthquake motion.



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TABLE I.—DAILY NUMBERS OF EARTHQUAKES AT KUMAMOTO,
FROM JULY 28th, 1889, TO DECEMBER 31st, 1890.

Day	JULY, 1889.						AUGUST, 1889.					
	violent shocks	strong shocks	weak shocks	feeble shocks or sounds	sum	activity	violent shocks	strong shocks	weak shocks	feeble shocks or sounds	sum	activity
1	1	6	4	11	12
2	1	..	7	8	9
3	1	3	18	13	35	40
4	2	11	9	22	24
5	5	6	11	11
6	1	5	5	11	12
7	1	3	..	4	5
8	5	6	11	11
9	1	1	2	2
10	4	2	6	6
11	4	2	6	6
12	4	1	5	5
13	4	5	9	9
14	1	5	6	6
15	1	4	6	11	12
16	6	7	13	13
17	3	3	6	6
18	1	5	6	6
19	2	1	3	3
20	2	6	8	8
21	3	4	7	7
22	2	1	3	3
23	2	4	6	6
24	2	1	3	6	8
25	1	1	2	4	5
26	1	2	3	3
27	1	4	5	5
28	1	1	3	1	5	6	6
29	..	23	14	33	70	93	2	2	4	4
30	..	5	10	12	27	32	2	2	2
31	..	1	12	2	15	16	3	3	3
Sum	1	29	36	47	113	144	1	13	103	126	243	259

Day	SEPTEMBER, 1889.					OCTOBER, 1889.				
	strong shocks	weak shocks	feeble shocks or sounds	sum	activity	strong shocks	weak shocks	feeble shocks or sounds	sum	activity
1	1	...	4	5	6
2	...	1	...	1	1	1	2	2	5	6
3	...	1	...	1	1	...	2	1	3	3
4	1	1	1	...	3	1	4	4
5	1	...	1	2	3	1	3	3	7	8
6	1	1	1	3	4	1	1	1	3	4
7	1	...	1	2	3	...	4	2	6	6
8	1	1	2	2
9	1	...	1	2	3	...	1	1	2	2
10	1	...	2	3	4	1	1	1
11	1	1	1
12	...	1	...	1	1
13	...	1	...	1	1	...	1	...	1	1
14	1	1	2	...	2	1	3	3
15	...	2	...	2	2	1	1	1
16	...	1	...	1	1	1	...	1	2	3
17	...	1	...	1	1	1	1	1
18	1	1	2	4	5
19	2	2	...	4	6	1	1	1
20	3	...	3	3
21	...	2	...	2	2	...	1	...	1	1
22	1	1	...	2	3	1	1	...	2	3
23	2	2	4	1	1	2	4	5
24	1	1	2	...	1	2	3	4
25	1	1	1	2	2	2
26	2	...	2	4	6	2	2	2
27	1	1	2	2
28	...	1	...	1	1	1	1	1
29	1	...	1	1
30	...	1	1	2	2	2	2	2
31	1	...	1	2	3
Sum	14	16	11	41	55	9	30	38	77	87

Day	NOVEMBER, 1889.					DECEMBER, 1889.				
	strong shocks	weak shocks	feeble shocks or sounds	sum	activity	strong shocks	weak shocks	feeble shocks or sounds	sum	activity
1	1	1	1	1	1	1
2	1	1	1	...	1	1	2	2
3	1	...	1	2	3	1	1	1
4	...	1	...	1	1	...	2	1	3	3
5	1	1	1	...	1	...	1	1
6	...	2	1	3	3
7	...	1	1	2	2
8	1	1	1	3	4
9	...	1	1	2	2	2	2	2
10	...	1	1	2	2	1	1	2	4	5
11	...	1	2	3	3	...	1	1	2	2
12	...	1	1	2	2	1	1	1
13	...	2	1	3	3	...	1	1	2	2
14	1	1	1	...	1	1	2	2
15	1	1	2	2
16	...	1	1	2	2	2	2	2
17	1	...	1	1
18	1	1	1	3	4	...	1	...	1	1
19	...	1	1	2	2	...	1	...	1	1
20	...	2	...	2	2
21	...	1	...	1	1	...	1	1	2	2
22	...	1	1	2	2	1	1	1
23	...	1	1	2	2	...	2	...	2	2
24	2	2	2
25	...	1	1	2	2	1	1	1
26	...	1	...	1	1
27	1	1	1	...	1	...	1	1
28	1	...	1	1
29	...	3	2	5	5	...	1	...	1	1
30	1	...	1	2	3
31						...	1	...	1	1
Sum	3	23	25	51	54	2	20	19	41	43

Day	JANUARY, 1890.			FEBRUARY, 1890.			MARCH, 1890.				APRIL, 1890.	MAY, 1890.			
	weak shocks	feeble shocks or sounds	sum	weak shocks	feeble shocks or sounds	sum	strong shocks	weak shocks	feeble shocks or sounds	sum	feeble shocks or sounds	strong shocks	weak shocks	feeble shocks or sounds	sum
1	2	2	1	1
2	1	1	1	3	5
3	...	1	1	3	3
4	...	2	2	1	...	1	1	2
5	...	2	2	1	1	2	1	1
6	2	...	2	4	2
7	1	1	2	2	2	1
8	2	2	1
9	1	1	1
10	1	1	1	1	1
11	1	...	1	1	1	1	1	1
12	2	2	1	1
13	...	1	1	1	1
14	1	1	2	...	1	1	2	2
15	1	1
16	...	1	1	1	1
17	2	2	1
18	1	...	1	2	1	4	4
19	...	1	1	...	1	1
20	...	1	1	1	1	1	1
21	1	2	2
22	...	1	1
23	1	2	3	1	1
24	2	2
25	1	1	2	1	1
26	4	4	1	1
27	4	4
28	...	1	1	1	1	...	1	1
29	4	4	1	...	1
30	...	1	1	1	...	3	4	1	1	2
31
Sum	2	14	16	1	7	8	4	1	35	40	16	2	4	27	33

Day	JUNE, 1890			JULY, 1890			AUG., 1890	SEP., 1890	OCT., 1890	NOV., 1890	DEC., 1890
	weak shocks	feeble shocks or sounds	sum	weak shocks	feeble shocks or sounds	sum	feeble shocks or sounds	feeble shocks or sounds	feeble shocks or sounds	feeble shocks or sounds	feeble shocks or sounds
1	2	1
2	1	4	5	...	1	1	2	1	2	1	...
3	1	1	2	2
4	1	2	3	...	3	3
5	...	1	1	...	2	2
6	1	1	2	1	2
7	1	1	1
8	1	1	2
9	...	1	1	2
10	1	1	...	1	1
11	2
12	2	2
13	1
14	...	1	1	...	1	1	1
15
16	...	1	1	1
17	...	1	1	1
18	1	1	2	1
19	1	1	2
20	...	2	2	1
21	2
22	...	3	3	...	1	1	1	3
23	1
24	1	...	1
25	1	1
26	...	1	1
27	...	1	1	1	1	...	2	...
28	1	1	2
29	1	...	1	1	1	...
30	1	1	2	1	2	...
31	1
Sum	6	21	27	3	17	20	19	14	6	6	3

TABLE II.—MONTHLY AND YEARLY NUMBERS OF EARTHQUAKES
AT KUMAMOTO, FROM 1889 TO 1893.

Month Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	sum
1889							113 ¹	243	41	77	51	41	566
1890	16	8	40	16	33	27	20	19	14	6	6	3	208
1891	8	19	6	9	7	7	2	1	...	12	9	9	89
1892	3	3	3	3	7	3	4	1	2	5	1	3	38
1893	2	1	1	1	...	1	4	7	3	...	20 ²
Sum	29	31	50	29	47	38	139	264	61	107	70	56	921

TABLE II.¹—AVERAGE MONTHLY NUMBERS OF EARTHQUAKES
AT KUMAMOTO.

Month Interval	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
1890-1893	7.3	7.8	12.5	7.3	12.0	9.3	6.0	5.5	5.0	7.5	4.8	4.0
1891-1893	4.3	7.7	3.3	4.3	4.7	3.7	2.0	0.7	2.0	8.0	4.3	4.0
1892-1893	2.5	2.0	2.0	2.0	3.5	2.0	3.0	0.5	3.0	6.0	2.0	1.5

1. The number for July, 1889, is that for four days, from 28th to 31st of that month.
2. Those shocks at Kumamoto which were evidently due to the Kagoshima earthquake of September 7th, 1893, are not included.

TABLE III.—HOURLY EARTHQUAKE NUMBERS AT KUMAMOTO, FROM JULY 31st TO AUGUST 13th, 1889.¹

Day Interval	JULY, 1889.	AUGUST, 1889.													sum
	31	1	2	3	4	5	6	7	8	9	10	11	12	13	
0—1 a.m.	2	1	1	...	1	1	1	7
1—2	1	1	3	1	...	5	1	1	13
2—3	3	2	...	6	2	...	1	1	2	...	1	18
3—4	1	1	...	4	1	...	1	1	1	...	1	...	1	...	12
4—5	1	1	1	5	...	1	1	...	1	11
5—6	1	...	1	4	1	1	1	1	1	11
6—7	1	1	1	3
7—8	...	2	...	1	...	1	1	1	6
8—9	1	2	1	4
9—10	...	1	1	2
10—11	2	1	1	4
11—12	1	1	...	1	3
0—1 p.m.	1	2	2	1	1	...	1	...	1	...	1	1	11
1—2	1	...	1	...	2	...	1	5
2—3	1	1	2
3—4	1	1
4—5	1	1	1	1	4
5—6	...	1	1	1	3
6—7	5	1	2	8
7—8	1	1	1	1	...	4
8—9	...	1	1	1	1	1	5
9—10	1	1	2
10—11	1	2	1	...	4
11—12	1	1	...	1	...	1	1	5
Sum	15	11	8	34	17	11	11	4	10	2	6	5	5	9	148 ²

{ During Night, or between 6 p.m. and 6 a.m., 100

{ „ Day, „ „ 6 a.m. and 6 p.m., 48

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1. The first great earthquake took place on July 28th, 11.48 p.m., and the times of occurrence of subsequent shocks were not noted down till 31st of the same month.

2 The times of occurrence of shocks were lost in a few cases, and therefore this number is slightly less than according to Table I.

TABLE IV—DAILY NUMBERS OF EARTHQUAKES AND BAROMETRIC HEIGHTS AT GIFU, FROM OCTOBER 28th, 1891, TO APRIL 30th, 1892.

Day	OCTOBER, 1891.								fluctuation, in mm.
	violent shocks	strong shocks	weak shocks	feeble shocks	sounds	sum	activity	mean ¹ bar. ht. in mm.	
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28	1	8	94	103	113	753.7	- 3.5, + 3.7
29	...	15	303	318	333	54.9	- 2. , + 2.2
30	3	14	147	...	9	173	193	61.3	+ 8.1
31	...	3	116	1	6	126	129	65.1	+ 1.3, - 2.3
Sum	4	40	660	1	15	720	768

1. The "Mean Barometric Height" was obtained by taking the mean of six observations on each day, at 2, 6, 10, a.m., and 2, 6, 10, p.m. The "Fluctuation" is the difference between these observations, + when rising, and - when falling. For instance, - 3.5, + 3.7 means that the pressure first fell by 3.5 mm. and then rose by 3.7 mm.

Day	NOVEMBER, 1891.								
	violent shocks	strong shocks	weak shocks	feeble shocks	sounds	sum	activity	mean bar. ht. in mm.	fluctuation, in mm.
1	...	5	94	99	104	762.7	-3. , +1.2
2	...	4	88	92	96	63.9	+1.6, -1.6, +2.2
3	81	81	81	63.6	-3.
4	...	2	75	1	...	78	80	59.1	-5.
5	...	1	52	53	54	55.3	+3.7
6	...	4	56	1	6	67	71	59.8	+3.9
7	...	3	42	45	48	64.2	+4.2
8	...	1	41	42	43	67.0	+1. , -2.6
9	...	1	43	44	45	63.0	-4.3
10	40	40	40	58.5	-6.3
11	37	...	1	38	38	60.5	+7.6
12	...	1	38	1	...	40	41	63.8	-1.7
13	...	1	32	1	1	35	36	62.3	-2. , +1.6
14	25	3	1	29	29	64.3	+2.1, -1.4, +2.
15	23	6	...	29	29	64.4	-3.2
16	27	1	...	28	28	59.9	-5. , +2.2
17	15	1	5	21	21	61.3	+2.5, -1.4, +2.
18	9	9	18	18	61.9	-2.6, +1.6
19	7	10	17	17	62.6	-1.2, +3.
20	...	1	...	18	14	33	34	65.1	-2.7, +2.3
21	1	1	10	5	4	21	24	65.0	-2.1, +1.6
22	9	3	12	12	65.4	+1.8
23	...	2	...	16	5	23	25	61.9	-6.2
24	14	4	18	18	69.1	+4.6
25	6	3	9	9	63.3	+1.2, -1.2, +1.4
26	3	3	9	15	15	63.8	-2.3, +2.3
27	...	1	4	...	6	11	12	66.6	+5.2
28	1	1	4	...	10	16	19	72.0	-1.4, +1.1
29	10	3	6	19	19	70.2	-2.2
30	12	1	1	14	14	66.7	-3.6
Sum	2	29	852	106	98	1087	1120

Day	DECEMBER, 1891.								
	violent shocks	strong shocks	weak shocks	feeble shocks	sounds	sum	activity	mean bar. ht. in mm.	fluctuation, in mm.
1	3	3	1	7	7	767.4	+3.8
2	3	9	4	16	16	70.5	+1.3, -2.1
3	1	1	9	4	2	17	20	61.3	-14.4
4	1	...	4	11	2	18	20	51.6	-1.4, +5.9
5	...	1	17	12	2	32	33	64.0	+10.5
6	6	1	7	7	71.5	+2.3, -2. , +2.2
7	...	1	...	6	1	8	9	72.4	-2.5
8	11	5	3	19	19	61.9	-15.4
9	...	2	10	8	2	22	24	57.9	+7.4
10	17	3	2	22	22	61.7	+1.3, -1.7
11	10	2	2	14	14	62.6	+3.7
12	18	2	3	23	23	63.6	+1.2, -4.7
13	5	1	8	14	14	61.5	+5.5
14	18	3	2	23	23	66.8	+3.5
15	14	1	15	15	67.3	-3.
16	...	2	17	...	1	20	22	63.7	-2.2, +1.6
17	10	8	...	18	18	64.7	+1.2, -1.4, +2.3
18	5	1	1	7	7	68.4	+5.4
19	3	2	5	5	70.8	-2.6, +1.8
20	5	1	2	8	8	67.3	-5.3
21	6	2	...	8	8	65.5	+1.8, -3.1
22	3	5	8	8	61.7	-4.2, +2.4
23	2	1	3	6	6	62.7	+3.2
24	1	1	10	...	3	15	18	63.7	+3.7
25	5	1	6	6	70.6	+2.4, -1.3
26	7	2	9	9	70.4	-2.6
27	2	1	3	3	69.0	+1.2, -2.5
28	1	6	...	7	7	69.3	+2.1
29	...	1	13	1	1	16	17	70.7	+1.4, -2.5, +1.6
30	10	2	3	15	15	71.2	-2.7
31	6	2	8	8	63.3	-7.3, +3.4
Sum	3	9	204	137	63	416	431

Day	JANUARY, 1892.								fluctuation, in mm.
	violent shocks	strong shocks	weak shocks	feeble shocks	sounds	sum	activity	mean bar. ht. in mm.	
1	4	1	1	6	6	766.3	+1.8, -2.4, +1.6
2	3	2	5	5	65.3	-2.2, +2.7
3	1	1	2	1	2	7	10	68.7	+3.2
4	...	1	6	3	2	12	13	68.0	-2.6
5	...	2	3	1	1	7	9	66.6	-1.3, +2.5
6	2	2	...	4	4	66.5	-2.7
7	4	5	9	9	63.2	-2.8, +3.
8	1	3	...	4	4	64.2	-4.5
9	2	1	...	3	3	60.9	+1.6, -2. , +1.8
10	4	3	3	10	10	62.2	+2.6
11	8	6	...	14	14	62.9	-4.4
12	...	1	2	2	...	5	6	54.1	-6.2, +3.5
13	...	1	5	3	2	11	12	61.4	+10.
14	67.3	+2. , -2.6
15	2	...	2	2	62.8	-5.8
16	3	...	3	3	58.1	-3.3, +1.8
17	1	1	...	2	2	58.7	+1.3, -1.5, +1.5
18	57.3	-2.2
19	4	2	6	6	59.9	+6.9
20	3	1	4	4	62.9	-3. , +3.5
21	2	...	2	2	66.6	+3.8
22	3	1	4	4	70.6	+2.4, -2.4, +2.2
23	1	...	2	3	3	71.6	+1.7, -3.2
24	1	2	3	6	6	69.6	0
25	1	1	1	3	3	68.3	+1.2, -3.9
26	...	1	...	4	1	6	7	62.7	-5.4
27	2	3	2	7	7	61.3	+4.5
28	...	1	...	5	1	7	8	65.2	+2.
29	4	1	5	5	63.6	-3.2, +1.2
30	2	2	2	65.2	+4.4
31	4	1	5	5	64.2	-5.1
Sum	1	8	45	74	36	164	174

Day	FEBRUARY, 1892.							
	strong shocks	weak shocks	feeble shocks	sounds	sum	activity	mean bar. ht. in mm.	fluctuation, in mm.
1	763.4	+ 2.1, - 1.5
2	3	1	4	4	61.5	- 3.1, + 3.4
3	7	1	8	8	64.9	+ 2.2, - 2.
4	1	1	2	2	59.3	- 11.2
5	1	1	3	1	5	6	60.7	+ 7.9
6	2	...	1	...	4	6	69.5	+ 3.5
7	2	2	4	4	71.8	- 3.3
8	1	2	3	3	56.9	- 18.5, + 5.9
9	4	...	4	4	60.9	+ 7.
10	1	1	2	2	66.4	+ 1.8, - 1.7
11	1	2	3	3	66.9	- 2.1,
12	2	1	3	3	62.3	- 5.4, + 1.5
13	1	1	2	2	62.5	+ 1.8, - 1.9
14	4	2	6	6	55.7	- 11.7, + 4.8
15	4	1	5	5	57.5	+ 3.6
16	2	2	4	4	62.7	+ 4.
17	1	1	2	2	6	7	64.1	- 2.8, + 4.4
18	1	...	1	1	64.5	- 3.4
19	4	...	4	4	59.4	+ 3.7
20	2	1	3	3	62.0	- 2.
21	5	1	6	6	60.3	- 4.2
22	1	...	2	1	4	5	58.5	+ 3.
23	...	1	...	2	3	3	58.5	- 7.5
24	...	1	3	1	5	5	54.1	- 1.5, + 6.9
25	3	...	3	3	59.5	0
26	...	1	2	...	3	3	56.6	- 4.5, + 1.6
27	3	...	3	3	59.4	+ 5.1
28	8	8	8	59.4	+ 6.5
29	3	3	6	6	65.8	- 4.3
Sum	5	5	67	37	114	119

Day	MARCH, 1892.							
	strong shocks	weak shocks	feeble shocks	sounds	sum	activity	mean bar. ht. in mm.	fluctuation, in mm.
1	2	3	5	5	760.4	- 4.9, + 5
2	2	1	3	3	63.4	- 1.4
3	3	4	7	7	60.1	- 2.5
4	3	2	5	5	58.9	- 2.3
5	58.6	- 1.7, + 2.4
6	53.0	- 11.3
7	1	2	3	3	53.1	+ 8.8
8	3	...	3	3	60.9	+ 5.3
9	1	1	1	67.0	+ 2.9
10	1	...	1	1	66.0	- 4.4
11	...	1	1	1	3	3	64.2	+ 4.
12	1	...	1	1	64.8	- 9.
13	4	1	5	5	51.5	- 5.8, + 10.6
14	5	2	7	7	58.5	+ 4.9
15	2	...	2	2	63.2	- 1.7, + 2.9
16	2	...	2	2	66.1	+ 1.5, - 1.5, + 2.7
17	2	1	3	3	67.4	+ 1.3, - 2.6
18	58.0	- 10. , + 2.8
19	57.6	+ 4.1
20	62.3	+ 1.6, - 1.0, + 3.1
21	2	2	2	65.9	+ 2.2, - 2.2, + 1.5
22	64.8	+ 1.3, - 2.4
23	3	...	3	3	60.8	- 4.6
24	1	1	1	55.9	- 3.1, + 5.3
25	3	...	3	3	63.4	+ 7.5
26	1	...	1	1	64.9	- 5.8
27	1	2	3	3	65.2	+ 3.5
28	1	...	2	3	6	7	64.5	- 3.9
29	3	1	4	4	58.9	- 4.9, + 3.6
30	...	3	5	2	10	10	65.2	+ 6.8
31	2	1	3	3	68.8	- 2.1, + 1.3
Sum	1	4	52	30	87	88

Day	APRIL, 1692.							
	strong shocks	weak shocks	feeble shocks	sounds	sum	activity	mean bar ht. in mm.	fluctuation, in mm.
1	2	...	2	2	764.4	-12.2
2	5	...	5	5	53.9	+8.2
3	1	...	1	...	2	2	57.1	-1.8,+7.
4	2	...	2	2	61.8	+1.3,-3.1
5	4	1	5	5	59.8	+3.1
6	...	1	1	1	60.9	-3.1
7	58.9	-2.3
8	...	1	7	1	9	9	57.6	-2.5
9	2	2	4	4	57.2	-1.3,+4.1
10	...	1	2	...	3	3	64.1	+7.8
11	66.4	-3.4
12	3	...	3	3	55.9	-12.4
13	2	...	2	2	55.6	+9.5
14	2	...	2	2	61.8	+2. , -3.6
15	2	2	2	59.2	+4.
16	64.8	+3.9
17	3	...	3	3	63.7	-4.6
18	4	1	5	5	58.4	-3.2
19	3	...	3	3	61.6	+6.2
20	...	1	3	1	5	5	65.9	+1.3,-2.2
21	1	...	1	1	58.8	-7.4
22	3	...	3	3	61.9	+8.9
23	2	...	2	2	65.6	-2.5,+1.6
24	1	1	3	...	5	6	59.9	-6.1
25	...	1	1	1	55.3	-1.4,+1.5
26	...	2	1	...	3	3	60.9	+5.6
27	...	1	...	2	3	3	64.6	+1.4
28	1	...	1	1	65.2	+1.2
29	...	1	5	...	6	6	69.2	-2.7
30	4	3	7	7	58.3	-3.3
Sum	2	10	65	13	90	92

TABLE V.—DAILY NUMBERS OF EARTHQUAKES AT GIFU,
FROM MAY, 1892, TO DECEMBER, 1893.

Day	MAY, 1892.				JUNE, 1892.				
	weak shocks	feeble shocks	sounds	sum	strong shocks	weak shocks	feeble shocks	sounds	sum
1	...	6	...	6	2	2
2	...	3	...	3	1	1
3
4	1	1	1	...	1
5	...	1	...	1
6	...	1	...	1
7	1	3	...	4	3	...	3
8
9	...	2	...	2	2	...	2
10	...	3	...	3	2	...	2
11	...	4	1	5	1	1	2
12	1	1	1	1	2
13	1	1
14	...	1	...	1
15	1	1	...	2	1	1	2
16	...	2	...	2
17	1	...	1
18	1	...	1
19	1	...	1
20	1	...	2	1	4
21
22	...	2	...	2
23	...	2	3	5	1	1	2
24	...	1	...	1	1	1
25	...	4	...	4	1	...	1
26	...	2	...	2
27	1	1	1	...	1
28
29	...	1	1	2
30	...	1	...	1	...	1	1
31	...	2	1	3					
Sum	5	42	7	54	1	1	19	9	30

Day	JULY, 1892.				AUGUST, 1892.				
	weak shocks	feeble shocks	sounds	sum	strong shocks	weak shocks	feeble shocks	sounds	sum
1	1	...	1	2	4	...	4
2	...	1	1	2	1	...	1
3	1	1	3	1	4
4	...	2	1	3	...	1	1
5	1	1	...	1	...	1	2
6	1	...	1
7	...	1	...	1
8
9	...	1	1	2
10
11	...	1	...	1
12	1	...	1
13	1	1	1	1
14	...	2	...	2	3	3
15	1	1
16	1	1
17
18	...	1	...	1
19	...	2	1.	3	1	...	1	...	2
20	...	1	...	1	...	1	3	2	6
21	...	1	1	2	2	1	3
22	1	...	1
23	...	1	...	1	2	1	3
24	1	1	1	...	1
25	1	1
26	...	1	1	2	1	...	1
27	1	3	...	4	1	2	3
28	1	1	2	1	3
29	4	1	5
30	...	1	...	1	2	...	2
31	...	2	...	2	1	...	1
Sum	3	21	11	35	1	3	31	17	52

Day	SEPTEMBER, 1892.						OCTOBER, 1892.					NOVEMBER, 1892.				
	violent shocks	strong shocks	weak shocks	feeble shocks	sounds	sum	strong shocks	weak shocks	feeble shocks	sounds	sum	weak shocks	feeble shocks	sounds	sum	
1	2	...	2	1	1	2	
2	2	...	2	1	1	...	2	
3	2	...	2	1	3	4	...	4	...	4	
4	2	1	3	2	...	2	...	3	...	3	
5	5	...	5	3	...	3	...	2	...	2	
6	5	...	5	1	1	...	1	...	1	
7	1	..	1	17	9	28	...	1	1	...	2	...	3	...	3	
8	6	2	8	...	1	3	...	4	
9	6	2	8	1	...	1	1	3	...	4	
10	5	...	5	3	...	3	
11	3	...	3	1	...	1	
12	2	...	2	1	...	1	...	2	...	2	
13	1	1	2	4	1	1	2	...	2	1	3	
14	2	1	3	3	...	3	
15	1	...	1	...	6	...	6	
16	2	1	3	2	...	2	...	2	...	2	
17	1	1	1	3	
18	4	...	4	1	...	1	1	1	
19	2	...	2	2	...	2	
20	1	1	2	1	1	2	
21	2	...	2	
22	2	...	2	2	...	2	...	1	...	1	
23	2	1	3	2	...	2	...	2	...	2	
24	
25	1	...	1	1	...	1	
26	2	...	2	1	1	2	
27	1	1	2	5	...	5	...	1	1	2	
28	1	1	2	...	2	
29	3	3	
30	1	1	2	
31	1	...	1	
Sum	1	...	3	79	24	107	1	2	37	7	47	3	40	5	48	

Day	DECEMBER, 1892.				JANUARY, 1893.				FEBRUARY, 1893.			
	weak shocks	feeble shocks	sounds	sum	weak shocks	feeble shocks	sounds	sum	weak shocks	feeble shocks	sounds	sum
1	1	1	4	2	6
2	1	1	...	3	...	3
3	1	1	2
4	2	...	2
5	3	...	3	...	1	...	1
6
7	3	...	3
8	1	1	...	1	...	1
9	1	4	...	5	1	...	1
10	...	2	...	2
11	1	2	1	4	...	1	...	1
12	...	2	...	2	...	1	...	1
13	...	2	...	2	...	1	...	1
14	...	2	...	2	...	1	...	1	1	1
15
16
17
18	...	1	...	1	...	1	...	1
19	...	5	1	6	1	1
20	...	2	...	2
21	...	1	...	1	1	1	...	2
22	...	1	...	1	...	2	...	2
23	...	1	...	1	...	2	1	3
24	1	1	...	3	1	4	...	1	...	1
25
26	...	1	...	1
27	...	1	...	1	...	1	...	1
28	...	1	...	1
29	...	1	...	1	1	1	2	4
30	1	1	...	2	...	2
31	...	1	1	2	...	2	...	2
Sum	2	30	7	39	1	23	7	31	1	16	3	20

Day	MARCH, 1893.				APRIL, 1893.			MAY, 1893.				JUNE, 1893.		
	weak shocks	feeble shocks	sounds	sum	feeble shocks	sounds	sum	weak shocks	feeble shocks	sounds	sum	feeble shocks	sounds	sum
1	...	2	...	2	2	...	2
2	...	2	1	3	2	...	2
3	...	2	...	2	3	...	3	...	1	...	1
4	3	...	3	...	2	2	4	1	...	1
5	6	...	6	1	...	1
6	...	2	1	3	3	...	3	1	...	1
7	...	2	1	3	2	...	2	1	1	2	...	2
8	...	5	...	5	1	1	1	...	1
9	2	..	2	...	1	...	1
10	...	1	1	2	3		3	1	1	1	...	1
11	...	1	...	1	2	...	2
12	2	1	3	1	1
13	2	1	3	...	3	...	3
14	...	1	...	1	8	...	8	1	...	1
15	...	4	...	4	2	...	2	...	1	...	1
16	...	2	...	2	2	...	2	...	1	...	1
17	...	2	...	2
18	...	1	...	1	2	...	2	...	4	...	4
19	...	1	...	1	1	...	1	1	...	1
20	...	1	...	1	3	...	3	1	...	1
21	...	4	...	4	2	...	2	..	2	1	3
22	...	2	...	2	2	2	4	1	...	1
23	1	1	...	2	1	...	1
24	...	5	...	5	1	...	1
25	...	3	...	3	2	...	2	1	...	1
26	...	1	...	1
27	1	...	1	...	2	...	2
28	...	1	...	1	1	...	1
29	...	1	...	1
30	1	1	...	1	...	1
31	1	...	1			
Sum	1	47	4	52	54	5	59	3	25	4	32	12	...	12

Day	JULY, 1893.			AUGUST, 1893.			SEPTEMBER, 1893.			OCTOBER, 1893.		
	feeble shocks	sounds	sum	feeble shocks	sounds	sum	feeble shocks	sounds	sum	feeble shocks	sounds	sum
1
2	1	...	1
3	1	...	1
4
5	2	...	2
6	1	...	1	1	...	1
7	1	...	1	1	...	1
8	1	...	1	1	...	1
9
10	1	...	1
11
12	...	1	1
13	1	...	1
14	2	...	2
15	1	1	2	...	2	...	1	1
16	2	...	2	1	...	1	2	...	2
17	2	...	2	1	...	1
18	2	...	2	...	1	1	1	...	1	1	...	1
19	1	...	1	2	...	2	1	...	1
20	1	...	1	1	...	1
21	1	...	1	1	...	1	1	...	1	1	...	1
22	3	...	3	2	...	2
23	2	...	2	1	1	2
24	1	...	1	1	...	1	1	...	1
25	1	...	1	1	...	1	1	...	1
26
27	1	...	1	1	1	2	1	...	1
28
29	3	...	3
30	1	...	1	...	1	1	1	...	1
31	1	...	1	2	...	2
Sum	17	1	18	10	3	13	18	2	20	18	1	19

Day	NOVEMBER, 1893.				DECEMBER, 1893.			
	weak shocks	feeble shocks	sounds	sum	weak shocks	feeble shocks	sounds	sum
1
2
3
4	...	1	...	1
5	...	1	...	1
6	1	...	1
7	1	...	1
8	...	1	...	1
9
10	1	1
11
12
13	1	1
14	...	2	...	2
15
16	...	1	...	1
17
18	...	2	...	2
19	...	1	...	1	...	1	...	1
20	2	...	2
21	1	1	...	4	...	4
22	...	1	...	1	...	2	...	2
23	...	1	...	1
24
25	1	...	1	2	1	1
26	1	1
27
28	1	...	1
29
30	1	1
31	1	1
Sum	2	11	3	16	2	12	2	16

TABLE VI.—DAILY NUMBERS OF EARTHQUAKES AT NAGOYA,
FROM OCTOBER 28th, 1891, TO APRIL 30th, 1892.

Day	OCTOBER, 1891.				NOVEMBER, 1891.				DECEMBER, 1891.			
	violent shocks	strong shocks	weak, feeble shocks	sum	violent shocks	strong shocks	weak, feeble shocks	sum	strong shocks	weak, feeble shocks	sum	
1	4	52	56	
2	5	25	30	...	5	5	
3	1	30	31	2	5	7	
4	2	18	20	1	9	10	
5	3	17	20	...	6	6	
6	2	15	17	...	2	2	
7	2	27	29	...	4	4	
8	1	17	18	...	3	3	
9	1	15	16	...	6	6	
10	12	12	...	4	4	
11	1	4	5	...	3	3	
12	1	6	7	...	4	4	
13	1	12	13	...	4	4	
14	12	12	...	4	4	
15	12	12	...	3	3	
16	13	13	...	6	6	
17	15	15	...	3	3	
18	9	9	...	1	1	
19	4	4	...	2	2	
20	9	9	...	2	2	
21	1	8	9	...	3	3	
22	5	5	...	5	5	
23	1	8	9	...	3	3	
24	9	9	...	4	4	
25	9	9	...	2	2	
26	5	5	...	2	2	
27	8	8	...	3	3	
28	1	4	121	126	1	1	5	7	...	1	1	
29	...	7	178	185	5	5	...	3	3	
30	...	4	89	93	2	2	...	3	3	
31	...	3	76	79	5	5	
Sum	1	18	464	483	1	27	388	416	3	110	113	

Day	JANUARY, 1892.					FEBRUARY, 1892.			MARCH, 1892.			APRIL, 1892.		
	violent shocks	strong shocks	weak shocks	feeble shocks	sum	weak shocks	feeble shocks	sum	weak shocks	feeble shocks	sum	weak shocks	feeble shocks	sum
1	1	1	1	1
2	2	2	1	...	1	...	1	1
3	1	3	4	...	2	2	1	...	1
4	2	7	9	1	1
5	2	3	5	1	...	1	...	1	1
6	4	4	1	1	2	1	...	1
7	1	1
8	2	2
9	3	3
10	1	1
11	1	1	1	1
12	4	4
13	1	1	...	1	1
14	1	1
15	1	1
16
17	1	1	...	2	2	1	...	1
18	1	1	...	1	1
19	1	1
20	1	1
21	1	1
22	1	1	1	4	5
23	1	...	1	...	2	2
24	2	...	2
25	1	1	2	...	1	1	...	1	1
26	...	1	...	1	2
27	2	2	1	1
28	1	1	...	2	2	1	1	2
29	1	1	2
30	1	1	1	2	3	...	1	1
31	2	2	1	1	2
Sum	1	1	4	37	43	6	23	29	3	13	16	6	5	11

TABLE VII.—DAILY NUMBERS OF EARTHQUAKES AT TSU,
FROM OCTOBER 28th TO DECEMBER 31st, 1891.

Day	OCTOBER, 1891.					NOVEMBER, 1891.				DECEMBER, 1891.		
	violent shocks	strong shocks	weak shocks	feeble shocks	sum	strong shocks	weak shocks	feeble shocks	sum	weak shocks	feeble shocks	sum
1	7	6	13
2	1	3	4	...	1	1
3	6	6	...	3	3
4	1	7	8	1	1	2
5	1	4	5	...	1	1
6	2	6	8
7	4	1	5	...	1	1
8	3	3	1	...	1
9	2	2	1	1	2
10	7	7
11	3	3	...	2	2
12	2	2
13	1	1
14	2	2	...	2	2
15	4	4
16	2	2
17	3	3
18
19
20	1	1
21	2	2
22
23	2	2
24	2	2
25
26	1	1
27	2	2
28	1	12	58	2	73	1	1	...	2	...	1	1
29	32	7	39	4	4	...	1	1
30	...	3	5	9	17	1	1
31	3	7	10	1	1
Sum	1	15	99	25	140	3	17	69	89	3	21	24

TABLE VIII.—DAILY NUMBERS OF EARTHQUAKES AT KYŌTO,
FROM OCTOBER 28th TO DECEMBER 31st, 1891.

Day	OCTOBER, 1891.				NOVEMBER, 1891.			DECEMBER, 1891.		
	strong shocks	weak shocks	feeble shocks	sum	weak shocks	feeble shocks	sum	weak shocks	feeble shocks	sum
1	3	1	4
2	1	...	1	...	1	1
3	1	...	1	2	1	3
4	1	...	1	1	...	1
5
6	1	...	1
7	1	...	1
8	1	...	1
9	1	...	1
10	1	...	1	1	...	1
11	1	1
12
13
14	1	1
15	1	...	1
16
17	1	...	1
18
19
20
21	1	1	2
22
23	1	...	1
24
25
26
27
28	1	11	26	38	1	2	3
29	...	4	13	17	1	...	1
30	...	5	3	8	1	...	1
31	...	2	4	6
Sum	1	22	46	69	16	4	20	6	4	10

TABLE IX.—DAILY NUMBERS OF EARTHQUAKES AT ŌSAKA,
FROM OCTOBER 28th TO DECEMBER 31st, 1891.

Day	OCTOBER, 1891.					NOVEMBER, 1891.			DECEMBER, 1891.		
	strong shocks	weak shocks	feeble shocks	feeble shocks	sum	weak shocks	feeble shocks	sum	weak shocks	feeble shocks	sum
1	2	...	2
2	1	...	1
3	1	1
4	1	...	1
5
6	1	...	1
7	1	...	1
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22	1	1
23
24	1	...	1
25
26
27
28	1	2	8	22	33
29	3	2	5	...	1	1	1	...	1
30	...	1	1	4	6
31	1	1	2
Sum	1	3	13	29	46	5	2	7	3	1	4

TABLE X.—MONTHLY NUMBERS OF EARTHQUAKES AT GIFU,
NAGOYA, TSU, KYŌTO AND ŌSAKA, FROM OCTOBER,
1891, TO DECEMBER, 1933.

Year,	Month	GIFU	NAGOYA	TSU	KYŌTO	ŌSAKA
1891,	10	720	483	140	69	46
	11	1087	416	89	20	7
	12	416	113	24	10	4
1892,	1	164	43	12	5	2
	2	114	29	10	5	2
	3	87	16	7	1	1
	4	90	11	4	2	...
	5	54	11	1
	6	30	12	1	3	...
	7	35	4	1
	8	52	15	7	1	...
	9	107	13	2	1	1
	10	47	8	4	2	...
	11	48	12
	12	39	14	4	2	2
1893,	1	31	5	2	2	...
	2	20	5	2	1	...
	3	52	10	1	...	1
	4	59	11	1	1	...
	5	32	14	1	...	2
	6	12	9	2	...	1
	7	18	14
	8	13	14	1
	9	20	8
	10	19	9	...	1	1
	11	16	4
	12	16	7	1
Sum		3398	1310	316	126	71

TABLE XI.—TIMES OF OCCURRENCE OF EARTHQUAKES AT GIFU,
FROM OCTOBER 28th, 1 p.m., TO NOVEMBER 10th, 12 p.m., 1891.

Oct. 28th, 1891	4.56 p.m.	8.41 p.m.	0.49 a.m.	3. 5 a.m.
1.55 p.m.	4.58	8.42	0.58	3. 9
2. 3	5.25	8.45	1. 1	3. 9½
2.11	5.30	8.50	1. 5	3.13½
2.12	5.50	8.53	1.10	3.19
2.16	6. 4	9. 3	1.12	3.20
2.27	6.11	9. 7	1.16	3.21
2.32	6.14	9.17	1.17	3.23
2.41	6.15	9.21	1.20	3.24
2.45	6.17	9.22	1.29	3.35
2.50	6.18	9.34	1.31	3.50
2.51	6.26	9.36	1.34	3.59
2.55	6.27	9.40	1.38	4. 0
3. 3	6.28	9.44	1.39	4. 5
3. 6	6.31	9.52	1.43	4. 7
3.10	6.34	9.58	1.44	4. 8
3.22	6.42	10.14	1.50	4.10½
3.32	6.44	10.18	1.53	4.15
3.51	6.47	10.21	1.54	4.18
3.52	6.48	10.21	1.56	4.19
3.53	7. 1	10.30	1.57	4.22
3.55	7. 2	10.40	2. 7	4.23
3.58	7. 8	11. 0	2. 9	4.27
4. 1	7.13	11.10	2.11	4.29
4. 6	7.14	11.14	2.19	4.32
4.13	7.18	11.32	2.28	4.33
4.20	7.26	11.41	2.33	4.35
4.22	7.38	11.49	2.34	4.36
4.24	7.40	11.53	2.36	4.38
4.25	7.50	October, 29th	2.39½	4.40
4.41	7.51	0. 1 a.m.	2.42	4.42
4.43	7.54	0.12	2.49	4.45
4.45	7.57	0.14	2.54	4.47½
4.48	8. 7	0.20	2.56	4.48½
4.51	8.22	0.28	2.57	4.51
4.52	8.23	0.37	2.30	4.52
4.54	8.32	0.46	3. 3	4.55½

October, 29th	8.23 a.m.	11.27 a.m.	0.54 p.m.	4.11 p.m.
5. 6 a.m.	8.24	11.33	0.56	4.17
5. 7	8.26	11.36	0.56 $\frac{1}{2}$	4.20
5. 8	8.33	11.40	1. 0	4.22
5. 9	8.35 $\frac{1}{2}$	11.42	1. 2	4.23
5.12	8.37	11.45	1. 2 $\frac{1}{2}$	4.24
5.14	8.40	11.46	1. 4	4.48
5.18	8.41	11.48	1. 4 $\frac{1}{2}$	4.51
5.24	8.43	11.49	1. 7	4.52
5.26	8.56	11.49 $\frac{1}{2}$	1.14	4.58
5.28	9. 6	11.51	1.17	5. 5
5.30	9.13	11.51 $\frac{1}{2}$	1.19	5. 6
5.31	9.16	11.54	1.21	5.15
5.52	9.21	11.55 $\frac{1}{3}$	1.21 $\frac{5}{6}$	5.23
5.53	9.33 $\frac{1}{2}$	11.58	1.30 $\frac{5}{6}$	5.29
5.54	9.34 $\frac{1}{2}$	0. 0 p.m.	1.35	5.34
6.13	9.36	0. 4	1.36	5.41
6.16	9.39	0. 6	1.38	5.45
6.41	9.45	0. 7 $\frac{1}{3}$	1.44 $\frac{5}{6}$	5.51
6.43	10. 1	0. 8	1.51	5.54
7. 4	10.27	0.10	2. 9	5.59
7. 7	10.29	0.13	2.10	6. 2
7.12	10.37	0.14	2.25	6.15
7.13	10.42	0.14 $\frac{1}{2}$	2.35	6.21
7.14	10.43	0.17 $\frac{1}{2}$	2.43	6.30
7.15	10.43 $\frac{1}{2}$	0.18	2.47	6.32
7.23	10.46	0.21	2.51	6.34
7.29	10.48	0.22	2.57	6.41
7.30	10.49	0.23 $\frac{1}{2}$	3.18	6.42
7.33	10.50 $\frac{1}{2}$	0.25 $\frac{1}{3}$	3.22	6.44
7.50	10.59	0.26 $\frac{1}{2}$	3.37	6.46
7.51	11. 0	0.28	3.39	6.47
7.52	11. 5	0.29	3.44	6.50
7.58	11. 7	0.31	3.45	6.53 $\frac{1}{2}$
8. 1	11.15	0.37	3.47 $\frac{1}{2}$	6.56
8.11 $\frac{1}{2}$	11.20	0.46	3.55	6.58
8.13	11.21	0.49	3.57	7. 7
8.22	11.22 $\frac{1}{3}$	0.53	4.10	7.13

October, 29th	10. 7 p.m.	2.14 a.m.	4.29 a.m.	11. 2 a.m.
7.17 p.m.	10.17	2.18	4.30	11.27
7.19	10.21	2.22	4.35	11.32
7.23	10.23	2.36	4.39	11.43
7.25 $\frac{1}{2}$	10.30	2.42	4.41	11.49
7.26	10.31	3. 2	4.42	11.51
7.27	10.50	3. 8	4.46	11.54
7.30	10.55	3.10 $\frac{1}{2}$	5. 5	0. 0 p.m.
7.39	11. 0	3.17	5. 8	0.12
7.45	11.10	3.18	5.12	0.26
7.45 $\frac{1}{2}$	11.15	3.26	5.32	0.43
7.46	October, 30th	3.29	5.39	0.58
7.47	0. 4 a.m.	3.33	5.42	1. 3
7.55	0. 5	3.34	5.47	1.11
7.56	0. 6	3.35	5.57	1.13
8. 4	0.13	3.35 $\frac{1}{2}$	6.10	1.15
8. 9 $\frac{1}{3}$	0.14	3.39	6.13	1.19
8.13	0.30	3.40	7.26	1.24
8.14 $\frac{1}{2}$	0.37	3.40 $\frac{1}{4}$	7.30	1.27 $\frac{1}{2}$
8.15	0.41	3.46	7.33	1.40
8.19	0.46	3.47	7.40	1.43
8.35	0.47	3.48	7.43	1.47
8.36	0.48	3.52	8. 0	2.17
8.41	0.53	3.55	8. 4 $\frac{1}{2}$	2.23
8.42	0.56	3.57	8.10	3. 6
8.49	1.10 $\frac{1}{2}$	4. 0	8.13	3.50 $\frac{1}{2}$
8.50	1.14	4. 1	8.14	4. 8 $\frac{1}{2}$
9. 5	1.17	4. 2	8.15	4.19
9.35	1.18	4. 5	8.21	4.35
9.40	1.22	4. 7	8.30	4.36
9.45	1.32	4. 8	8.50	4.47
9.46	1.37	4.8 $\frac{1}{2}$	9. 6	4.58
9.47	1.39	4. 9	9.35	5. 8
9.56	1.40	4.19	9.49	5.14
9.57	1.49	4.24	9.57	5.19
9.59	1.50	4.25	10.12	5.23
10. 1	1.54	4.27	10.14	6. 4
10. 4	2. 6	4.27 $\frac{1}{3}$	10.49	6.26

October, 30th	2.40 a.m.	6. 8 a.m.	2.53½ p.m.	10.16 p.m.
6.34 p.m.	2.46	6.14	3. 1	10.29
6.35	2.48	6.28	3. 8	10.34
6.40	2.50	6.35	3.29	10.36
7. 0	2.51	6.48	3.42	10.48
7. 2	2.56	6.57	3.51	10.59
7. 5	2.57	6.59	4. 1	11. 6
7. 7	2.59	7. 7	4.12	11.43
7.25	3. 5	7.13	4.40½	11.52
7.27	3.10	7.22	4.51	Nov., 1st
7.31	3.22	7.24	5.12	0. 7 a.m.
7.42	3.25	7.40	5.45	0. 8
7.43	3.46	7.47	6. 5	0.13
7.49	4. 4	8.18	6.12½	0.18
7.53	4. 8	8.26	6.14	0.19
7.57	4.10	8.38	6.16	0.42
8. 1	4.14	8.51	6.22	0.55
8. 2	4.17	8.53	6.24	1. 2
8. 7	4.19	8.55	6.57	1.23
8.12	4.31	8.57½	7. 8	1.29
8.14	4.37	9.11	7.12	1.30
8.22	4.39	9.27	7.16	1.34
8.28	4.43	9.34	7.17	1.35
8.30	4.49	9.39	7.21	1.44
8.47	4.55	10.23	7.27	1.45
8.50	5. 4	10.30	7.29	1.57
8.53	5. 9	10.35	7.32	2.16
9. 0	5.15	11.14	7.34	2.27
9.17	5.17	11.19	7.35	2.34
9.22	5.21	11.27½	7.55	2.42
9.25	5.25	0. 5 p.m.	8.11	2.44
9.30	5.29	0. 8	8.14	2.47
10.30	5.30	0.36	8.15	2.50
11.47	5.34	1. 2½	8.24	2.54
October, 31st	5.38	1.35	8.35	2.57
0.20 a.m.	5.40	1.48	8.46	3.22
2.20	5.49	2.22	9.14	4.33
2.38	5.58	2.32	9.35	4.40

Nov. 1st	3.10 p.m.	0.33 a.m.	10.45 a.m.	8.19 p.m.
4.50 a.m.	3.47	0.37	10.53	8.44
5. 7	3.51	0.40	11.19	8.52
5.10	4. 6	0.45	11.46	9. 5
5.17	4.55	0.47	11.50	9.26
5.19	5.28	0.55	0.11 $\frac{1}{2}$ p.m.	9.35
6. 6	5.38	1.15	0.24	9.52
6.29	5.55	2.12	0.47	10.14
6.40	5.58	2.25	0.51	10.17
6.43	6.13	2.36	1. 7	10.37
6.44	6.28	2.58	1.20	10.42
6.47 $\frac{1}{2}$	6.31	3. 7	1.20 $\frac{1}{2}$	11.43
6.52	6.45	3.15	1.37	11.57
7.12 $\frac{1}{2}$	6.49	3.26	2. 9	Nov., 3rd
7.21	6.55	3.29	2.23	0.12 a.m.
7.30	6.59	3.47	2.32	0.31
7.41	7.10	3.50	2.34	1.47
8.31	7.16	3.52	2.38	1.52
8.46	7.45	4. 2	2.45	2. 7
9.17	7.50	4. 5	3. 1	3. 0
9.37	7.57 $\frac{1}{2}$	4.10	3. 5	3. 3
10. 1	8.20	4.29	4.24	3.13
10.15	8.35	4.29 $\frac{1}{2}$	4.50	4. 3
10.21	8.38	4.54	4.58	4.15
10.38	9.18	5.46	5.40	4.28
10.43	9.26	5.48	5.50	4.47
10.48	9.27	6. 0	6.29	4.55
11.42	9.30	6.43	6.36	4.58
11.45	9.33	7.11	7. 7	5. 2
11.46	9.35	7.21	7.12	5.32
11.56	9.38	7.32	7.16	5.56
0.46 p.m.	9.58	7.33	7.19	6.48 $\frac{1}{2}$
1. 5	10.20	8.53	7.29	6.53
1.14	10.21	9.17	7.45	6.56
1.47	Nov., 2nd	9.30	7.48	7. 6
2.43	0. 2 a.m.	9.46	7.50	7. 7
2.55	0.16	9.55	7.52	7.12
3. 0	0.21	10.20	8. 8	7.37

Nov., 3rd	7. 3 p.m.	5. 0 a.m.	5.31 p.m.	6.24 a.m.
7.41 a.m.	7.28	5.29	6.19	6.33
8. 4	7.29	5.34	6.58	7.10
8.20	7.38	5.54	7. 0	7.28
8.32	7.50	6. 8	7.10	7.48
8.53	7.55	6.26	7.16	8. 9
8.55	8.12	6.35	7.19	8.46
9.15	8.15½	6.36	7.45	9.16
9.45	8.40	6.47	8.30	9.18
10.20	8.49	7. 4	8.40	9.58
10.21	8.52	7.47	8.51	10.24
10.32	9.23	8.15	9.24	11.16
11. 2	9.34	8.17	9.35	11.32
11.11	9.35	9.20	9.50	0.15 p.m.
11.30	9.47	9.22	9.54	1.48
11.41	10.14	9.38	10.17½	2.51
0.38 p.m.	10.31	9.55	10.39	3.35
0.44	10.36	9.56	10.41	3.59
0.58	11.43	10.27	10.59	5.10
1. 0	11.50	10.44	11. 0	5.13
1. 8	Nov., 4th	10.48	11.10	5.28
1.16	0.10 a.m.	10.57½	11.17	5.38
2.13½	0.32	11. 4½	11.32	6.48
2.56	0.52	11.38½	Nov., 5th	6.52
3.44	0.57	11.51	0.22 a.m.	6.54
3.54	1. 3	0.34 p.m.	1. 1	7.39
3.58	1.18	0.35	1.14	7.50
4. 7	1.27	0.37	1.52	8. 8
4.47	1.40	1. 0	2. 3	8.16
4.49	1.57	1.17	2.22	8.23
4.50	1.58	1.49	3. 2	8.36
5. 7	2.12	2. 7	3.22	9.10
5.28	2.25	2.19	3.48	9.19
5.34	2.39	2.20	4. 4	9.25
5.42	3. 0	3.15	5. 0	9.28
6.19	3. 1	3.34	5.15	10.24
6.36	4. 5	4. 0	5.22	10.58
6.45	4.20	5. 2	5.24	11.35

Nov., 5th	1.14 p.m.	1.45 a.m.	11.28 p.m.	10.26 p.m.
11.37 p.m.	1.25	3.29	11.30	10.58
Nov., 6th	2.16	4.21	Nov., 8th	11.38
0.10 a.m.	2.17	4.55	0.22 a.m.	11.39
0.12	2.39	5.52	0.53	11.45
1. 4	2.45	6. 7	1. 5	11.48
1.17	3.42	6. 9	1. 7	11.56
2. 7	3.50	6.47	2.32	Nov., 9th
2.47	3.55	6.57	4.15	1.18 a.m.
2.49	5. 7	7.35	4.35	1.27
3.17	5.35	7.47	5.50	1.40
3.26	5.47	8.12	5.55	1.46
3.30	5.57	8.26	6.39	2.47
4. 4	6. 7	8.57	7.26	3.25
4. 8	6.42	9.27 $\frac{1}{2}$	7.48	3.36
4.20	7. 2	9.49	7.50	4. 0
4.25	7. 6	11.44	8. 3	4.14
4.32	7.11	0.14 p.m.	8.24	4.20
4.39	7.23	2. 0	9.12	4.22
4.56	7.51	2.30	10. 7	4.32 $\frac{1}{2}$
5. 9	8. 8	2.36	10.41	4.43
5.32	8.10	2.48	10.44	5.25
5.52	8.16	3.49	10.58	5.27
6.18	9.25	4.34	11. 8 $\frac{1}{2}$	6. 8
6.57	10.10	5.12	11.17	6.20
7. 7	10.30	5.23	11.27 $\frac{1}{2}$	6.23
9.42	10.50	5.47	11.45 $\frac{1}{2}$	6.37
9.58	11. 2	6. 6	2.44 p.m.	6.47
10.24	11. 5	7. 8	4. 6	7.24
11. 2	11.32	8. 5	4.15	8. 0
11.15	11.35	8.10	5. 1	9.30 $\frac{1}{2}$
11.22	11.41	8.43	6.33	10.59
0. 9 p.m.	Nov., 7th	9. 1	6.36 $\frac{1}{2}$	11.48
0.25	0. 9 a.m.	9.13	7. 2	0.34 p.m.
0.36	0.25	9.40	8.10	2.13
0.39	0.36	9.58	8.14	2.52
0.44	0.48	10.52	8.40	3. 7
0.52	0.58	11. 7	9.44	3.39 $\frac{1}{2}$

Nov., 9th	10.42 $\frac{1}{2}$ p.m.	3.35 a.m.	9.39 a.m.	7. 8 p.m.
4. 4 $\frac{1}{2}$ p.m.	11. 0	5. 0	10.42	7.12
4.40	11. 4	5.39	11. 7	7.21
5. 8	Nov., 10th	6. 8	11.38 $\frac{1}{2}$	8. 1
5.33	0. 0 a.m.	6.23	0.46 $\frac{1}{2}$ p.m.	9.13
6. 3	1.24	6.34	3.55	9.59
7.25	1.45	7.32	5.37 $\frac{1}{2}$	10.10
9. 6	1.57	7.45	5.38 $\frac{1}{2}$	10.43
9. 5	2.48	7.58	5.40	11.10
9.41 $\frac{1}{2}$	2.57	8.51	5.51	11.30
10.22	2.58	9.31	6.47	11.33

TABLE XII.—TIMES OF OCCURRENCE OF EARTHQUAKES AT NAGOYA,
FROM OCTOBER 28th, 1 p.m., TO NOVEMBER 10th, 12 p.m., 1891.

Oct. 28th, 1891	4.24.30 p.m.	8. 7.40 p.m.	11.13. 0 p.m.	3.30.50 a.m.
1.9.45 p.m.	4.24.52	8. 9.35	11.18.31	3.34.30
1.16.5	4.28.10	8.10.46	11.21.26	3.39. 0
1.22.45	4.33.59	8.23.24	11.40.30	3.40.50
1.43.14	4.44.32	8.32. 0	11.44.11	3.41.30
1.54.13	4.51.40	8.32.36	October. 29th,	3.46.55
1.54.32	4.56. 8	8.36. 6	0. 2. 0 a.m.	3.55.35
1.55.00	4.57.10	8.41.49	0.19.45	3.57.28
2. 2.24	5. 0.30	8.46.41	0.22.30	4. 2.10
2. 2.50	5. 4.40	8.52.36	0.34.11	4.13.40
2.10.31	5.15. 2	8.53. 4	0.38.39	4.21. 0
2.19.33	5.17. 1	8.55.15	0.46.29	4.21.40
2.21. 8	5.31.25	8.57.49	0.58.26	4.28.30
2.22.30	5.33. 8	9. 3. 0	1. 1.43	4.35. 0
2.25.46	5.39.55	9. 4. 8	1. 7.38	4.36.30
2.31.40	5.42.15	9. 8.12	1.16. 0	4.39.30
2.35.18	5.45.40	9.21.12	1.20.20	4.44.15
2.29.44	5.51.59	9.24. 9	1.30.29	4.50.30
2.41.37	6. 4.10	9.33.36	1.49.33	4.55. 0
2.48.56	6.14.35	9.41. 3	1.53. 1	5. 3.30
2.50. 2	6.16.39	9.46.10	1.57. 0	5. 7.30
2.56.13	6.19. 5	9.44.13	2.18. 8	5.12.30
3. 2. 5	6.26.25	9.52.40	2.24.25	5.23.50
3.21. 0	6.27.50	10. 7.59	2.29.26	5.28. 0
3.22. 5	6.38.27	10.11.33	2.33.46	5.50. 0
3.23.35	6.47.53	10.17.45	2.35.10	5.53.30
3.24.54	7.12.45	10.30. 0	2.38.21	6. 4.30
3.31.48	7.13.40	10.30.34	2.46.58	6.13.30
3.49.55	7.16.40	10.39.14	2.48.24	6.13.59
3.51.23	7.17.45	10.44.12	3. 0.59	6.43.25
3.52.14	7.19.25	10.47.21	3. 2.57	6.45.20
3.53.12	7.26.30	10.53. 2	3. 7. 0	7. 0. 0
3.55.22	7.29.31	11. 2.12	3. 4.20	7. 3.30
4.13.15	7.34. 5	11. 3.36	3.11.40	7.12. 0
4.15. 8	7.41.14	11. 7.26	3.19.10	7.20.35
4.21.25	7.42. 7	11. 9.32	3.20.55	7.30.45
4.22.16	7.46.30	11.11.21	3.28.50	7.40. 0

7.52.35 a.m.	0.59.22 p.m.	7. 7.30 p.m.	0.40. 4 a.m.	11.31.25 a.m.
7.56.10	1.19.39	7.30.20	0.47.12	11.50.25
7.58.32	1.24.12	7.39.10	0.49.29	0.29.32
8. 4.45	1.34. 4	7.45.20	0.51.20	0.43.27 p.m.
8.11.28	1.34.22	7.46.10	1. 7.32	1. 2.30
8.14.30	2. 8.23	7.55.24	1. 9.50	1. 4.19
8.26.10	2. 8.50	8.18.21	1.13.31	1.47.21
8.27.15	2.10.15	8.33.22	1.15.50	2.24.12
8.33.37	2.42.30	8.31.50	1.25.15	3. 6. 5
8.36. 7	2.53. 0	8.41.15	1.28. 0	3.17.28
8.37.53	2.59.40	8.48. 0	1.30. 0	3.59.25
8.41. 1	3. 3.50	8.51.37	1.30.22	4.28.35
8.50.57	3. 5.32	8.58.26	1.39.45	4.39. 5
9. 7.20	3.14.44	9. 1.45	1.51.25	5. 8.29
9.15.56	3.16. 5	9.34.36	1.54.10	5. 9.19
9.21.25	3.22.25	9.43.33	1.56.21	5.10.10
9.24.18	3.33.10	9.51.49	2. 6.20	5.24.15
9.40. 5	3.35. 0	10.29.25	2.27.18	6. 1. 6
9.44. 0	3.44.25	10.44.30	2.36.15	6. 7.32
9.54.40	3.45. 0	10.48.37	3.35.15	6.27.12
10.20.50	4.19.50	10.53.25	3.40. 0	6.36. 2
10.37.40	4.22.20	11. 2. 0	4.16.45	6.42.32
10.44.20	4.37.15	11. 9.16	4.35. 0	7. 7.22
10.46.31	4.43.40	11.14.50	4.53.42	7.17.17
10.49.42	4.49.20	11.32.29	5. 7. 5	7.49.22
11. 7.20	5. 3.10	11.44.19	5.42.17	7.53.10
11.12.30	5.25.50	11.46. 0	5.52.22	8.49.45
11.22.36	5.28.10	October 30th	5.57. 2	9. 6.52
11.27.40	5.30. 0	0. 4.12 a.m.	6.10. 0	9.25.51
11.34.45	5.43.45	0. 5. 0	6.17.31	9.36.32
11.51.50	5.47.25	0. 6.51	7.19.20	10. 4. 7
0. 6.25 p.m.	5.57.27	0.12.35	7.54.26	10.10. 7
0.13.40	6. 4. 0	0.14.40	8.17.20	10.21.32
0.17.15	6.32.12	0.16.10	8.17.20	10.23.22
0.46.37	6.39. 0	0.29. 0	8.30.22	10.27.35
0.47.50	6.45.30	0.29.40	9. 5. 5	10.30.50
0.51. 0	6.51.20	0.32. 0	10. 5.27	10.37.52
0.56.40	7. 4.23	0.37.15	11. 2.40	10.58.35

10.54.54 p.m.	9. 2. 5 a.m.	11.44.15 p.m.	9.41.15 a.m.	3.20. 4 p.m.
11.10. 0	10.33.25	Nov., 1st	10.41. 2	4.58.31
11.10.36	10.44.10	0. 8.27 a.m.	10.52.20	5.30.30
11.16.21	11. 0.36	0.47. 0	11.12.55	8. 9.45
11.28. 5	0.25. 0 p.m.	0.48.46	11.33.31	8.14.25
11.52.28	0.34.10	0.50.40	0.24.50 p.m.	8.19.50
11.53.10	0.37. 0	1.18.55	3. 0.22	9.39.10
11.55.10	0.44.39	1.28.40	4.34.45	9.46.55
October, 31st	1. 4.45	1.36.34	5.18.21	9.48. 5
0. 1.30 a.m.	1.25. 0	1.44.32	5.57.51	10.18.40
0. 6.31	1.56.15	1.46.10	6.30. 8	10.42. 5
0.15.13	2.17.35	1.48.44	7.56.27	11.25. 0
0.29. 7	2.25.41	1.50.44	8.19.52	11.55.30
0.34.28	2.37.25	1.52.56	8.27.10	Nov., 3rd
0.50. 0	2.48. 6	2.13. 0	8.39.38	0. 0.15 a.m.
0.56.50	3. 5.10	2.28.20	9.11.55	0. 2. 5
0.58.12	3.29.59	2.35.13	9.19.21	0.35. 0
0.59.24	3.51.35	2.41.52	9.28.45	0.51.45
1.11. 5	4.18. 8	3. 7.36	9.37.22	0.55. 0
2. 5.52	4.39.20	3.20.24	10.51.40	1.17.35
2.27.32	6.13.28	3.46.17	Nov., 2nd	1.27.30
2.48.50	6.29.25	3.47.58	0. 7.50 a.m.	1.25.10
2.58. 0	7.18. 0	3.52. 0	0.24.20	3. 5.35
3.10.14	7.37. 5	4.24.10	0.34.36	3.24.10
4. 0.55	7.55.52	4.27.15	1.53.10	5.33. 0
4. 9. 5	8.34.25	4.37.40	1.56.45	5.43.42
4.21.30	8.48.20	4.44.25	2. 8. 0	8.16.25
5.16.41	9.12.30	4.51.31	2.27.45	8.22.49
7. 0. 0	9.27.50	5. 9.21	3. 6.15	8.32.56
7.15.25	9.43.14	5.15. 0	4.14.32	8.36.40
7.23.30	9.49.55	5.19.10	5. 0.30	9.16.59
7.49.30	10. 0.20	5.37.25	6.12.59	9.32.34
7.51.27	10. 8.50	6. 2. 0	9.50.40	9.46. 8
8. 4. 2	10.28. 0	7. 7.15	10.52. 0	10. 4.45
8.22.14	10.49. 5	7.12.46	11.52.34	10.23. 2
8.27.58	11.20.35	8. 3.10	1.58.23 p.m.	11. 0.29
8.56.30	11.24.25	8.30.15	2.12.25	11.43.10
8.58.31	11.43. 5	9. 4.40	3. 4.58	11.43.37

0. 1.25 p.m.	Nov., 5th.	11.19.41 a.m.	3. 0. 0 p.m.	4.20.10 a.m.
1.22.30	3. 1.30 a.m.	1.25.15 p.m.	7.39.59	4.34. 2
4.50.40	3.54. 0	3.31.21	9. 3. 4	5. 7.12
4.51.42	4.45.50	7.43.30	11.14.30	5.13.40
7.29.20	6.33.15	7.53. 0	Nov., 8th	6.25. 0
8.51.20	6.34. 5	8.35.15	0.54.40 a.m.	7.17.25
9.54.20	7. 7.35	8.39. 0	1. 0. 5	8.20. 2
Nov., 4th	7.49. 0	9.23.40	1.24.25	0.38.14 p.m.
0.18.29 a.m.	9.58.40	10.11.15	1.52.40	2. 1.10
0.45.25	10.24.25	10.32.25	3. 8.45	7.32.10
1.20.34	11.57.12	10.41.52	3.24.43	7.52.10
1.27.45	0.44.52 p.m.	Nov., 7th	5.23.12	8.55. 0
1.33.46	3.35.13	0. 1.20 a.m.	9. 2.59	Nov., 10th
2. 0. 0	5. 9.56	0. 9.21	11.10.58	2.44.30 a.m.
4. 2.10	5.42.45	0.10. 0	1.28. 5 p.m.	5. 2.50
4.12.21	7.10.21	0.12. 9	1.30. 0	6. 3.45
4.21.50	9.29.56	1. 1. 5	1.47.15	6.24.27
6.37.45	10.16.15	1.46.12	6.37.50	8.34. 0
8.19.15	10.41.18	2.45.30	9.34.20	9.32.35
11.59.30	10.45.38	3.16.40	10.38.50	11.58. 7
2.22.31 p.m.	11.43. 5	4.18.20	11.36.50	5.10.24 p.m.
3.15.18	Nov., 6th	4.18.50	11.46.39	5.41.34
3.56.45	0.50. 0 a.m.	5.47.55	11.58.15	6.14.19
4.18. 8	1. 0.15	5.55. 0	Nov., 9th	7.27. 7
4.39.20	1.19. 7	6.38.50	0.36.25 a.m.	8. 4. 0
4.44.35	4.25.40	9.18.15	2.48.47	
7. 1.20	7.26.45	1. 9.22 p.m.	3.25. 0	
8.22. 5	9.59.45	2.48.58	3.37.20	

TABLE XIII.—HOURLY NUMBERS OF EARTHQUAKES AT GIFU,
FROM OCTOBER 28th TO NOVEMBER 16th, 1891.

Interval \ Day	OCTOBER, 1891.				NOVEMBER, 1891.										sum
	28	29	30	31	1	2	3	4	5	6	7	8	9	10	
0—1 a.m.	...	9	13	1	7	9	2	4	1	2	5	2	...	1	56
1—2	...	19	12	...	9	1	2	6	3	2	1	2	4	3	64
2—3	...	15	6	10	9	4	1	3	2	3	...	1	1	3	58
3—4	...	13	20	5	1	7	3	2	3	3	1	...	2	1	62
4—5	...	25	20	12	3	6	6	2	1	7	2	2	6	...	92
5—6	...	15	8	13	4	2	3	4	4	3	1	2	2	2	63
6—7	...	4	2	7	7	2	3	5	2	2	4	1	5	3	47
7—8	...	14	5	6	4	4	5	2	3	1	2	3	1	3	53
8—9	...	14	9	7	2	1	5	2	2	...	3	2	1	1	49
9—10	...	9	4	4	2	4	2	5	3	2	2	1	1	2	41
10—11	...	12	3	3	6	3	3	4	1	1	...	4	1	1	42
11—12	...	22	7	3	4	3	4	3	2	3	1	3	1	2	58
0—1 p.m.	...	26	5	3	1	4	3	3	1	6	1	1	54
1—2	...	17	10	3	3	4	3	3	1	2	1	1	48
2—3	11	8	2	3	2	6	2	3	1	4	4	1	2	...	38
3—4	10	9	2	5	4	2	3	2	2	3	1	...	2	1	36
4—5	16	11	6	4	2	3	4	1	1	2	2	...	36
5—6	3	11	4	2	4	2	4	2	4	4	3	1	2	4	47
6—7	15	15	5	7	7	2	3	2	3	2	1	2	1	1	51
7—8	13	16	12	11	5	9	6	5	2	5	1	1	1	3	77
8—9	9	12	11	6	3	4	5	3	4	3	3	3	...	1	58
9—10	11	9	5	2	8	4	4	4	4	1	4	1	3	2	51
10—11	6	10	1	6	2	4	3	4	2	3	1	2	2	2	42
11—12	7	3	1	3	...	2	2	4	2	5	3	5	2	3	35
Sum	101	318	173	126	99	92	81	78	53	67	45	42	43	40	1258

Note. In forming the hourly "sums," the record for October 28th has not been taken into account.

{ During Night, or between 6 p.m. and 6 a.m., 709
 { " Day, " " " 6 a.m. and 6 p.m., 549
 1258

TABLE XIV.—HOURLY NUMBERS OF EARTHQUAKES OF NAGOYA,
FROM OCTOBER 28th TO NOVEMBER 10th, 1891.

Interval \ Day	OCTOBER, 1891				NOVEMBER, 1891										sum
	28	29	30	31	1	2	3	4	5	6	7	8	9	10	
0—1 a.m.	...	7	14	9	4	3	5	2	...	1	4	1	1	...	51
1—2	...	8	12	1	8	2	3	3	...	2	2	3	44
2—3	...	8	3	4	4	2	...	1	1	...	1	1	25
3—4	...	16	2	1	5	1	2	...	2	...	1	2	2	...	34
4—5	...	11	3	3	5	1	...	3	1	1	2	...	2	...	32
5—6	...	7	4	1	4	1	2	2	1	2	1	25
6—7	...	5	2	...	1	1	...	1	2	...	1	...	1	2	16
7—8	...	9	2	5	2	2	1	1	...	22
8—9	...	10	2	5	2	...	4	1	1	1	26
9—10	...	7	1	1	2	1	3	...	1	1	1	1	...	1	20
10—11	...	5	1	2	2	1	2	...	1	14
11—12	...	6	3	1	2	1	3	1	1	1	...	1	...	1	21
0—1 p.m.	...	8	2	4	1	...	1	...	1	1	...	18
1—2	7	4	3	3	...	1	1	1	1	3	17
2—3	14	6	1	4	...	1	...	1	1	...	1	...	15
3—4	11	9	3	3	1	2	...	2	1	1	1	23
4—5	12	5	2	2	1	1	2	3	16
5—6	10	7	4	...	2	1	2	2	18
6—7	8	5	5	2	1	1	...	1	15
7—8	11	7	4	3	1	...	1	1	1	2	1	...	2	1	24
8—9	13	7	1	2	3	3	1	1	...	2	1	1	22
9—10	10	4	3	4	4	3	1	...	1	1	1	1	23
10—11	9	4	9	4	1	2	3	3	...	1	27
11—12	10	6	7	4	...	2	1	...	1	3	24
Sum	115	171	93	68	56	30	31	20	20	17	20	18	16	12	572 ¹

Note.—In forming the hourly "sums," the record for October 28th has not been taken into account.

{ During Night, or between 6 p.m. and 6 a.m., 346
 { " Day, " " 6 a.m. " 6 p.m., $\frac{226}{572}$

1. The times of earthquake occurrence were lost in a few cases, and therefore this number is slightly less than according to Table VI.

TABLE XV.—DAILY NUMBERS OF EARTHQUAKES AT CHIRAN,
FROM SEPTEMBER 7th, 1893, TO JANUARY 31st, 1894. ¹

Day	SEPTEMBER, 1893.					OCTOBER, 1893.				
	strong shocks	weak shocks	feeble shocks or sounds	sum	activity	strong shocks	weak shocks	feeble shocks or sounds	sum	activity
1	2	2	2
2	2	...	2	2
3	2	2	2
4	2	2	2
5
6
7	1	1	1
8	4	7	34	45	49
9	5	12	27	44	49
10	1	4	23	28	29
11	...	9	14	23	23
12	...	3	15	18	18	...	1	...	1	1
13	1	1	14	16	17
14	3	2	16	21	24
15	...	4	8	12	12	1	1	...	2	3
16	8	8	8	1	1	1
17	1	1	3	5	6	1	1	1
18	...	2	5	7	7
19	5	5	5
20	...	3	3	6	6
21	4	4	4
22	3	3	3
23	...	1	3	4	4
24	...	1	1	2	2	...	1	1	2	2
25	...	1	2	3	3	1	1	1	3	4
26	...	1	3	4	4	...	1	7	8	8
27	2	2	2
28	1	2	3	3
29	...	2	1	3	3	...	1	1	2	2
30	2	2	2
31
Sum	15	54	194	263	278	2	9	23	34	36

1. The Kagoshima earthquake took place on September 7th at 2.46 a.m., and the record of after shocks was not taken till about 9 p.m. of the same day. The figures in this Table are the numbers of earthquakes during successive twenty-four hours between 9 p.m. of each day and 9 p.m. of the next.

Day	NOVEMBER, 1893.				DECEMBER, 1893.				JANUARY, 1894.				
	weak shocks	feeble shocks or sounds	sum	activity	weak shocks	feeble shocks or sounds	sum	activity	strong shocks	weak shocks	feeble shocks or sounds	sum	activity
1	1	1	1
2
3
4	1	2	...	3	4
5	1	6	7	7
6	...	1	1	1	3	3	3
7	1	...	1	1
8	...	2	2	2	1	1	1
9	1	3	4	4
10	1	1	1
11	2	2	2
12	1	1	1
13	1	1	2	2	2	2	1
14	2	...	2	2	5	5	5
15	2	2	2
16	1	1	1
17	1	1	1
18	2	2	4	4
19	3	3	3
20
21	1	1	1
22	2	2	2	1	1	1
23	1	1	1
24	1	1	1
25	4	1	5	5	1	1	1
26	2	...	2	2	4	4	4
27
28	...	2	2	2	1	...	1	1
29	2	2	2
30	2	...	2	2	2	4	6	6
31	2	...	2	2
Sum	7	7	14	14	10	4	14	14	1	6	47	54	55

TABLE XVI.—HOURLY NUMBERS OF EARTHQUAKES AT CHIRAN,
FROM 8th TO 21st, SEPTEMBER, 1893.

Interval	Day	SEPTEMBER. 1893.														sum
		8	9	10	11	12	13	14	15	16	17	18	19	20	21	
0—1 a.m.		1	1	1	3
1—2		5	1	1	2	1	1	4	15
2—3		5	2	2	2	1	12
3—4		1	1	1	1	1	1	2	1	9
4—5		1	1	2	...	1	2	...	1	1	9
5—6		...	3	3	4	...	1	2	2	1	1	17
6—7		1	1	2	...	1	1	1	...	1	...	1	9
7—8		4	2	...	2	1	2	1	12
8—9		2	3	1	1	1	...	1	1	10
9—10		4	7	...	1	2	1	15
10—11		...	1	1	...	1	2	...	1	...	6
11—12		3	3	2	1	1	1	...	11
0—1 p.m.		1	1	1	1	4
1—2		1	2	1	1	5	1	...	1	12
2—3		...	2	3	1	2	1	2	1	1	13
3—4		1	3	2	1	1	8
4—5		3	4	...	1	1	1	10
5—6		...	1	1	...	1	1	4
6—7		3	2	...	2	1	...	1	1	10
7—8		3	2	3	1	1	1	...	11
8—9		1	...	2	3
9—10		2	1	1	2	1	1	...	8
10—11		3	1	...	1	1	1	4	1	12
11—12		4	1	3	1	1	10
Sum		42	41	30	21	20	14	25	6	7	6	6	5	6	4	233

{ During Night, or between 6 p.m. and 6 a.m., 119
 { „ Day, „ „ 6 a.m. „ 6 p.m., 114
 233

TABLE XVII.—MONTHLY AND YEARLY NUMBER OF EARTHQUAKES
INSTRUMENTALLY RECORDED IN TOKIO FROM JANUARY,
1876, TO DECEMBER, 1893.

Month Year	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	sum
1876	3	4	6	11	5	3	3	5	3	3	4	6	56
1877	4	5	6	5	8	9	6	4	1	8	6	9	71
1878	3	8	7	2	5	4	4	1	2	4	6	4	50
1879	6	7	14	...	9	4	3	4	1	7	6	9	70
1880	9	9	6	6	2	9	8	4	1	3	10	10	77
1881	13	8	8	8	4	3	3	3	2	3	3	8	66
1882	4	7	15	6	3	2	2	1	1	4	1	...	46
1883	6	...	3	3	6	2	3	1	...	1	3	4	32
1884	5	2	8	2	9	4	1	4	2	8	8	15	68
1885	7	9	8	4	3	6	...	3	8	10	3	7	68
1886	3	3	3	2	8	4	2	8	7	4	2	8	54
1887	10	4	3	8	13	5	6	2	10	...	5	14	80
1888	4	15	7	7	11	9	9	7	11	4	13	4	101
1889	5	16	11	18	13	7	5	8	7	8	9	6	113
1890	5	5	6	15	14	5	12	7	4	8	10	2	93
1891	1	4	6	7	10	7	8	4	4	45	12	15	123
1892	9	9	2	7	8	6	7	2	4	10	4	5	73
1893	6	4	3	7	11	9	4	3	5	3	3	1	59
sum	103	119	122	118	142	98	86	71	73	133	108	127	1300
Average	5.8	6.6	6.8	6.6	7.9	5.4	4.8	4.0	4.0	7.4	6.0	7.0	72.2

Note.—The greatest monthly earthquake number, namely 15, occurred in October, 1891. Of this however, 23 took place within the last four days of the month, being due to the residual effect of the great Mino-Owari Earthquake. We may take $(15-28) \times \frac{31}{27} \doteq 19$ as the proper number of shocks for October, 1891. Making this modification, the total number for 1891 becomes 97, and the average monthly number for October becomes 6. These latter values have been used in drawing curves, Figs. 18, (4) and 20.

TABLE XVIII.—HOURLY DISTRIBUTION OF 1168 EARTHQUAKES
RECORDED INSTRUMENTALLY IN TOKYO FROM
1876 TO 1891.

Month Interval	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	sum
a.m.													
0—1	4	6	2	3	3	3	3	2	4	3	7	7	47
1—2	1	1	3	1	4	3	1	1	3	3	4	3	28
2—3	1	7	3	5	5	2	5	3	4	4	8	2	49
3—4	1	...	4	2	2	4	3	3	1	6	3	4	33
4—5	2	2	7	5	4	...	3	4	2	3	3	7	42
5—6	2	4	4	7	4	2	...	3	6	3	2	9	46
6—7	3	6	8	2	5	2	2	...	2	7	2	7	46
7—8	5	4	4	4	4	4	2	4	1	12	...	7	52
8—9	3	4	2	4	11	6	...	1	6	2	3	2	44
9—10	4	5	4	3	11	6	4	5	1	7	8	6	64
10—11	5	4	2	4	6	5	2	3	4	4	4	2	45
11—12	4	3	4	3	4	3	2	3	1	4	1	6	38
p.m.													
0—1	2	2	2	4	9	8	4	2	2	1	6	7	49
1—2	3	5	5	4	4	4	3	4	3	7	9	4	55
2—3	3	9	6	6	3	4	6	4	1	4	...	4	50
3—4	7	12	6	8	6	3	6	2	4	4	9	0	67
4—5	3	1	11	8	3	4	5	3	3	4	5	4	54
5—6	4	4	4	4	3	2	5	2	3	3	2	3	44
6—7	4	4	6	2	6	6	1	4	1	5	2	3	44
7—8	6	3	7	3	2	2	3	8	5	4	43
8—9	4	4	4	4	4	...	7	3	4	13	7	11	65
9—10	4	5	5	7	6	5	5	4	1	4	6	6	58
10—11	8	4	7	6	3	2	3	2	1	6	2	9	53
11—12	5	7	7	5	7	5	3	2	1	3	3	4	52
Sum	88	106	117	104	124	83	75	66	63	120	101	121	1168

{ During Night, or between 6 p.m. and 6 a.m., 560
 { „ Day, „ „ 5 a.m. „ 6 p.m., 608
 1168

TABLE XIX.—HOURLY DISTRIBUTION OF 3842 EARTHQUAKES
IN JAPAN DURING 6 YEARS, FROM 1885 TO 1890.

Month Interval	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	sum
a.m. 0— 1	8	13	5	11	11	13	10	11	7	7	28	9	133

ERRATUM AND NOTE.

Page 150, line 7, *for* September 8th to 21st *read* 8th to 21st, September.

NOTE TO TABLE XIX.—The numbers given in this table include those of the after-shocks of the Kumamoto earthquake of July 28th, 1889. When the shocks which happened at Kumamoto during the latter half of 1889 are excluded, the monthly distribution of earthquakes becomes as follows.

Month.	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Number of eqkes.	51	57	51	49	65	43	43	42	43	45	57	18

The curve of monthly earthquake frequency [Fig. 21, (5)] has been drawn from the above modified data.

11—12	16	21	13	10	11	10	11	19	12	16	19	13	171
Sum	303	340	308	293	391	259	270	392	298	307	369	312	3842

{ During Night, or between 6 p.m. and 6 a.m., 1902
 { „ Day, „ „ 6 a.m. „ 6 p.m., 1880
 3842

TABLE XX.—HOURLY DISTRIBUTION OF 5333 EARTHQUAKES
IN JAPAN.

Interval	GIFU.	CHIRAN	WHOLE JAPAN	sum
0— 1 a.m.	56	3	133	192
1— 2	64	15	153	232
2— 3	58	12	211	281
3— 4	62	9	176	247
4— 5	92	9	139	240
5— 6	63	17	166	246
6— 7	47	9	153	209
7— 8	53	12	162	227
8— 9	49	10	151	210
9—10	41	15	157	213
10—11	42	6	139	187
11—12	58	11	155	224
0— 1 p.m.	54	4	151	209
1— 2	48	12	186	246
2— 3	38	13	188	239
3— 4	36	8	162	206
4— 5	36	10	144	190
5— 6	47	4	132	183
6— 7	51	10	135	196
7— 8	77	11	156	244
8— 9	58	3	153	214
9—10	51	8	184	243
10—11	42	12	185	239
11—12	35	10	171	216
Sum	1258	233	3842	5333

1. The data in this table are collected from Tables XIII, XVI, and XIX.

TABLE XXI.—YEARLY SEISMIC “ACTIVITIES” IN SWITZERLAND, THE VESUVIAN DISTRICT, SICILY, AND THE BALKAN PENINSULA AND NEIGHBOURING ISLANDS, FROM 1865 TO 1883.¹

District Year	SWITZERLAND	VESUVIAN DISTRICT	SICILY	BALK. PENIN.	sum
1865	8	45	21	38	112
1866	7	24	19	104	154
1867	13	26	8	100	147
1868	23	16	8	19	66
1869	14	62	12	28	116
1870	19	51	6	25	101
1871	25	27	6	23	81
1872	1	3	...	9	13
1873	1	...	1	53	55
1874	3	23	16	9	51
1875	3	8	5	5	21
1876	7	55	33	12	107
1877	7	17	6	4	34
1878	9	11	3	6	29
1879	11	8	18	11	48
1880	33	9	26	14	82
1881	65	8	7	17	97
1882	40	...	5	17	62
1883	24	3	21	23	71
Sum	313	396	221	517	1447

1. The data are taken from Fuchs' "Statistik der Erdbeben." It is difficult to count exact numbers of earthquakes from the Catalogue, and the figures in the table, which are intended to represent seismic "activities," are merely the numbers of days in successive years on which one or more shocks have been recorded.

TABLE XXII.—YEARLY NUMBERS OF EARTHQUAKES IN MINO, OWARI, AND MIKAWA, FROM 1887 TO 1890.

PLACE, DISTRICT	1887	1888	1889	1890	sum
(MINO)					
Gifu (Met. Station.)	7	20	19	16	62
Nakatsugawa, Ena	11	11	7	10	39
Mitake, Kani.	8	9	7	13	37
Ōta, Kamo.	6	7	14	16	43
Takayama, Toki.	10	11	12	9	42
Kozuchi, Mugi.	7	4	6	7	24
Kitagata, Motosu.	5	7	7	6	25
Ibi, Ōno.	7	8	7	4	26
Ōgaki, Ampachi.	13	5	8	6	32
Takata, Taghi.	3	2	7	4	16
Takasu, Shimo-Ishizu.	15	3	8	4	30
Hachiman, Gujō.	5	5	6	2	18
Kasamatsu, Haguri	1	3	7	6	17
Takatomi, Yamagata.	2	6	5	6	19
Tarui, Fuwa.	5	3	5	...	13
(OWARI)					
Nagoya (Met. Station)	8	5	9	13	35
Atsuta, Aichi.	3	7	6	8	24
Katsukawa, Higashi-Kasugai.	4	5	9	13	31
Shimo-Otai, Nishi-Kasugai	10	10	8	10	38
Handa, Chita.	12	11	6	15	44
Tsushima, Kaito	8	4	5	7	24
Inazawa, Nakajima.	6	3	8	5	22
Koori, Niwa.	7	8	5	4	24
(MIKAWA)					
Toyohashi, Atsumi.	2	2	3	2	9
Shinshiro, Minami-Shidara	8	5	3	7	23
Okazaki, Nukada.	9	10	12	8	39
Chiryu, Aomi.	5	11	9	8	33
Koromo, Nishi-Kamo.	2	12	9	9	32
Nishio, Hazu.	6	10	3	10	29
Tomioka, Yana.	2	3	3	3	11
Taguchi, Kita-Shidara.	2	3	8	5	18
Asuke, Higashi-Kamo.	5	11	12	14	42
Gou, Hoi.	4	7	7	7	25

Fig. 3.—Frequency of Earthquakes at Kumamoto.
(between Aug. 5th-6th and Dec. 30th-31st, 1889).

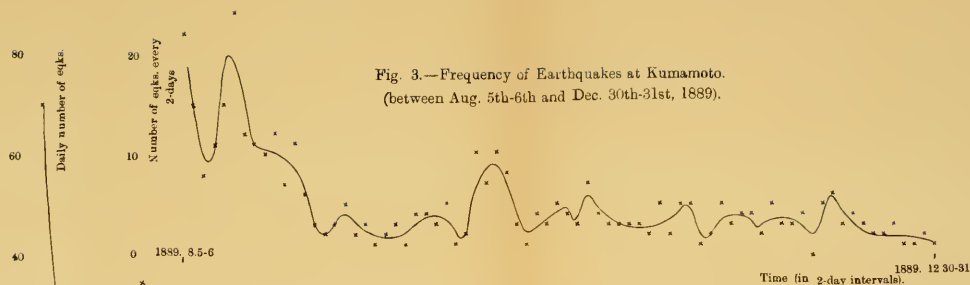


Fig. 2.—Frequency of Earthquakes at Kumamoto,
(between July 29th and Nov. 19th 1889).

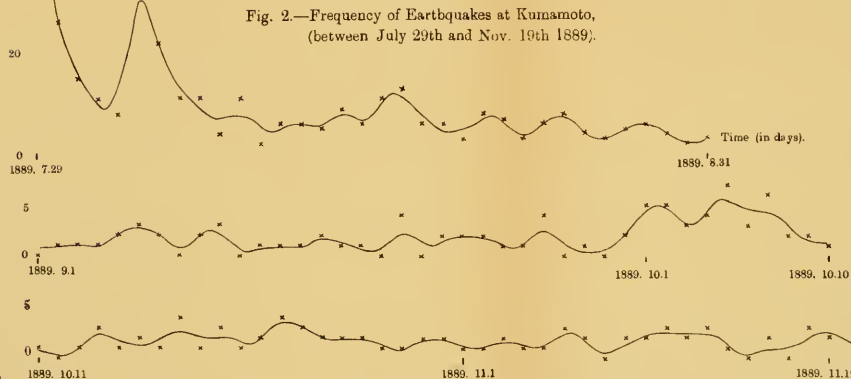
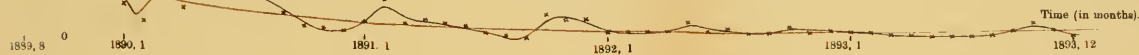


Fig. 1.—Frequency of Earthquakes at Kumamoto,
(between Aug., 1889, and Dec., 1893).



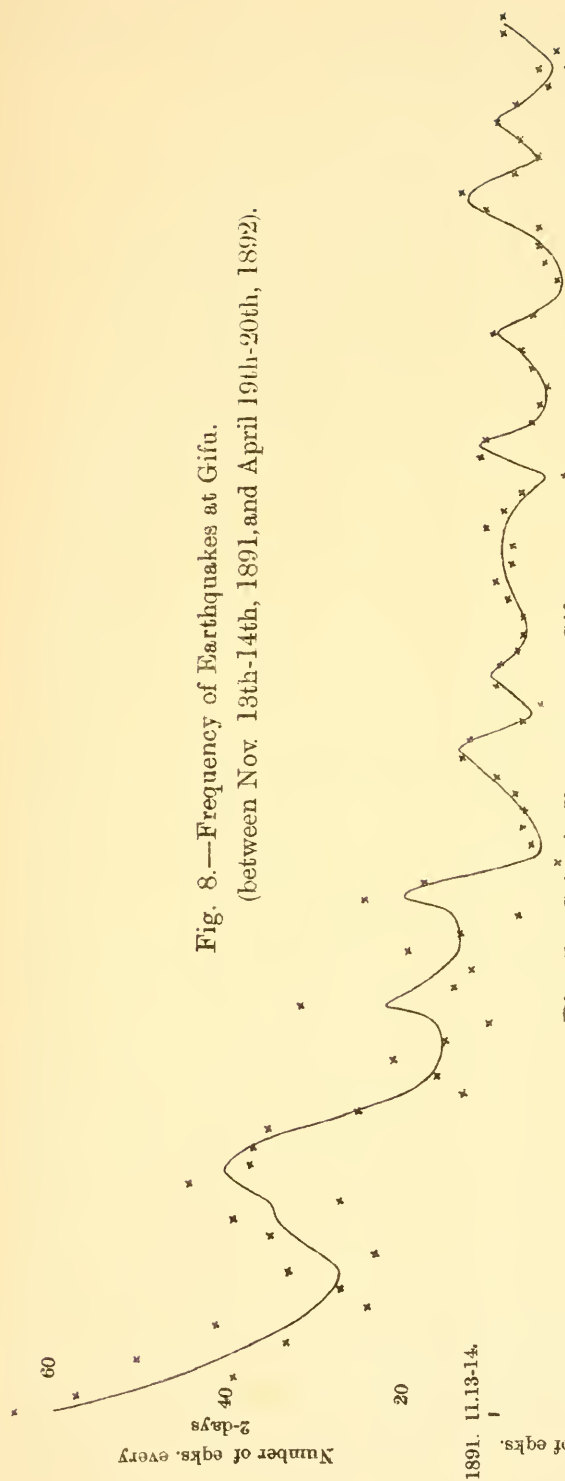


Fig. 8.—Frequency of Earthquakes at Gifu.
(between Nov. 13th-14th, 1891, and April 19th-20th, 1892).

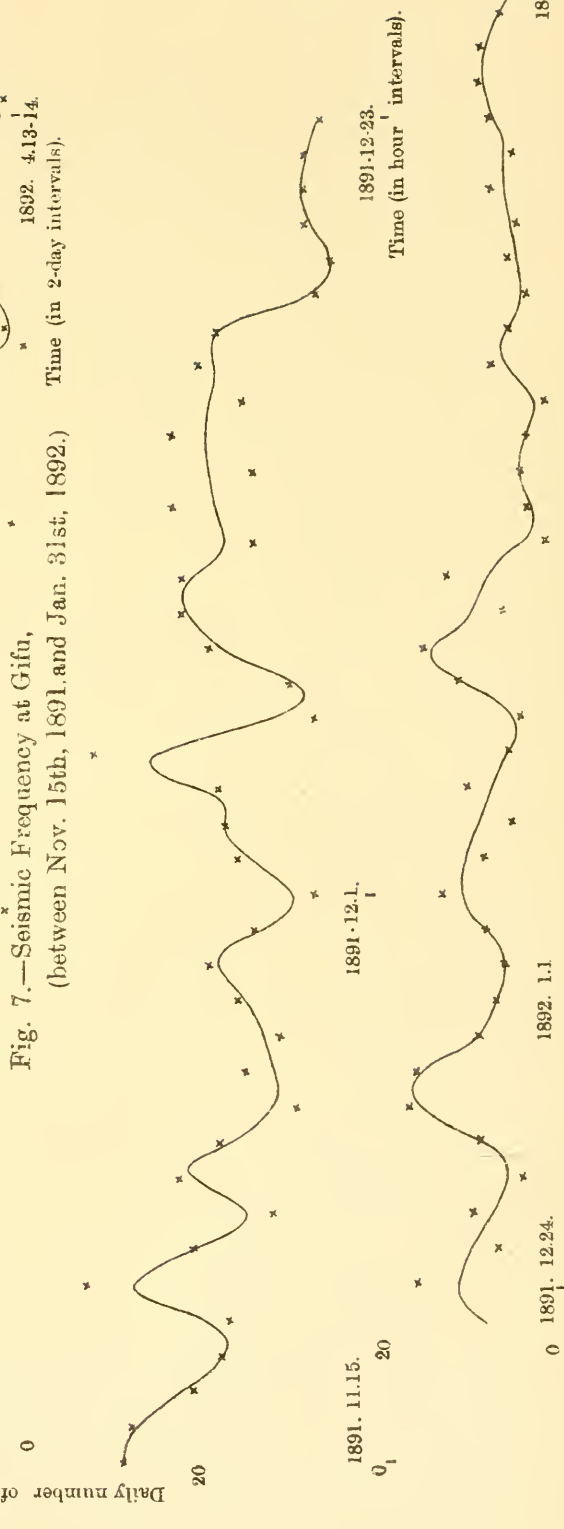


Fig. 7.—Seismic Frequency at Gifu,
(between Nov. 15th, 1891, and Jan. 31st, 1892.)

Time (in hour intervals).

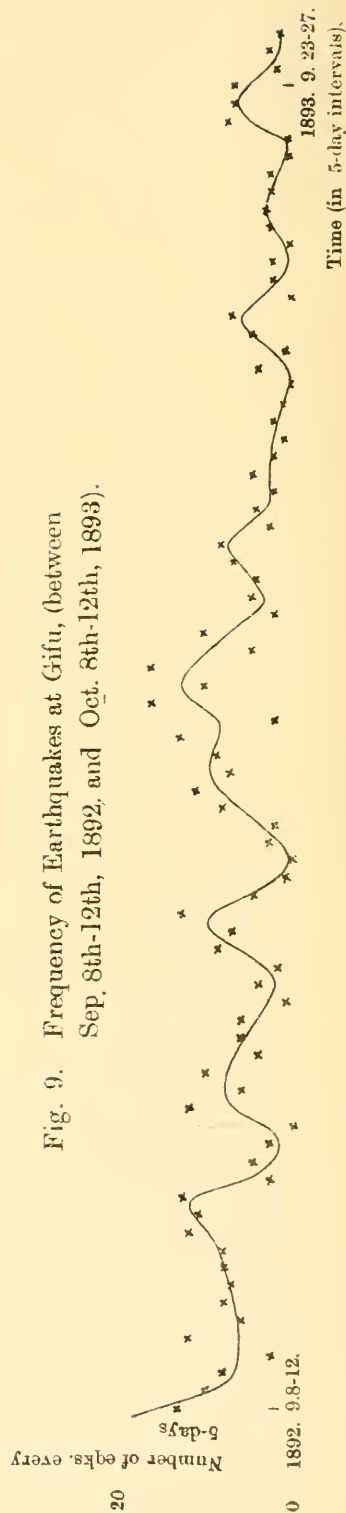
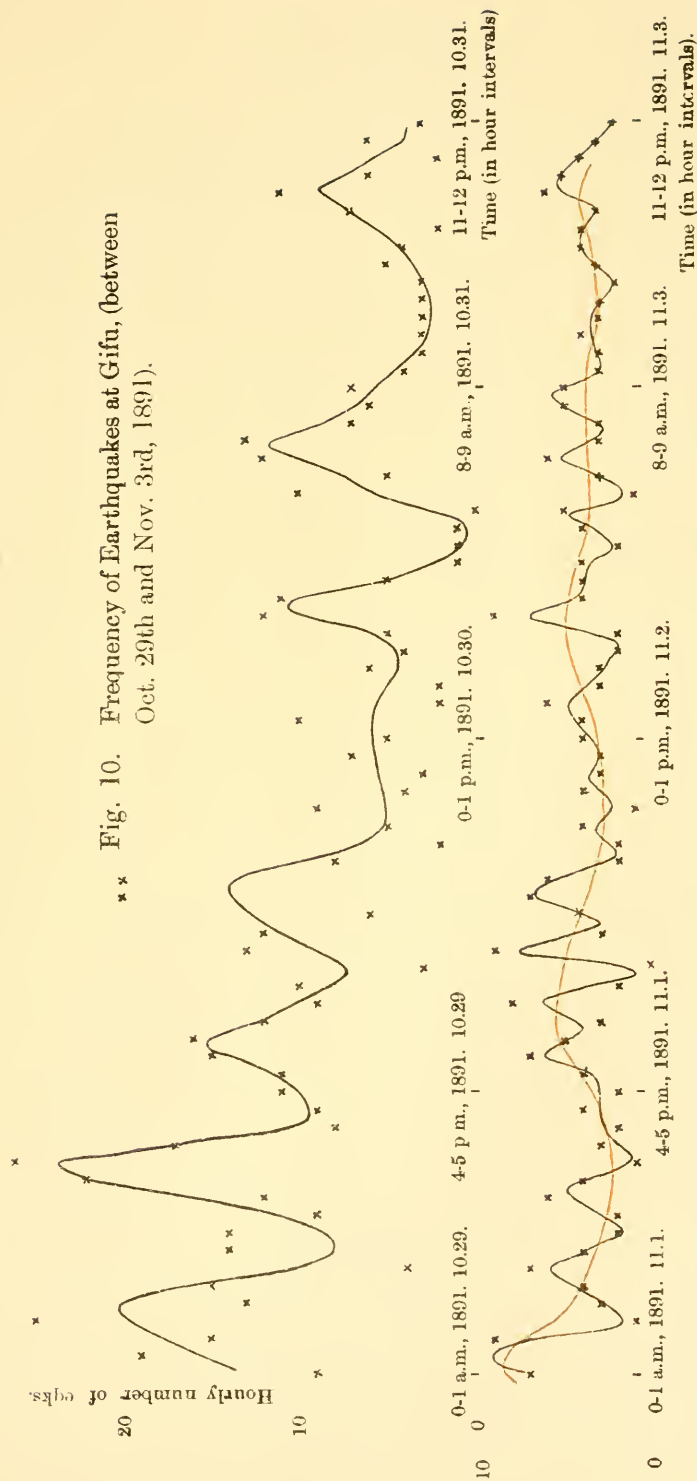


Fig. 11. Frequency of Earthquakes at Gifu.
(between Oct. 29th and Nov. 9th, 1891).

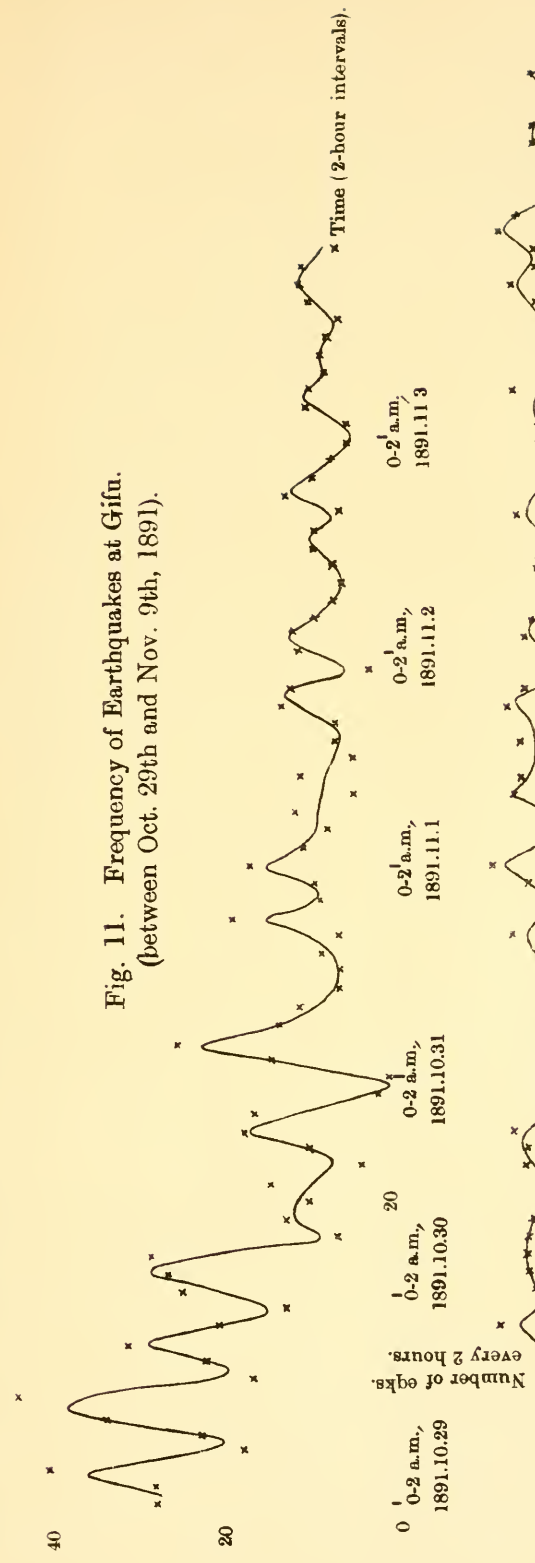
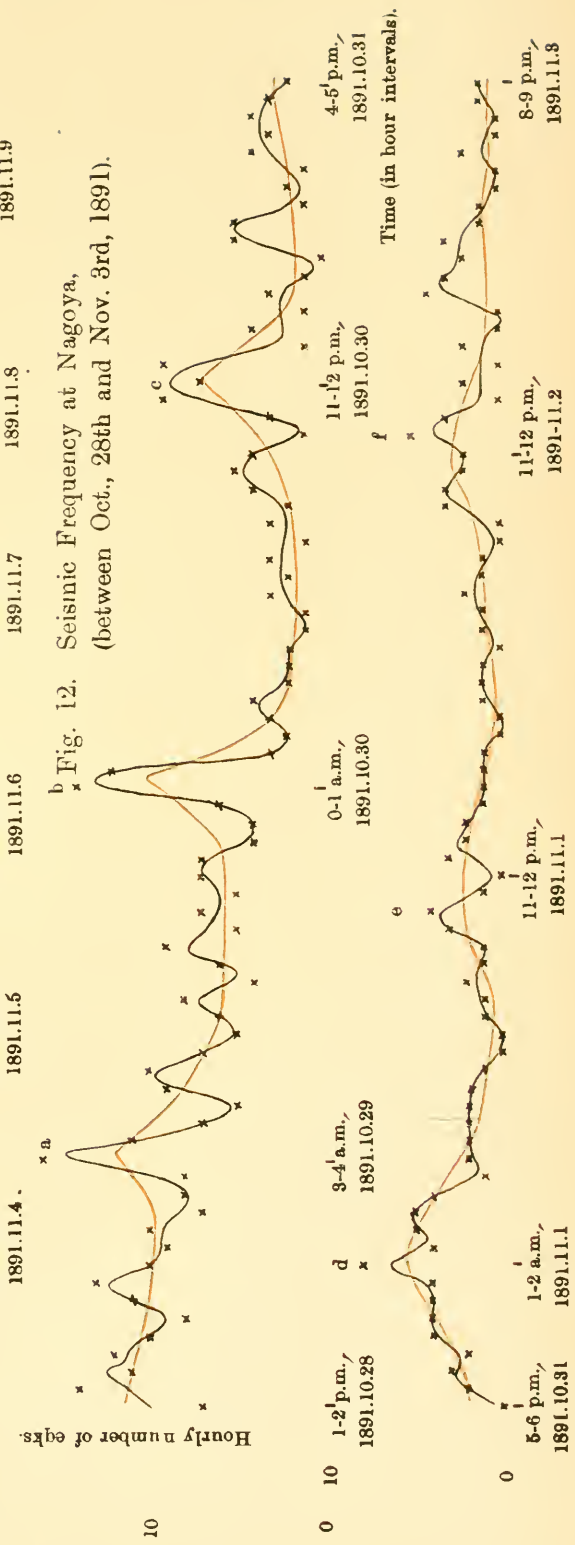


Fig. 12. Seismic Frequency at Nagoya,
(between Oct., 28th and Nov. 3rd, 1891).



60

Fig. 14. Frequency of Earthquakes at Nagoya,
(between Oct. 29th and
Nov. 7th, 1891).

Number of eqs. every
6 hours.

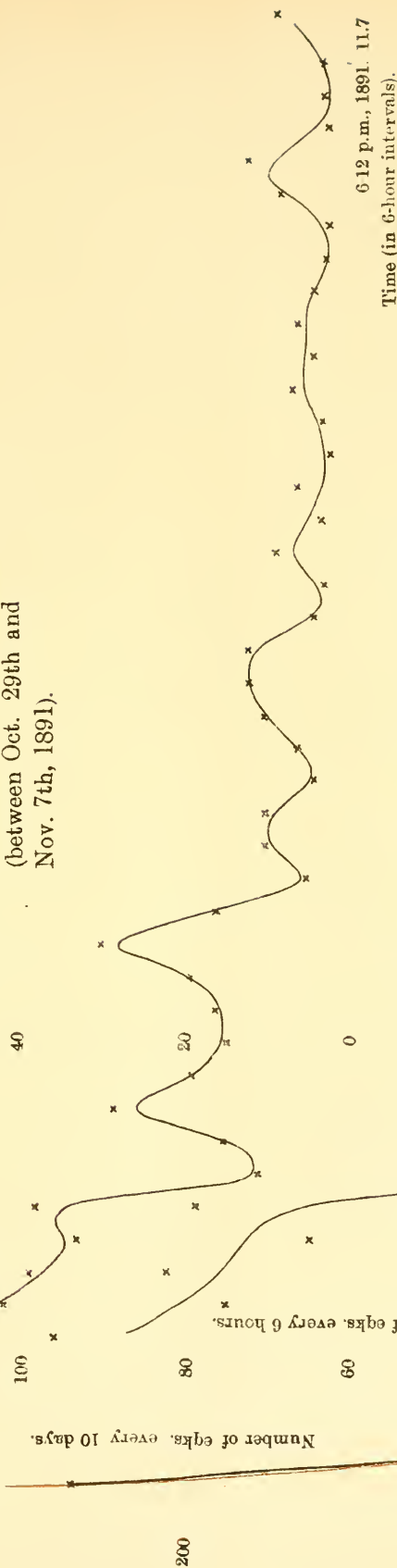


Fig. 13. Frequency of Earthquakes at Gifu,
(between Oct. 29th and Nov. 7th, 1891).

100

Fig. 15. Frequency of Earthquakes at Chiran, (be-
tween Sep. 8th -17th. and January 22nd-31st,
1894).

0-6 a.m.,
1891.10.29

20

0

Time (in 5-day intervals).

1893. 9.8-17

1893. 12.7-16

Time (in 6-hour intervals).

6-12 p.m.,
1891.11.7



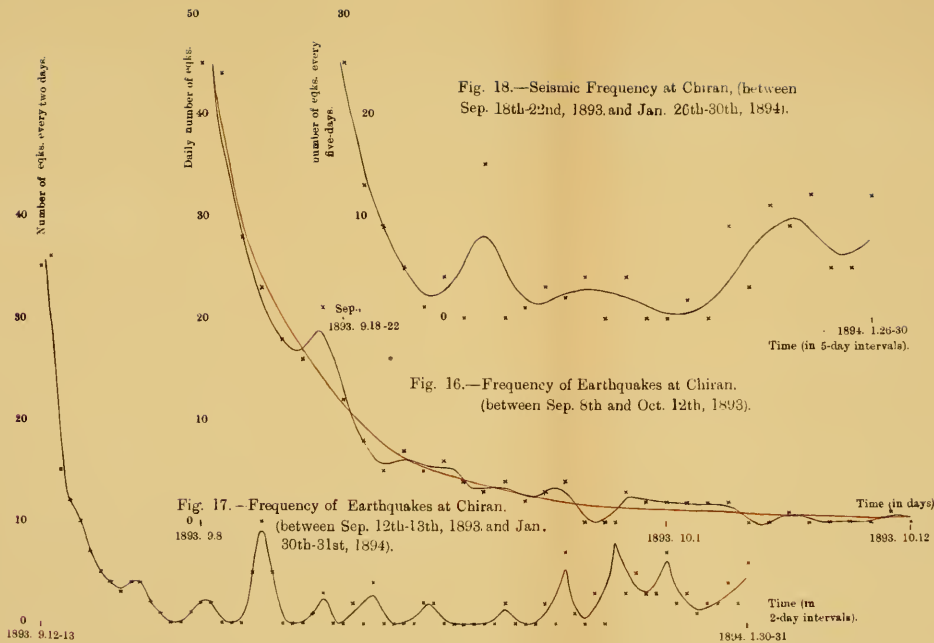


Fig. 19

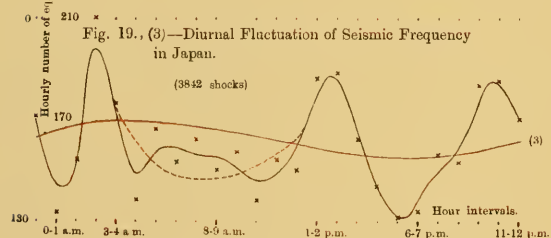
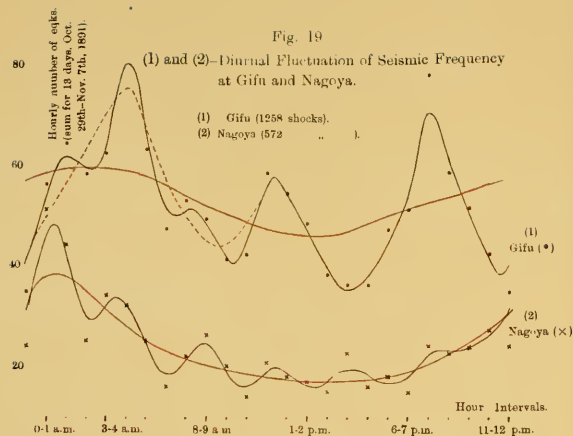


Fig. 19.

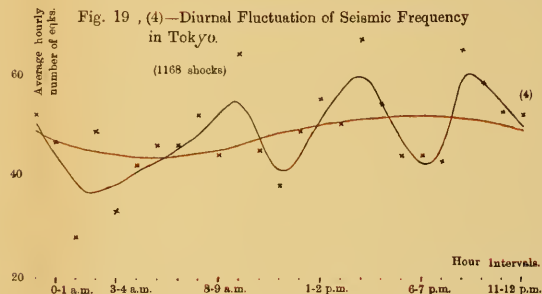
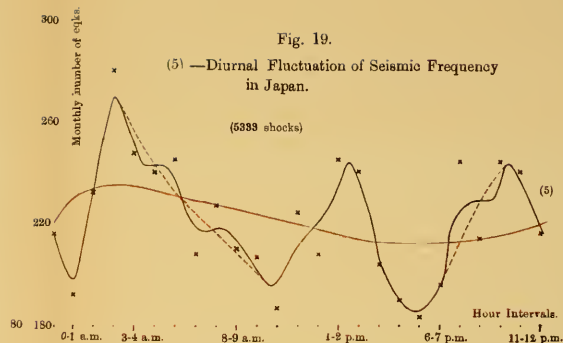


Fig. 21



Fig. 21, (5)—Annual Fluctuation of Seismic Frequency in Japan.

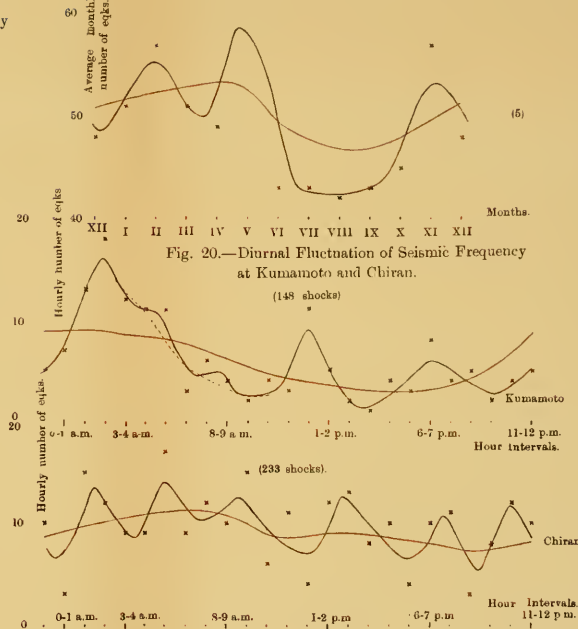


Fig. 24.—Frequency of Earthquakes at Gifu and Okawara.

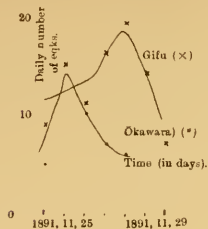


Fig. 23.—(B).
Seismic Intensity for the Vesuvian District, (between 1865 and 1883).

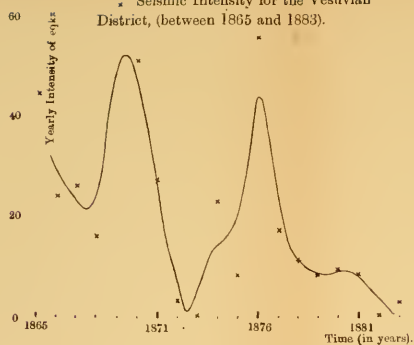


Fig. 23.
Seismic Frequency in Tokyo, (between 1876 and 1893).

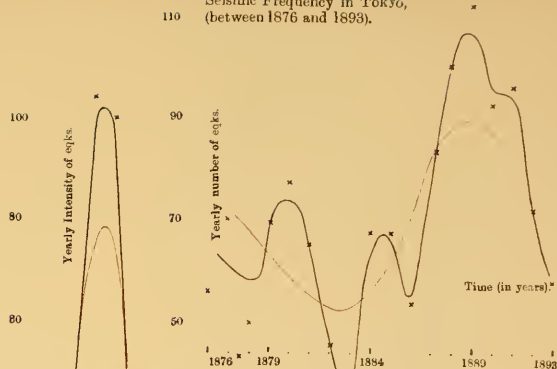


Fig. 25.—Frequency of Earthquakes at Gifu and Midori.

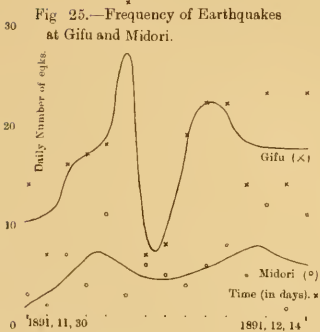


Fig. 23.—(C).
Seismic Intensity for Sicily, (between 1865 and 1883).

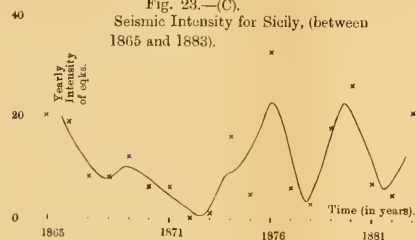


Fig. 23.—(D).
Seismic Intensity for the Balkan Peninsula and the Archipelago, (between 1865 and 1883).

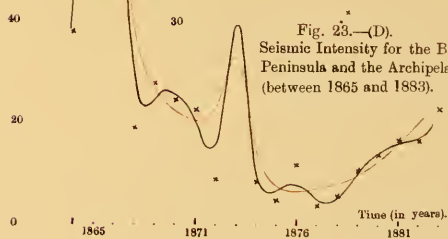


Fig. 22.—Seismic Frequency in Tokyo, (between Jan. 1876 and Dec. 1882).

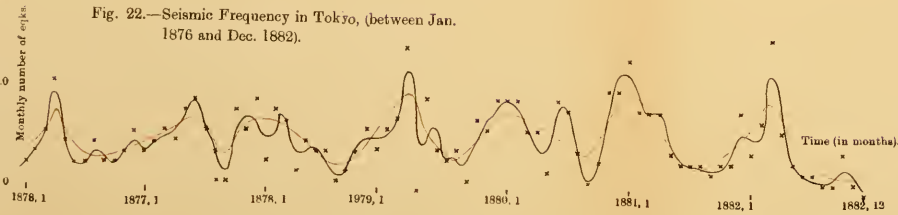
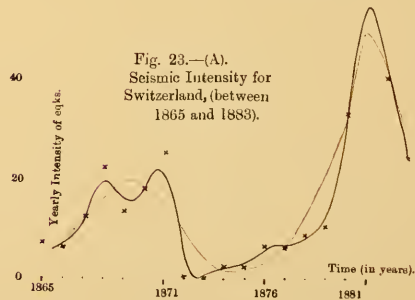


Fig. 23.—(A).
Seismic Intensity for Switzerland, (between 1865 and 1883).



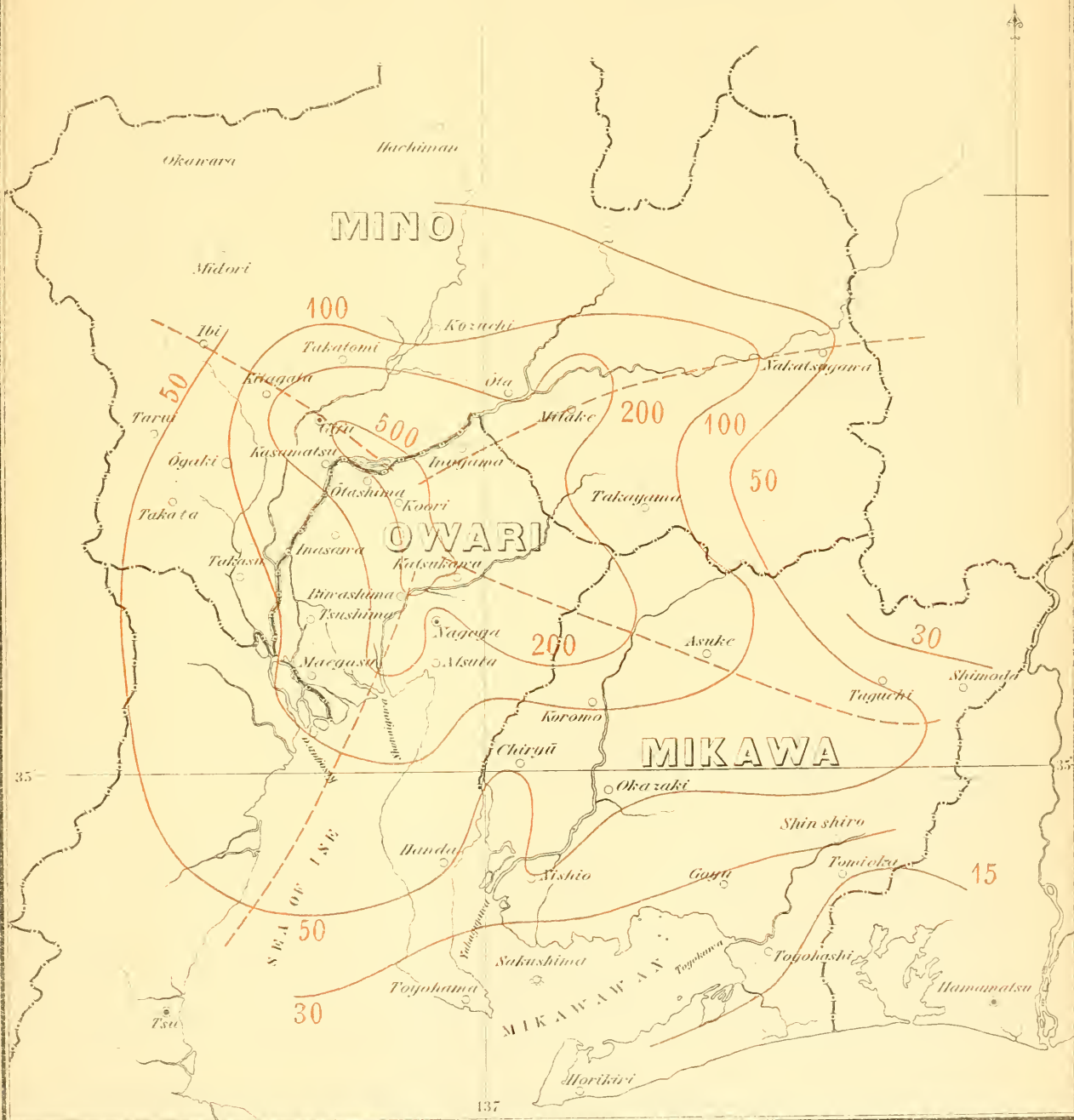


137

Fig. 27.—Distribution of Earthquakes
during 1892 in Mino, Owari, and Mikawa.

- observing station.
● meteorological station.

Curves are lines of equal earthquake numbers.

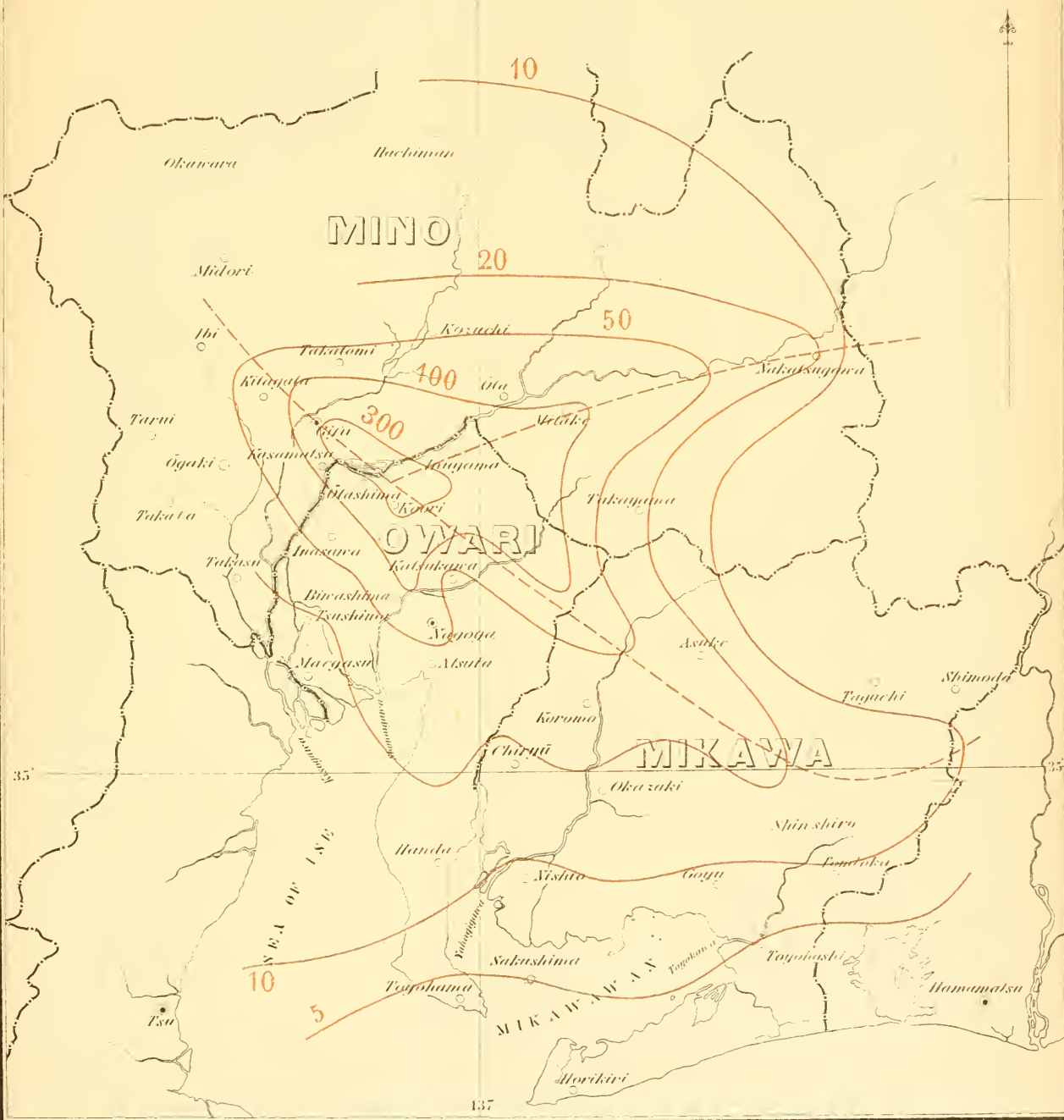


137

Fig. 28.—Distribution of Earthquakes
during 1893 in Mino, Owari, and Mikawa.

○ observing station.
● meteorological station.

Curves are lines of equal earthquake numbers.



137

Fig. 29.—Distribution of Earthquakes
during January, 1894, in Mino, Owari, and Mikawa.

- observing station.
● meteorological station.

Curves are lines of equal earthquake numbers.

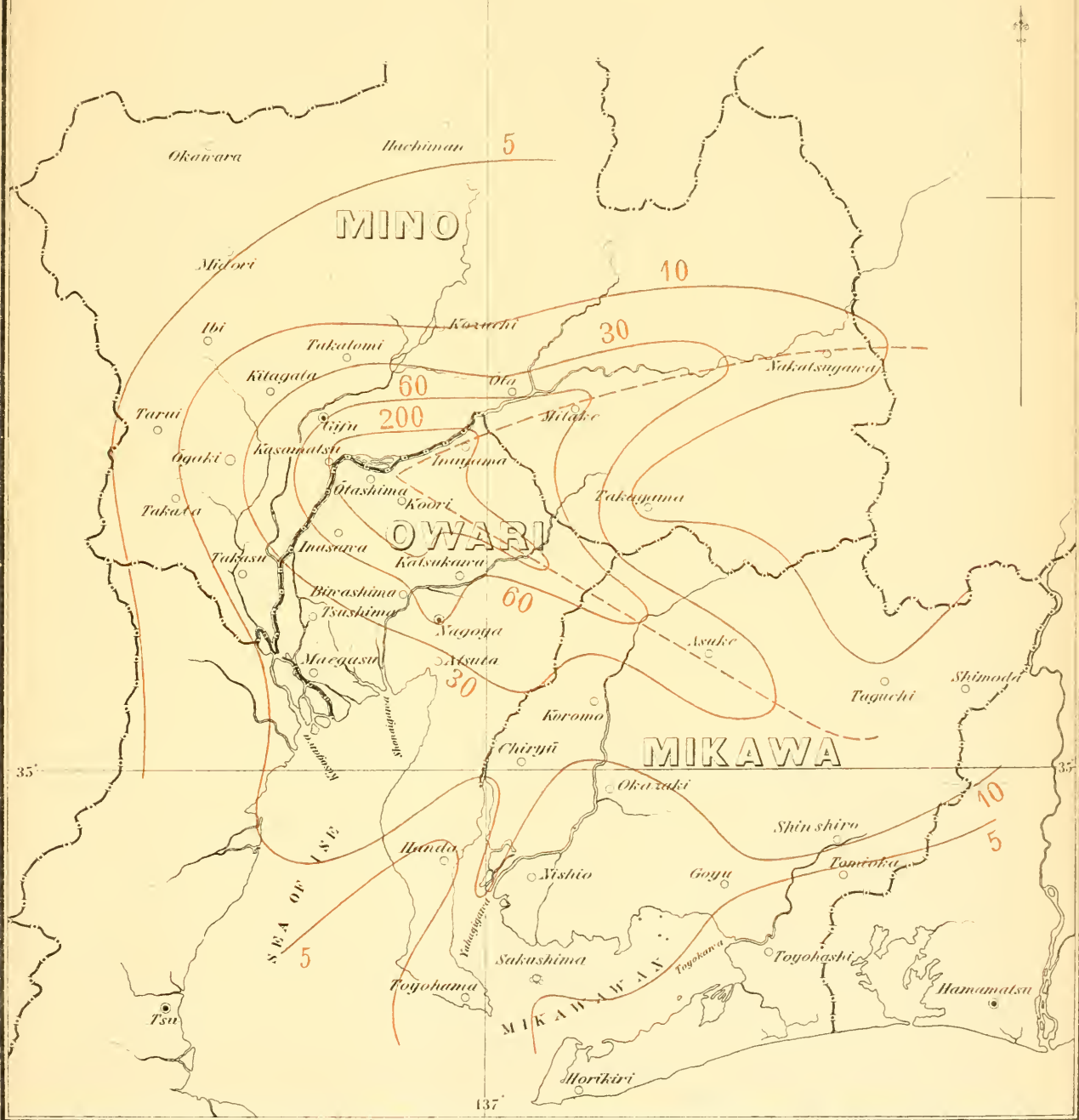


Fig. 30.—Distribution of Earthquakes
during 4 years, 1887-90, in Mino, Owari, and Mikawa.

{ ○ observing station.
● meteorological station.

Curves are lines of equal earthquake numbers.

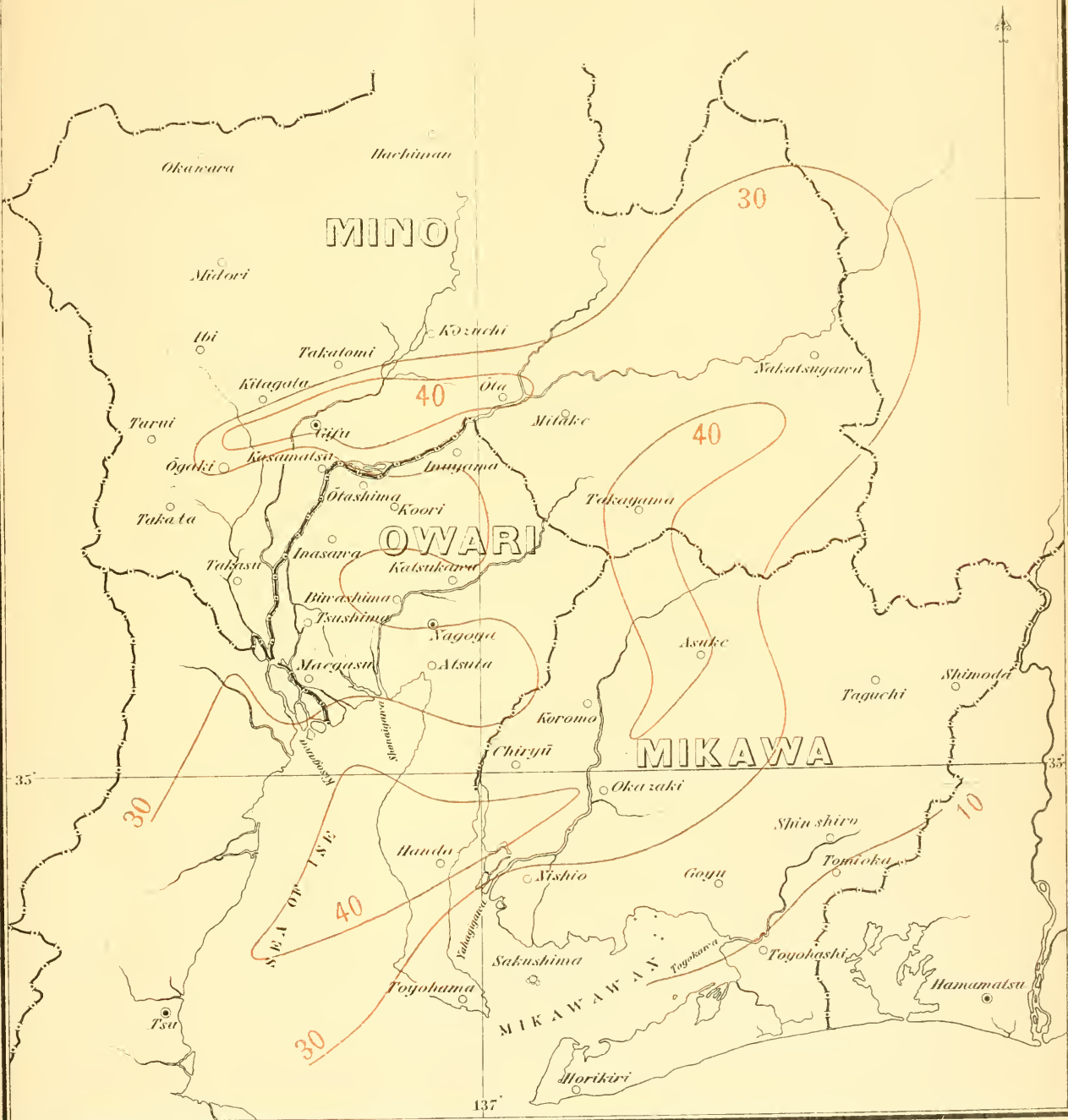
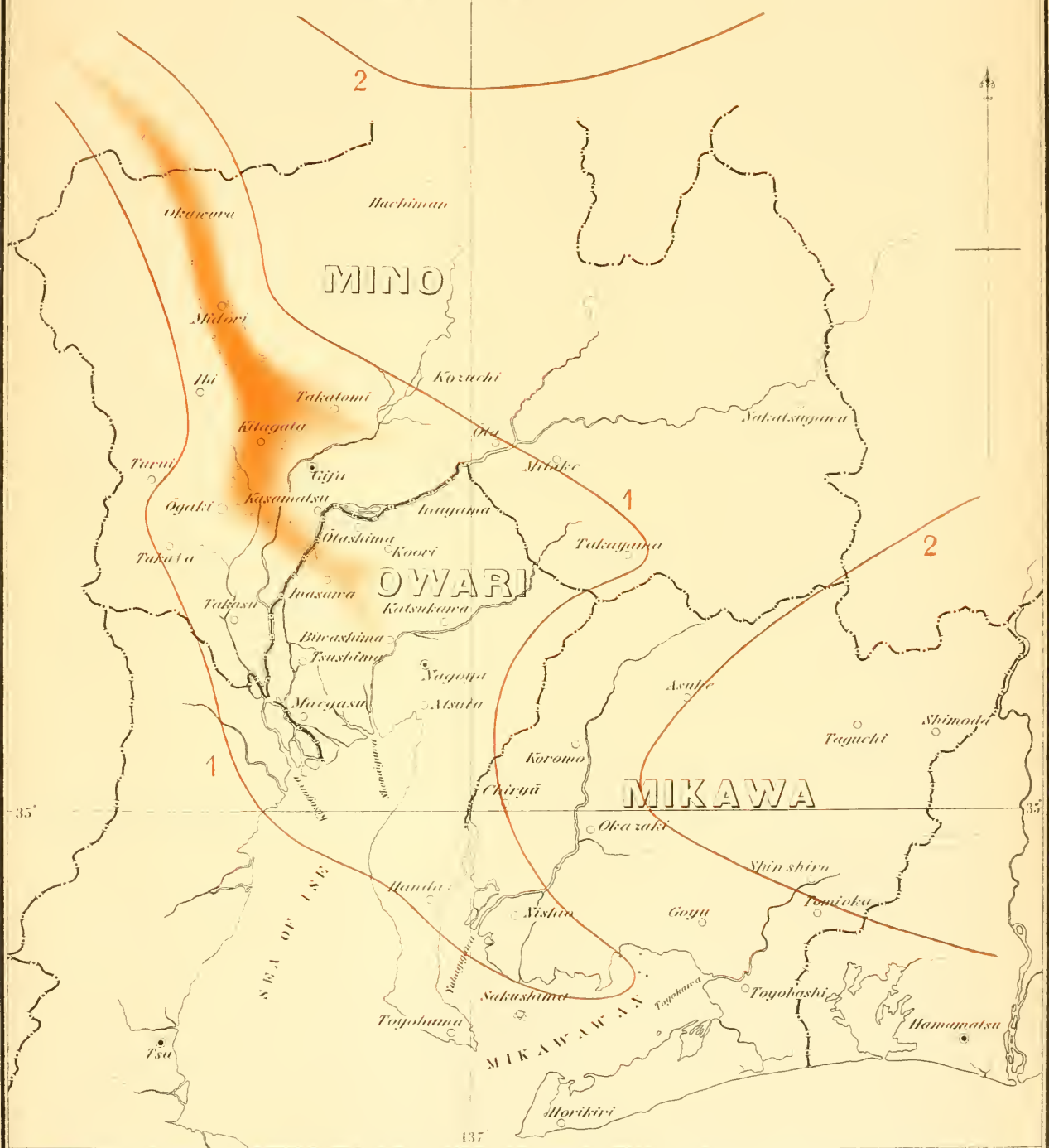


Fig. 31.—The Great Earthquake of Oct. 28th, 1891.

○ observing station.
● meteorological station.

(1) and (2) are isoseismal lines along which the maximum accelerations of earthquake motion were respectively 2000 and 800 mm. per sec. per sec.
The area most strongly shaken is indicated by red shades.



Mesozoic Plants from Kōzuke, Kii, Awa, and Tosa.

By

Matajiro Yokoyama. *Rigakushi, Rigakuhakushi,*

Professor of Palæontology, Imperial University.

General Remarks.

In 1890, Prof. A. G. Nathorst, of Stockholm, published a very valuable paper on the fossil plants of Shikoku entitled *Beiträge zur mesozoischen Flora Japans*¹⁾ The present treatise deals with the same subject, and indeed partly with the fossils therein described.

When I wrote my memoir on the Middle Jurassic flora of Kaga and its neighbouring provinces,²⁾ I thought that the plants occurring in Shikoku also belong to the same geological epoch; but as meanwhile the investigations of the Swedish palæobotanist had shown them to be decidedly younger than the Kaga flora, namely, either Upper Jurassic or Wealden, I deemed it advisable to extend the investigations further than he had carried it, and if possible, obtain a more precise knowledge as to the age in which the Shikoku plant-beds were deposited. With this view I have been engaged for some years in gathering as many specimens as possible, not only from the localities whence Nathorst obtained his material, but also from several other places where similar

1) *Deukschriften der mathematisch-naturwissenschaftlichen Classe der kaiserlichen Akademie der Wissenschaften, Wien*, Bd. LVII, 1890.

2) Yokoyama.—*Jurassic Plants from Kaga, Hida, and Echizen. Journal of the College of Science, Imperial University, Japan*, vol. III, 1889.

fossils occur. These specimens, together with those already in the museum of the Imperial University, Tōkyō, form the subject of the following pages.

Before entering, however, into the consideration of the general character of the above fossils, it may be well to give a brief account of the geological nature of each locality in which plants are found.

The places whence I obtained my material are the following :

1. Kagahara, province Kōzuke.
2. Yuasa, province Kii.
3. Sakamoto, Fujikawa, and Tannō, in the Katsuragawa basin, province Awa (Ashū).
4. Kataji, Ishiseki, and Tōgodani, near Ryōseki, province Tosa.
5. Kaisekiyama, not far from Sakawa, province Tosa.
6. Yoshida-Yashiki, near Sakawa, province Tosa.
7. Chōja, in the Shiraishigawa-valley, province Tosa.

1. Kagahara.

In a long and narrow Mesozoic depression in the northern part of the Chichibu Mountains, commonly known as the Sanchu-Ditch, there occurs a thick series of shales and sandstones with subordinate layers of conglomerate. The greater part of this formation was found to belong to the Gaulto-Cenomanian epoch. From beneath these Cretaceous rocks, there peeps out, in the valley of the Hachimanzawa, a set of strata consisting of conglomerates in the lower part, and of shales and sandstones in the upper. The shales and sandstones contain in their lower horizon innumerable remains of fresh-water shells, of which *Cyrena* forms the most important part. Plants occur

1) Yokoyama.—*Versteinerungen aus der japanischen Kreide. Palaeontographica, vol. XXXVI, 1890. On some Cretaceous Fossils from Shikoku. Journal of Coll. Science, Imp. Univ., Japan, vol. IV, pt. II, 1891.*

in a horizon higher than the shells, and are imbedded in a dark, soft, often sandy, and at the same time micaceous, shale, easily splitting into slabs. The state of preservation of the fossils leaves much to be desired, most of them being changed into a black coaly substance. The number of species which I could distinguish is 7, among which *Cyparissidium* (?) *japonicum* is by far the most abundant.

2. Yuasa.

In 1881, while I was engaged in reconnoitring the geology of Kii, I discovered fossil plants on the northern shore of the Bay of Yuasa, Yuasa being a town about 7 ri south of Wakayama. The spot is locally known as Mizutani, and lies between low and high water marks. Here a well stratified sandstone crops out from beneath the water, steeply dipping towards the north. This sandstone passes above into a conglomerate overlaid by a dark-blue shale. The rock in which I found the fossils is the sandstone. It is soft, fine-grained, greyish to yellow, brittle and often argillaceous, and easily splitting into thin plates. The fossils are generally in good preservation, but owing to the brittle nature of the rock, large specimens are difficult to obtain. The number of species found is 13, among which *Pecopteris Geyleriana* is the most frequent. I obtained here also a species of *Estheria*.

3. Sakamoto, Fujikawa, and Tanno.

These three places all lie in the valley of the Katsuragawa, in Awa, and very close to one another. The discoverer of the fossils was my lamented colleague, the late Prof. Y. Kikuchi, who surveyed the district in 1882. According to his report,¹⁾ the valley is composed of the Mesozoic plant-bearing series, overlaid by the Cretaceous sandstone

1) *Geology of Awa, 1883 (M.S.)*.

in which I had already recognised the occurrence of the Middle Cretaceous *Trigoniae*.¹⁾ The plant-bed consists of shales and sandstones, and is sometimes in such close relation with the *Trigonia* sandstone that it is difficult to distinguish the boundary between the two.

From Sakamoto I possess many small fragments of a dark grey shale in which badly preserved molluscan shells are found, and a big block of the same rock containing a single species of *Zamiophyllum Buchianum* Ett. sp.

At Fujikawa the plant-bearing rock is a dark-grey, hard, fine-grained sandstone which is sometimes clayey and passes to shale. Owing to the more or less rough nature of the sandstone, the preservation is far from satisfactory, the minute details of the plants being in most cases obliterated. The number of species found at this locality is 5.

The plants of Tanno are found in a dark brittle shale, exposed in a valley called Kashiwaradani, deeply cut by a stream. The upper part of this shale becomes sandy, and on it is superposed the Cretaceous sandstone. Kikuchi found in pebbles probably derived from the sandy part of the shale some remains of fresh-water molluscs. The preservation of the plants is generally excellent, but we have only four species from this locality and all in small fragments.

From Hiura in Mitani, near Sakamoto, Nathorst obtained 3 species, but in my collection there is none which comes from the same place.

4. Kataji, Ishiseki, and Tōgodani.

These localities all lie in the Ryōseki "Hügelland" of Dr. Naumann. The geological nature of this district has been recently studied by Mr. M. Yamagami, now a geologist in the Imperial

1) *On some Cretaceous Fossils from Shikoku. Op. cit.*

Geological Survey. According to his report,¹⁾ the lowest Mesozoic stratum in the vicinity of Ryōseki consists of a black shale upon which there is a coarse conglomerate occupying the greater part of the district. The plant-bearing rocks are shales and sandstones which overlie this conglomerate. Immediately over the plant-bed, there rests the Cretaceous formation with its characteristic *Trigonia pocilliformis*, and in the midst of this formation there peeps out at Okuminodani a dark compact limestone, exactly similar to the Upper Jurassic limestone of Torinosu in Sakawa, and containing the spines of the same glandiferous *Cidaris* as occurs in it. The relation of this limestone to the plant-bed is not exactly known, but from its direction of dip, it seems to underlie the latter.

The fossil locality of Kataji is in a river bed. The rock is a dark-grey, hard, fine-grained sandstone, more or less clayey and easily splitting into large slabs. Fossils are numerous, but rare in species. Furthermore, owing to the smoothly polished and carbonised nature of the vegetable substance, the details of the plants are for the most part effaced. The number of species and varieties obtained is 13, among which *Pecopteris Geyleriana* and *Zamiophyllum Buchianum* seem to play the most important part.

The fossil layer at Ishiseki which is a place very near to Kataji, is exposed in a little stream-bed, and is a sandstone similar to that of the preceding place, although somewhat lighter-coloured. The number of plants is many, but of species few. Moreover, the state of preservation is exactly like that of Kataji, so that the two localities seem to belong to exactly the same horizon. The number of species here obtained is 6; *Chladophlebis Nathorsti* is the most frequently observed.

In the upper part of Tōgodani, a small side-valley of the Kasanogawa, is a cliff which is the fossil locality here. The rock is a

1) On the Plant-bearing Beds of the Ryōseki Basin, 1892 (M.S.).

greenish-grey sandy shale which passes into a shaly sandstone. It is fine-grained and tolerably firm, but where it is exposed to air, it is yellowish and more brittle. The plants are very well preserved. I obtained 8 species, but principally *Pecopteris Browniana*, *Zamiophyllum Buchianum* and *Nilssonia schauburgensis*.

The majority of the fossils from the above three localities are those sent by Mr. Gyōken Otsuka of Ryōseki at the request of Prof. Kotō.

Nathorst in his paper described also some from Yakyō, Ueno, Torikubi, Ōtani, and Haginotani, places all in the neighbourhood of Ryōseki.

5. Kaisekiyama.

This is a mountain north of Sakawa, also called Kompirayama from a temple dedicated to Kompira standing on its top. The spot where plants occur is on the southern flank of this mountain, in a soft, yellow, sandy shale, where there are numerous impressions of plants from which collection has been made partly by Mr. T. Nasa, and partly by myself. The preservation is excellent, but the rock is so soft and brittle that unless the greatest care is taken, the specimens are apt to fall to pieces. The number of species which I could distinguish from this locality amounts to 8, besides 1 variety, among which *Pecopteris Browniana* is the most abundant.

6. Yoshida-Yashiki.

At this place close to the town of Sakawa a series of sandstones and shales covers unconformably a dark, compact, bituminous, so-called Torinosu-limestone containing fossils which are referable to the Malm.¹⁾ The plants occur in shales which are ash-grey and

1) Naumann und Neumayr—Zur Geologie und Palaeontologie von Japan. Denksch. d. mathem.-naturw. Cl. d. Kaiserl. Akad. d. Wissensch., LVII Band, 1890.

sandy, and easily split into thin plates. The only species which I could find in a collection made by Mr. Nasa is that already described by Nathorst, viz., *Pecopteris Browniana*. However from the specimens, which are present in large numbers, I could convince myself how variable the form of the pinnules is in different parts of the frond. In the underlying limestone we find also now and then impressions of plants which have been very probably drifted into it. I have been able to distinguish only two forms, one a *Chladophlebis*, and the other a *Nilssonia* resembling *N. pterophylloides*. However, owing to their unfavourable state of preservation precise determination is not possible.

7. Chōja.

Chōja is a mountain village in the valley of the Shiraishigawa, many kilometers to the west of Sakawa. I got only a single piece of stone from this locality, collected by Mr. Toyama, a zealous collector of fossils at Sakawa. It is an ash-grey sandy shale, quite similar to that of Yoshida-yashiki, and contains also only a single species, *Pecopteris Browniana*. According to Toyama and others, a limestone occurs in the locality.

Conclusion.

The number of fossils which I have been able to obtain from the above enumerated places amounts to 24 species and 1 variety. Of these 24 species, 23 are plants and 1 a phyllopod. Adding to these, 3 species and 1 variety described by Nathorst, viz. *Macrotænipteris marginata*, *Lycopodites* sp., *Ptilophyllum* cf. *cutchense* and *Podozamites lanceolatus* var. *latifolia*, the total number becomes 26 species and 2 varieties of plants, and 1 species of animal. Before proceeding, however directly to the conclusion which is to be drawn from the occurrence

of these fossils, it is very necessary to examine whether the floras of all the localities represent one and the same geological horizon. In the first place, that the 8 localities around Ryōseki, namely, Kataji, Ishiseki, Tōgodani, Yakyō, Ueno, Torikubi, Ōtani, and Haginotani, palæontologically belong to the same formation is not to be the least doubted; for out of 15 species which were obtained from all the places together, 12 are found at Kataji, so that Ishiseki with 6, Tōgodani with 8, Ueno with 4, Yakyō, Torikubi, Ōtani, and Haginotani, each with 2, may be fairly considered as a part of the Kataji flora. Of the 3 species which are not found at Kataji, 1 is from Ueno, 1 from Tōgodani, and 1 from Yakyō, the last of which, however, is represented in several other spots out of the Ryōseki District.

Also that the four places in the valley of the Katsuragawa, viz., Sakamoto, Fujikawa, Tanno, and Mitani, belong to the same terrain, although the number of species is not so large, and the relation of the respective floras to one another is not so evident as in Ryōseki, there is every reason, both geological and palæontological, to believe. We have therefore in all 7 distinct localities from which fossils were got. These localities show close affinity in their floras, as will be seen from the following table:—

NAME OF LOCALITIES.	Total no. of spec. found.	Number of species in common with.							Total no. of spec. found else- where.
		Kagahara.	Yuasa.	Katsura- gawa.	Ryōseki.	Kaiseki- yama.	Yoshida- yashiki.	Chōja.	
Kagahara	7	—	5	4	4	3	0	0	5
Yuasa	13	5	—	4	9	6	0	0	8
Katsuragawa	9	4	4	—	6	4	1	1	7
Ryōseki	15	4	9	6	—	7	1	1	10
Kaisekiyama	8	3	6	4	7	—	1	1	7
Yoshida-yashiki ...	1	0	0	1	1	1	—	1	1
Chōja	2	0	0	1	1	1	1	—	1
Unknown locality...	1	0	0	0	0	0	0	0	0

Thus, Kagahara has 7 species, 5 in common with others ; Yuasa 13, 8 in common with others ; Katsuragawa 9, 7 in common with others ; Ryōseki 15, 10 in common with others ; and Kaisekiyama 8, 7 in common with others ; while Yoshida-yashiki has only 1, represented in many other localities, and Chōja 2, 1 of which is the same as that of the preceding place. Such being the case, the floras of all the places except the last may be safely looked upon as belonging to the same epoch. Of Chōja we shall speak more later on.

Having thus proved the close relationship existing between the floras of the respective localities, the next question is their age. Except Chōja, and an unknown locality yielding *Macrotaniopteris marginata*, the number of plants collected in various localities amounts to 25 species and 2 varieties. Of these 25 species, 3 are not determinable, and 15 are peculiar to Japan and indeed to the formation in question ; so that what might be available, if ever available, for the fixing of the age would be the following :

1. *Ouychiopsis elongata* Geyl. sp.
2. *Pecopteris Browniana* Dunk.
3. *Pecopteris* cf. *virginiensis* Font.
4. *Podozamites lanceolatus* Lindl. var. *minor* Heer.
5. *Podozamites lanceolatus* Lindl. var. *latifolia* Heer.
6. *Podozamites pusillus* Velenor.
7. *Zamiophyllum Buchianum* Ett. sp.
8. *Zamiophyllum Buchianum* Ett. sp. var. *angustifolia* Font.
9. *Nilssonia schauburgensis* Dunk. sp.
10. *Nilssonia Johnstrupi* Heer.

Let us now speak of each species separately. *Podozamites lanceolatus* has a very wide geographical range, but its vertical distribution is equally as wide, for it appears in the Rhætic and goes up as high as the Cenomanian, according to Velenovsky. It is not improbable that we have here to deal, in many cases, with forms which are in reality specifically different, but as long as we are left to rely only on leaves in their determination, we can not but consider these forms as belonging to one and the same species. Therefore this cycle only tells us that we have here a formation which is referable to the Mesozoic group. *Ouychiopsis elongata* has hitherto been found only in the Dogger of Japan where, however, it forms one of the most abundant plants. On this account we do not know yet how wide its vertical range may be. The case is different with the three species of *Pecopteris Browniana*, *Nilssonia schauburgensis*, and *Zamiophyllum Buchianum*. The first two are characteristic Wealden plants in Europe, and the first has been also described from the Potomac Formation in America which Fontaine considers as Neocomian. The third one is Urgonian in Europe and Potomac in America, and is very abundant in the latter region. These three species are therefore very important for Japan, especially because they occur in many localities and sometimes

also in great profusion. *Pecopteris* { *virginicusis* and *Zamiophyllum* *Buchianum angustifolia* are also Potomac, and if the latter is really identical with *Dioonites abietinus*, as Fontaine asserts, it occurs also in the European Wealden. The two remaining forms, *Podozamites pusillus* and *Nilssonia Johnstrupi* are hitherto only Cretaceous, the former being found in the Cenomanian of Bohemia, and the latter in the Kome-beds of Greenland, considered by Heer as of Lower Cretaceous age. Thus, the greater part of the fossils point to the Wealden or to the Lower Cretaceous. But as the Wealden is generally looked upon as the fresh-water equivalent of the lowest Cretaceous, I go a step further than Prof. Nathorst, and say that the plant-bearing beds of Kōzuke, Kii, and Shikoku represent the whole Neocomian series, corresponding to the Potomac of America. This assertion is in accordance with their geological position. On the one hand they show a close relationship to the *Trigonia* Sandstone of the Gaulto-Cenomanian, and on the other to the so-called Torinosu or *Cidaris* Limestone, which has been considered as Upper Jurassic. As to Chōja, which has yielded only *Pecopteris Browniana* and *Ptilophyllum cf.utchense*, it is difficult at present to make any definite statement; but the occurrence of a *Pecopteris* so characteristic of Wealden seems to show, in spite of the other, that we have here to deal with strata which in all probability belong to the same geological formation as the other localities.

In contrast to the Middle Jurassic flora of Kaga, a marked feature in the one above discussed, is the comparative rarity of species already found in other countries. This is no doubt due, not only to the preponderance of marine deposits in the early part of the Cretaceous period and consequent scarcity in rocks containing vegetable remains, but also to the fact that the already known floras of this epoch such as the Wealden and the Potomac belong to provinces widely distant from

ours ; whilst in the Dogger flora we have a comparatively rich one from Siberia.

The determination of the age of the plant-beds leads to the conclusion that on the outer or convex side of Japan, as far as our present investigation goes, no Middle Jurassic fresh-water formation is developed. But whether the converse holds true, that no Younger Mesozoic plant-beds have been deposited on the inner or concave side of Japan is at present difficult to say and must be left for future investigation to determine.

For convenience' sake I propose the name of *Ryōseki Series* for all those strata containing Younger Mesozoic plants in distinction to those containing Middle Jurassic ones. For the latter I also suggest the name *Tetori Series*, from the valley of the river Tetori in Kaga, where they were first discovered.

	Kô-zoku													Kii.	Awa.					Tosa (Ryôseki district).							Tosa.			Unknown Locality.					
	Kugedara.	Yusa.	Sakanoto.	Fujikawa.	Tanno.	Mitani.	Kataji.	Ishioeki.	Togodani.	Yakyo.	Ueno.	Torikubi.	Otani.	Hagimotani.	Kaisôkyûm.	Yashida-Yashiki.	Chôja.																		
FILICES.																																			
1. <i>Thyrsopteris</i> sp.	+															
2. <i>Dicksonia tosaana</i>	+	+	...	+															
3. <i>Dicksonopteris Naumanni</i>															
4. <i>Onychiopsis elongata</i>	+	+	...	+	...	+	+	+	+	+	...	+	+	+	+															
5. " <i>elegans</i>	+															
6. <i>Adiantites yuasensis</i>	+															
7. <i>Pteris</i> (?) sp.	+	?															
8. <i>Sphenopteris tenticula</i>	+	+	+	+															
9. <i>Pecopteris Browniana</i>	+	+	...	+	?	+	+	+															
10. " <i>Geyleriana</i>	+	...	+	+	+	+															
11. " cf. <i>virginiensis</i>	+	?															
12. <i>Chladophlebis Nathorsti</i>	+	?	...	+	+	+	...	+	+															
13. <i>Macrotaniopteris</i> (?) <i>marginata</i>	+															
LYCOPODIACE.																																			
14. <i>Lycopodites</i> sp.	+															
CYCADEACE.																																			
15. <i>Podocarpites</i> sp.	+															
16. " <i>laureolatus</i> var. <i>minor</i>	+															
16 a. " " <i>latifolia</i>	+															
17. " <i>pusillus</i>	+															
18. <i>Zamiophyllum Buchianum</i>	+	+	+	+	+	+	+	...	+	+	+	...	+															
18 a. " " var. <i>angustifolia</i>	+	+	+															
19. " <i>Naumanni</i>	+	+	+	+	+															
20. <i>Glossozamites parvifolius</i>	+															
21. <i>Nilssonia Johnstrupi</i>	+	...	+															
22. " <i>schaumburgensis</i>	+	+	+	+	...	+															
23. " <i>pteryfolioides</i>	+	+	+															
24. <i>Ptilophyllum</i> cf. <i>cutchense</i>	+															
CONIFERÆ.																																			
25. <i>Cyparissidium</i> (?) <i>japonicum</i>	+	+	+															
26. <i>Torreya venusta</i>	+															
PHYLLOPODA.																																			
27. <i>Estheria rectangulara</i>	+															

Middle Jurassic of Kaga, Hida, etc.

Wealden of Europe. Potomac of America.

Potomac of America.

Rhætic, Jurassic and Cretaceous of Europe.

Middle Jurassic of Kaga etc.

Cenomanian of Bohemia.

Urgonian of Europe, Potomac of America.

Potomac of America.

Kome beds of Greenland.

Wealden of Europe.

Rajmahal of India.

Description of the Species.

Filices.

1. *Thyrsopteris* sp.

Pl. XXIII, Figs. 2, 3.

On a piece of stone, represented in fig. 2, there are many elongated pinnae apparently belonging to a twice pinnated frond. They taper very gradually forward, and seem to be tolerably distant. The pinnules are long, alternate, more or less directed forwards, close together, and sharply toothed. The general appearance of the fern reminds us of the many species of *Thyrsopteris* hitherto described from the Jurassic and Cretaceous rocks. It also closely resembles *Ongchiopsis elongata* Geyl. (see further on), which possesses, however, more acutely directed pinnules. It is to be regretted that the lateral veins are so indistinct as not to permit of any stricter comparison with the already known forms being made.

Fig. 3 probably represents an apical part of the same fern. Pinnules are here linear and not unlike those of *Pecopteris virginicensis* Font. described below.

Loc.—Fujikawa, in the Katsuragawa basin.

2. *Dicksonia tosana* m.

Pl. XXV, Figs. 13, 13a.

Frond tripinnated; rachises of various orders rather slender; primary rachis bent somewhat zigzag, others nearly straight; primary pinnae elongated, distant, rising at nearly right angles to the rachis; secondary pinnae comparatively short, alternate, close together and a little overlapping, those on the back of the rachis being more acutely directed forward and

bearing more elongated pinnules than those on the front pinnules linear to elliptical, acute at apex, directed more or less forwards and close together; veins fine, few, indistinct, an evanescent midvein sending off a few simple lateral veins.

This slender and elegant fern is, I believe, to be brought under *Dicksonia*, and indeed close to *D. acutiloba* Heer (Yokoyama, Jurassic Plants. p. 24, Pl. I. figs. 2, 2a, 1b), from which however it is distinguished in having shorter ultimate pinnae, which on the back of the secondary rachis are more acutely directed forward than on the front. The pinnae with the linear pinnules look also not unlike those of *Onychiopsis elongata* Geyl. Veins are in most cases not distinctly visible, in one or two cases however they were observed, and then arranged as in fig. 13a.

Loc.—Tōgodani near Ryōseki. Only a single specimen.

3. *Dicksoniopteris Naumanni* Nath.

Pl. XXV, Fig. 4.

Dicksoniopteris Naumanni.—Nathorst, Beitr. zur mesoz. Flora Japans, p. 11, pl. V, fig. 4.

What Nathorst described from Haginotani under the above name was found also at two other localities cited below. The fern is rather slender, with distant, opposite pinnae rising at an angle of 60–65° from a straight main (?) rachis. Pinnules are elongated, finger-like, obtuse, more or less crenulate and quite close together, although not overlapping. The margins of these pinnules show on each side 4 or 5 round fruit-dots, most of which however have left only slight impressions on the stone. Veins have been nearly obliterated, the only thing which I can now and then observe being a very fine midvein which near the apex is indistinct.

Loc.—Kataji and Ishiseki, near Ryōseki. Rare.

4. *Onychiopsis elongata* Geyl.

Pl. XX, Fig. 8. Pl. XXI, Figs. 1, 4.

Onychiopsis elongata—Yokoyama, Jurassic Plants, p. 27, pl. II, fig. 1-3, III 6d, XII, 9,10. Nathorst, Beitr. z. mesoz. Flora Japans, p. 4, pl. I, fig. 1-3, p. 8, p. 10, p. 13, p. 14, pl. VI, fig 5.

Thyrsopteris elongata—Geyler, Ueb. Foss. Pflanzen a. d. Juraform. Japans, p. 224, pl. XXX, fig. 5, XXVI, 4, 5. Schenk, in Richthofen's China, vol. IV, part X, p. 263 pl. LIV, fig. 1.

This fern, which is so numerous in the Middle Jurassic of Kaga, Hida, and Echizen, is also abundantly represented in several of the localities in Tosa and Awa. It is easily distinguished from the nearly related forms by the characteristic long linear pinnules acutely directed forward. Sterile as well as fertile fronds were obtained, the fertile ones being quite similar to those figured by me in the above mentioned work.

Nathorst's opinion that *Sphenopteris Mantelli* Schenk of the Wealden and *Thyrsopteris capsulifera* Velen. from the Cenomanian of Bohemia belong to *Onychiopsis*, is, I believe, quite justified.

Loc.—Numerous at Kaisekiyama near Sakawa, and Fujikawa in the Katsuragawa basin, less so at Yuasa and Kagahara. It also occurs at Kataji, Tōgodani, and Ishiseki, and by Nathorst it has been also described from Yakyō, Ueno, Ōtani, Haginotani and Hiura (Mitani).

5. *Onychiopsis elegans* m.

Pl. XXVIII, Fig. 7, 7a.

Frond twice pinnated ; pinnæ elongated, rachis slender with a terminal pinnule ; pinnules tolerably close together, thin, opposite or alternate, directed forwards, lanceolate, broadest at base and gradually tapering above, entire in the lower half and coarsely toothed at the upper, with apex obtusely pointed.

Veins rather numerous with distinct but evanescent midvein; lateral veins acute, simple or once forked.

The general features of this graceful fern are those of *Ongeliopsis*, although in some respects it also resembles *Dicksonia*. The generic determination is therefore not settled.

Loc.—Kaisekiyama. Only one specimen.

6. *Adiantites yuasensis*, n.

Pl. XXI, Fig. 15.

I have only a single and not quite perfect specimen of this fern. It shows a pinna with a slender rachis on which we see subopposite, distant, very thin, oval pinnules cuneate at base and furnished with a short stalk. The upper margin is not well preserved. Veins are very typical, being fine, very numerous, equal, divergent, and repeatedly dichotomous. Therefore, there is not the least doubt that we have here to deal with a fern closely akin to our recent *Adiantum*.

Schenk described from the Albours Chain a similar form under the name of *Adiantum Tietzei* (Die von E. Tietze in der Albourskette gesam. foss. Pflanzen, p. 3. pl. II, fig. 2) which possesses larger and more closely set pinnules.

Loc.—Yuasa in Kii.

7. *Pteris* (?) sp.

Pl. XX, Fig. 9. Pl. XXI, Fig. 6, 7. (?)

A fragment of a fern from Kagahara (Pl. XX, fig. 9) having thin, parallel-sided, spatulate pinnules, which measure a little over 2 cm. in length and 8–9 mm. in breadth. One of the pinnules is seen attached to the slender rachis by the greater part of the base. Veins are fine and numerous. The midvein is distinct. The lateral

veins are very fine, once or twice forked, and rise at first acutely, but soon bend outward so as to be nearly perpendicular to the midvein.

The fragments in Figs. 6 and 7, pl. XXI, from Kii with evanescent midvein resemble in many respects the fern just described. But the pinnules seem to be broader and the veins coarser.

The specimens represented in the figures above cited are at all events too imperfect to allow even a correct generic determination.

Loc.—Kagahara, Yuasa (?). Rare.

8. *Sphenopteris tenuicula* m.

Pl. XX, Fig. 11. Pl. XXI, Figs. 2, 2a. Pl. XXVIII, Fig. 6.

Frond tripinnated; *rachises* of various orders slender; *ultimate pinnæ* short; *pinnules* thin, remote, obtuse, very acutely directed forward, cuneate at base, entire or lobed or even pinnately parted; *lobes* or *partitions* few, also obtuse and some of them may again be furnished with 2 or 3 coarse crenations; *veins* few, equal, fine, divergent, and several times forked.

This fern is present only in small fragments, yet is sufficiently characterised to be treated as a new form. It seems to have been tolerably delicate, as can be judged from the slender rachises and thin pinnules.

Sphenopteris Auerbachi Trautschold (Der Klin'sche Sandstein, p. 19, pl. XVIII, fig. 5) and *Sphenopteris Goeperti* Dunker (Monographie der norddeutschen Wealdenbildung, p. 4, pl. I, fig. 6. IX, 1-3) are near relatives of our Japanese species. But in the former the venation is obsolete, while in the latter the lobes are single-veined. *Thyrsopteris brevipennis* Fontaine (The Potomac or Younger Mesozoic Flora, p. 124, Pl. XXXIV, fig. 3, XXXVI, 2) is also not unlike our plant, but possesses more numerous veins.

What Nathorst calls *Sphenopteris cf. Goepperti* Dunker (Beiträge, p. 11, pl. VI, fig. 2, 3) probably pertains to the same fern.

Loc.—Yuasa (numerous fragments), Kaisekiyama (rare) and Kagahara (rare).

9. *Pecopteris Browniana* Dunk.

Pl. XXIV, Fig. 2, 3. Pl. XXVII. 1-4, 5*ed.*

Pecopteris cf. Browniana—Nathorst, Beitr. z. mesoz. Flora Japans, p. 13, pl. V, fig. 5.

Pecopteris Browniana—Dunker, Monographie d. norddeutschen Wealdenbildung, p. 5, pl. VIII, fig. 7. Schenk, Fossile Flora der nordwestdeutschen Wealdenformation, p. 215, pl. XXVI, figs. 2, 2*a*. Fontaine, The Potomac or Younger Mesozoic Flora, p. 88, pl. XXII, fig. 10, 11, XXIII, 2-7, XXVI, 3, 13.

Aethopteris Browniana—Schenk, Zur Flora der nordwestdeutschen Wealdenformation, p. 159, pl. XXVI, fig. 3-5.

Frond tripinnated; *rachises* of the first and second order comparatively slender, but rigid; *primary pinnae* subopposite; *secondary pinnae* elongated, straight or lightly curved, opposite, subopposite or alternate, more or less directed forward; *pinnules* leathery, opposite or alternate, quite close or a little separate, finger-shaped, obtuse or acute, sometimes lightly falcate at the end of the pinna, and attached to the rachis with the whole base; they are generally entire in the middle part of the frond, but in its posterior part, or in its anterior part where secondary pinnae are reduced to pinnules, they are crenate or toothed, much more elongated even to linear, and now and then, strongly falcate; *veins* in smaller entire pinnules indistinct, in larger and crenate or toothed ones, with a distinct midvein which sends off acute, simple or sometimes even twice to thrice forked lateral veins.

This is the most abundant fern at Kaisekiyama and Fujikawa. Nathorst had already compared it with *Pecopteris Browniana* Dunker. The shape of the pinnules in this fern is very different in different

parts of the frond, and this has been already noticed by Fontaine in his American specimens. *Pecopteris brevipennis* Font. (The Potomac Flora, p. 88, pl. XXVI, fig. 4) seems to be only a part of the frond of this fern with toothed pinnules. *Pecopteris* sp. of Nathorst from Haginotani (*l. c.* Pl. VI, fig. 4) also appears to belong to it.

In one of the fragments apparently belonging to this fern, pinnules were observed bearing sori (pl. XXVII, fig. 1, 1a) which are preserved as black round dots in number of 2-4 on each side of the midvein. This mode of fructification strongly reminds us of the recent genus *Aspidium*.

Loc.—Kaisekiyama (most abundant), Yoshida-Yashiki (numerous), Chōja, Fujikawa (frequent), Tōgodani.

10. *Pecopteris Geyleriana* Nath.

Pl. XXI, Fig. 12. Pl. XXIII, Fig. 1, 1a. Pl. XXVIII, Fig. 5.

Pecopteris Geyleriana—Nathorst, Beiträge zur mesoz. Flora Japans, p. 8, pl. IV, fig. 1, VI, 1.

This plant first described by Nathorst from Ryōseki has been since found in several other localities. As this author had already pointed out, many of the pinnules are eared, not only on the anterior side, but also often on the posterior side, so that they become more or less triangular in shape. And it is a peculiar character of this fern that the lowest pinnules on the front of the rachis are often falcate backward instead of forward. The pinnules are in general blunt, but in those which are elongated are often pointed. Veins are in most cases indistinct save the evanescent midvein; in some cases, however, dichotomous lateral veins were observed (pl. XIII, fig. 1a).

Nathorst's assumption, that the specimens with smaller and more pointed pinnules like those represented in fig. 2, 6, pl. IV of his work

may represent a thrice pinnated portion of a frond, has been verified in several specimens in my collection. That the pinnae were not all in one plane is also shown by having them more or less overlapping.

This species, which exhibits a close relationship to *Pecopteris exilis* Phillips (Yokoyama, Jurassic Plants, p. 35, pl. I, fig. 8-10.), is distinguished from it by auriculate and at the same time more falcate pinnules.

Loc.—Fujikawa (very frequent), Ynasa (do.), Kataji (do.), Fujikawa (less frequent), Tōgodani. By Nathorst it has been also described from Torikubi.

11. *Pecopteris* cf. *virginiensis* Fontaine.

Pl. XXIV, Fig. 1.

Pecopteris virginiensis.—Fontaine, The Potomac or Younger Mesozoic Flora, p. 82, pl. VIII, figs. 1-7, IX, 1-6, XXIV, 2, CLXIX, 3.

A fragment of a pinna with alternate or subopposite, distant, long linear, toothed pinnules no doubt belongs to a fern which, if not identical with, is at least very closely allied to, *Pecopteris virginiensis* Fontaine, from the Potomac Formation of America. The lateral veins, owing to the thick consistence of the leaf are not quite distinct, but so far as I can see, they seem to be acutely directed forward and at least once forked.

Loc.—Fujikawa. Tanno (?).

12. *Chladophlebis* Nathorsti m.

Pl. XXVIII, Figs. 3, 4, 10, 11.

Chladophlebis sp.—Nathorst, Beiträge z. mesoz. Flora Japans, p. 4, pl. I, figs. 1-3, p. 8, 13.

Frond bipinnated; *rachises* comparatively slender; *pinnae* alternate, elongated, narrowed at base; *pinnules* coriaceous, opposite or alternate, close

together, often confluent at base, triangular to lanceolate, entire, falcate, obtuse or pointed, and shorter in the posterior than in the middle part of the pinna; veins distinct; midvein disappearing near the apex; lateral veins directed acutely forward and once dichotomous.

I have no doubt that what Nathorst calls *Chladophlebis* sp. in his Beiträge refers to this fern. This author had already recognised its difference from the closely allied *Asplenium* (*Chladophlebis*) *whitbiense* Brgt. in having the pinnae narrowed at base. The reason why he left it unnamed was the indistinct venation which characterises most of the fossils of Ryōseki. At Kaisekiyama, however, where the state of preservation is much more favourable, the veins are well preserved, and are quite similar to those of *Asplenium Roesserti* Presl. from the Rhaetic of Yamanoi. (Yokoyama, on some Fossil Plants from the Coal-bearing Series of Nagato, pl. XXIV, figs. 1, 2, 5). The difference between the latter and *A. whitbiense*, which species are often very difficult to distinguish, has been given in the work just mentioned. In truth, *Chladophlebis Nathorsti* shows such a close affinity to *Asplenium Roesserti* that it would be often quite impossible to distinguish the two, especially when the specimens are present only in fragments.

Loc.—Kaisekiyama; numerous. Yuasa; frequent but in fragments; Kataji and Ishiseki; numerous. According to Nathorst, the plant occurs also at Ueno, Tōgodani, and Hiura.

13. *Macrotaeniopteris* (?) *marginata* Nath.

Macrotaeniopteris (?) *marginata*.—Nathorst, Beiträge, *loc. cit.*, p. 14, pl. VI, fig. 6, 6a.

What Nathorst has described under the above name from an unknown locality is not represented in my collection.

Lycopodiaceæ.

14. *Lycopodites* sp.

Lycopodites sp.—Nathorst, Beiträge, *loc. cit.*, p. 10, pl. II, fig. 3.

This species found by Nathorst at Ueno is also not represented in my collection.

Cycadeaceæ.

15. *Podozamites lanceolatus* Lindley et Hutton.

Pl. XXIII, Figs. 4, 5.

Podozamites lanceolatus.—Heer, Beitr. zur. Jurafloora Ostsib. u. d. Amurl., 1876, p. 110, pl. XXVII, figs. 6, 7, 8, 5*ab*. Yokoyama, Jurassic Plants, *loc. cit.* p. 45, pl. V, fig. 8. Velenovsky, Die Gymnospermen d. böhm. Kreideform. p. 11, pl. II, figs. 11–19, 24.

There are only two leaflets in my collection which are referable to the well known *Podozamites lanceolatus*. One of them (fig. 5) has the petiole preserved, and the other (fig. 4) the apex. Both seem to belong to the variety *minor* of Heer.

Loc.—Tanno.

Nathorst mentions and describes in his Beiträge p. 10, pl. IV, fig. 7, another variety *latifolia* of the same species from Kataji.

16. *Podozamites pusillus* Velenov.

Pl. XX, Fig. 2, 3*b*, 4, 5, 7.

Podozamites pusillus.—Velenovsky, Die Gymnospermen der böhm. Kreideform., p. 11, Pl. II, figs. 20–22, 24*a*.

A number of small cycadeous leaflets, mostly isolated, but rarely attached to the rachis, oval or elongate oval in shape, rounded or blunt at apex, and when well preserved, furnished with a short petiole.

These are no doubt to be identified with *Podozamites pusillus*, a species founded by Velenovsky on a Cretaceous form from Bohemia. Fig. 5 represents a leaflet about 22 mm. long and 6. mm. broad belonging to a longer form. It possesses 22 distinct elevated parallel veins between which a single interstitial vein is visible. Fig. 4 represents one attached to a strong rachis. It also shows distinct veins. In other specimens they are more or less defaced. A leaflet represented in fig. 2 left, is much slenderer than others and resembles *Podozamites lanceolatus*, so that it is not impossible that this species may afterwards turn out to be only a variety of the latter.

Podozamites lanceolatus var., which I described from Kaga (Jurassic Plants, Pl. V, fig. 5), is not unlike the longer forms of *Podozamites pusillus*, although more abruptly tapering above and acutely ending.

Loc.—Kagahara; frequent.

17. *Podozamites* sp.

Pl. XXV, Fig. 8–12.

Many fragments of a leaf with small, elongate oblong to lanceolate leaflets which look very much like those of the preceding species. In most of them, however, the veins are completely obliterated, and when slightly visible they seem to be much coarser than in *Podozamites pusillus*. In this latter respect, the plant resembles some species of the genus *Nageiopsis* described by Fontaine from the Potomac Formation, e.g., *Nageiopsis heterophylla* (Fontaine, *loc. cit.* pl. LXXXVI, fig. 6).

Loc.—Katagi.

18. *Zamiophyllum Buchianum* Ett sp.

Pl. XX, Fig. 1. Pl. XXII, Figs. 1, 2. Pl. XXIII, Fig. 6.

Pl. XXVII, Fig. 5*ab*. Pl. XXVIII, Fig. 1, 2.

Zamiophyllum Buchianum.—Nathorst, Beiträge z. mesoz. Flora Japans, p. 6, pl. II, figs. 1–2, III, V, 2, p. 9.

Pterophyllum Buchianum.—Ettingshausen, Beitrz. Flora d. Wealdenperiode, p. 21, pl. I, fig. 1. Schenk, Die Fossilen Pflanzen d. Wernsdorferschichten i. d. Nord Carp. p. 8, Pl. III, fig. 5.

Dioonites Buchianus.—Fontaine, The Potomac or Younger Mesozoic Flora, p. 182, pl. LXVIII, fig. 1, LXIX, 1, 3, LXXI, 1, LXXII, 1, 2, LXXIII, 1-3, LXXIV, 1-3.

This plant first pointed out by Nathorst as occurring at Ryōseki is profusely represented in my collection. A specimen shown in fig. 6, pl. XXIII is from Sakamoto. It represents a leaf belonging to its upper part. The leaflets are narrower than those figured by Nathorst, the broadest being about 5 mm. with 15-18 veins. It looks more like fig. 2, pl. LXX of Fontaine. The specimens from other localities show no essential difference from those described by Nathorst. In some, however, the apex of the leaflets were observed, and in one from Kaisekiyama it was bluntly pointed (fig. 5*ab*, Pl. XXVII), while in that of Yuasa it was acute (fig. 2, Pl. XXII.).

Fontaine in describing this species from the Potomac, where it seems to be very abundant, used the generic name of *Dioonites*, but as this name is now applied to a cycad whose leaflets or segments are attached in front of the rachis with their whole base, it would be advisable to retain that proposed by Nathorst.

Loc.—Kaisekiyama, Yuasa, Sakamoto, Tanno, Kagahara, Tōgodani, Ishiseki, Katagi. According to Nathorst, the plant occurs also at Ōtani, Torikubi, and Ueno.

18*a*. *Zamiophyllum Buchianum* Ett. sp.
var. *angustifolia* Font.

Pl. XXII, Fig. 4. Pl. XXV, Fig. 5. Pl. XXVIII, Figs. 8, 9.

Dioonites Buchianus var. *angustifolius*.—Fontaine, The Potomac or Younger Mesozoic Flora, p. 185, pl. LXVII, fig. 6, LXVIII, 4, XVI, 2.

None of the specimens which I refer to this form are quite well preserved; still I think I am sufficiently justified in placing them under it, as the leaflets exhibit essentially the same characters as the foregoing species. They are long, linear, 2–2.5 mm. broad, acute at apex, and somewhat narrowed at base. They are tolerably close together, directed acutely forward, and one of them seems to terminate the leaf. Fig. 4, pl. XXII shows a specimen from Yuasa. Its leaflets possess 7–9 parallel veins. Two fragments from Kaisekiyama show also 7–9 veins. A specimen from Kataji (fig. 5, pl. XXV) has them quite obliterated.

Fontaine considers *Dioonites abietinus* Miq. (Schenk. Fossil Flora d. norddeutsch. Wealdenform. p. 32, pl. XVI, fig. 1) as referable to this variety of *Zamiophyllum Buchianum*, to which indeed it shows a great resemblance.

Loc.—Kataji, Kaisekiyama, Yuasa; rare.

19. *Zamiophyllum Naumanni* Nath.

Pl. XXII, Fig. 3. Pl. XXVI.

Zamiophyllum Naumanni—Nathorst, Beitr. z. mesoz. Flora Japans, p. 7, pl. V, fig. 1.

I possess a large specimen of a *Zamiophyllum* from Ishiseki which is undoubtedly referable to the species founded by Nathorst. The leaflets are distant, opposite and not tapering towards the base at which place however they are a little contracted. The breadth measures up to 20 mm. Besides this undoubted specimen of *Zamiophyllum Naumanni*, there are many fragments of leaflets from other localities which measure 12–15 mm. in breadth, and are broader than those of *Z. Buchianum* figured by Nathorst and Fontaine, in which the breadth seems never to exceed 10 mm. These are therefore to be

considered as those of *Z. Naumanni*. However as the difference between the two species lies not in the breadth, but in the form of the leaflets and in their mode of attachment to the rachis, it is in many cases impossible to decide which of the two species we are really dealing with, especially when the specimens are in small fragments. Moreover there is much doubt whether *Z. Naumanni* is not a species founded on the lower part of a leaf of *Z. Buchianum*.

Loc.—Kaisekiyama, Kataji, Tōgodani, Ishiseki, Yuasa.

20. *Glossozamites parvifolius* n.

Pl. XXI, Figs. 5, 5a.

I possess a small fragment only of a pinna in which the rachis dichotomises, with small, distant, opposite or subopposite, entire, obovate leaflets, about 5 mm. in length and attached to the rachis almost perpendicularly. Veins are few, equal, divergent, those in the middle part of the leaflet dichotomous, and those near the lateral margin simple.

This plant reminds us of *Podozamites* (*Glossozamites*) *oboratus* Schenk (Die Foss. Pflanzen d. Wernsdorferschichten i. d. Nordcarpathen, p. 10, pl. II, figs. 7-10, III, 1-3), which however possesses much larger leaflets.

Loc.—Yuasa.

21. *Nilssonia Johnstrupi* Heer.

Pl. XXV, Figs. 1-4.

Nilssonia Johnstrupi.—Heer, Flora der Komeschichten, Flora Fossilis Arctica, Vol. VI. p. 44, pl. VIII, figs. 1-6.

Nilssonia cf. orientalis.—Nathorst, Beiträge. l. c., p. 5, Pl. I. fig. 4-5.

Now and then there occur at Ryōseki oval or linear oval, entire leaves of a cycad which exhibits a close resemblance to *Nilssonia ori-*

entalis Heer (Yokoyama, Jurassic Plants, Pl. XIV, figs. 4-9). These leaves are in general larger than those from Echizen, some attaining 4 cm. in breadth and up to 20 cm. in length. In this respect they resemble more those of *Siberia* figured by Heer. But the veins in the Ryōseki plant are doubly as coarse, there being only 2 in a millimeter, while in the other we can count 4 in the same space. Therefore we have here to deal with another species of *Nilssonia*, and indeed with *N. Johnstrupi* Heer from the Lower Cretaceous of Greenland, which agrees exactly with the Japanese form both in number of veins and also in the more oval shape of the leaves.

Nilssonia bohémica Velenov. (die Gymnospermen d. böhm. Kreideform. p. 11, pl. II, fig. 25-28) seems also to be a very nearly related species. It has however the leaves longer and narrower.

Loc.—Kataji and Tōgodani ; rare.

22. *Nilssonia schauburgensis* Dkr.

Pl. XX, Figs. 12, 14. Pl. XXI, Fig. 14. Pl. XXII, Figs. 5-7.

Nilssonia cf. schauburgensis.—Nathorst, p. 5, pl. I, figs. 6-9a, p. 9, 13.

Pterophyllum schauburgense.—Dunker, Ueb. d. Norddeutsch. Wälderthon. Progr. d. höheren Gewerbeschule in Cassel, p. 116. Monogr. d. Norddeutsch. Wealdenform., p. 15, pl. I, fig. 7, II, 1, VI, 5-10. Ettingshausen, Beitr. z. näheren Kenntn. d. Flora d. Wealdenperiode, p. 22.

Anomozamites schauburgensis.—Schenk, Die Flora d. Nordwestdeutsch. Wealdenform. p. 231, pl. XXXIII, figs. 1-9.

Nathorst has rightly referred this plant to the well known Wealden form *Nilssonia schauburgensis* Dkr. sp. The only difference existing between the Japanese and European forms is the entire absence of segments with rounded apex, the segments here being more or less pointed in all the specimens. But as there are also such forms in the European plant (*ex. gr.*, figs. 4, 7, Pl. XII of Schenk), so there is no

reason why we can not unite the two, especially when we consider that the shape of the segments in this genus is generally very variable.

All the specimens hitherto found in Japan belong to the form with narrow leaves, that represented in fig. 14, Pl. XX being the broadest.

Loc.—Yuasa, Kataji, Tōgodani; numerous. Kagahara; rare. The plant is also mentioned as from Hiura by Nathorst.

23. *Nilssonia pterophylloides* m.

Pl. XXII. Figs. 8–10 Pl. XXV. Fig. 7.

Leaf elongated gradually tapering anteriorly and ending in a terminal segment, deeply and regularly segmented; segments alternate, directed slightly forward, linear, parallel sided, apex obtuse or obtusely pointed; veins fine, parallel, simple, 8–10 in number.

Although this plant looks like *Pterophyllum*, the insertion of the leaf on the rachis distinguishes it from the latter genus.

The segments are all regular, narrow, parallel margined, 3–4 mm. in breadth, and standing inclined at about 70° to the moderately strong rachis. The apex is obtuse in those at the posterior part of the leaf and more pointed in those at the anterior part of it, and their line of meeting on the surface of the rachis appears to have been zigzag, as seen in fig. 10, pl. XXII.

I am acquainted with no form of the genus which can be directly compared with our species. Indeed, *Zamites gracilis* Kurr (Beitr. z. foss. Flora d. Juraform. Württ., p. 11, pl. I. fig. 4) and *Pterophyllum pecten* Lindley and Hutton (Fossil Flora of Great Britain, vol. II, p. 102) greatly resemble the Japanese form, as far as their external appearance is concerned, but according to Schimper (Zittel, Handbuch der Palaeontologie, Abth. Palaeophytologie, p. 223) they both belong

to *Ctenophyllum* in which the veins are partly forked and those near the margin run unto it.

A specimen represented in fig. 7, pl. XXV is a splendid one from Kataji, showing the under side of the leaf, although the veins are not well visible. The other figures represent fragments found at Yuasa.

Loc.—Kataji, Ishiseki, Yuasa; not rare.

24. *Ptilophyllum* cf. *cutchense* Morris.

Ptilophyllum cf. *cutchense*.—Nathorst, Beitr. z. mesoz. Flora Japans, p. 12, Pl. IV, fig. 8.

What Nathorst has described from Chōja under the above denomination, I have not been able to find in my collection.

Coniferæ.

25. *Cyparissidium* (?) *japonicum* m.

Pl. XX, Figs. 3a, 6, 6a, 13. Pl. XXIV, Fig. 4.

Branches copious, alternate, rising at an acute angle, slender, and cord-like, straight or very slightly curved, ultimate branches about 1 mm. in breadth; leaves imbricated, closely appressed, acute at apex, with a longitudinal ridge on their backs.

In Kagahara there are remains of a coniferous plant whose branches thickly cover the faces of stones. Unfortunately however they are so flatly pressed and smoothed that the minute details of the leaves are not well visible; but in a specimen from Tanno which is much better preserved, the imbricate leaves, as they are always partly covered by the two preceding ones, appear lozenge-shaped. The longitudinal ridge is distinct and tolerably elevated, and in external impressions of leaves, it is present as a deep groove. It is not possible to decide as to what genus the plant belongs, there being several genera with a similar type of leaves. I have brought it therefore provisionally

under the genus *Cyparissidium* Heer on account of its great resemblance to *C. gracile* Heer (Kreideflora der Arktischen Zone, p. 74, pl. XIX). It is also not unlike some of the *Arthrotaxis* described by Fontaine from the Potomac.

Loc.—Kagahara; in great abundance. Tanno, Fujikawa ; rare.

26. *Torreya venusta* m.

Pl. XXII, Figs. 11, 12, 12a.

Stem slender; leaves tolerably close together, subopposite, directed somewhat forward, small, linear, straight or slightly curved outwards, gradually tapering in front into an acute apex, base abruptly rounded and distichous; no distinct midrib.

There occur fragments of a little* conifer whose leaves measure 5–6 mm. long and 1 mm broad, and are directed forwards at an angle of about 60° to the stem. They possess no midrib; but in its stead, two strong cord-like longitudinal lines were observed in one or two leaves. Therefore it is not wholly improbable that we have here to deal with a plant at least closely akin to the recent genus *Torreya*. The leaf-substance, however, as it appears on stone, seems not quite so firm as might be expected in this genus. Therefore the generic determination must be left still unsettled.

Fontaine describes a similar conifer from the Potomac under the name of *Torreya virginica* (l. c. p. 2234, pl. CIX, fig. 8).

Loc.—Yuasa ; not frequent.

Phyllopoda.

27. *Estheria rectangular* m.

Pl. XXI, Fig. 13.

Carapace-valves small, quadrate, moderately ventricose, broader than high with umbo terminal; the dorsal margin straight and parallel with the

anterior portion of the ventral margin, the posterior part of the latter obliquely ascending to the straight posterior margin, and forming at the point of junction a rounded angle; anterior margin truncate, parallel to the posterior and at right angles to the dorsal as well as to the anterior portion of the ventral margin, so that the valves assume a decidedly quadrate shape; from the umbo run two flat diagonal edges, one to the postero-ventral angle and one, which is flatter than the other, to the middle part of the ventral margin, which at this point ascends backwards; surface with about twelve short concentric ridges, between which there are still finer interstitial lines. Length 7.5 mm. height 3.5 mm., thickness of the left valve, 2 mm.

I possess only a single specimen of this species in the form of an external impression. My figure was drawn after a clay cast of it. The four-sided shape and two oblique edges are characters which remind us more of the Carboniferous Genus *Leaia* Jones (A Monograph of the Fossil *Estheriæ*. Appendix, p. 117, pl. I, fig. 19-21. Palæontogr. Soc. London, vol. XV, 1862) than any of the hitherto described *Estheriæ*.

Loc.—Yusa.

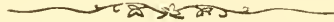


PLATE XX.

Plate XX.

KAGAHARA.

- Fig. 1.*—*Zamiophyllum Buchianum* Ett. sp.
,, *2, 3b, 4, 5, 7, 10.*—*Podozamites pusillus* Vel.
,, *3a, 6, 6a, 13.*—*Cyparissidium* (?) *japonicum* Yok.
,, *8.*—*Onychiopsis elongata* Geyl. sp.
,, *9.*—*Pteris* (?) sp.
,, *10.*—*Sphenopteris tenuicula* Yok.
,, *12, 14.*—*Nilssonia schauburgensis* Dunk. sp.

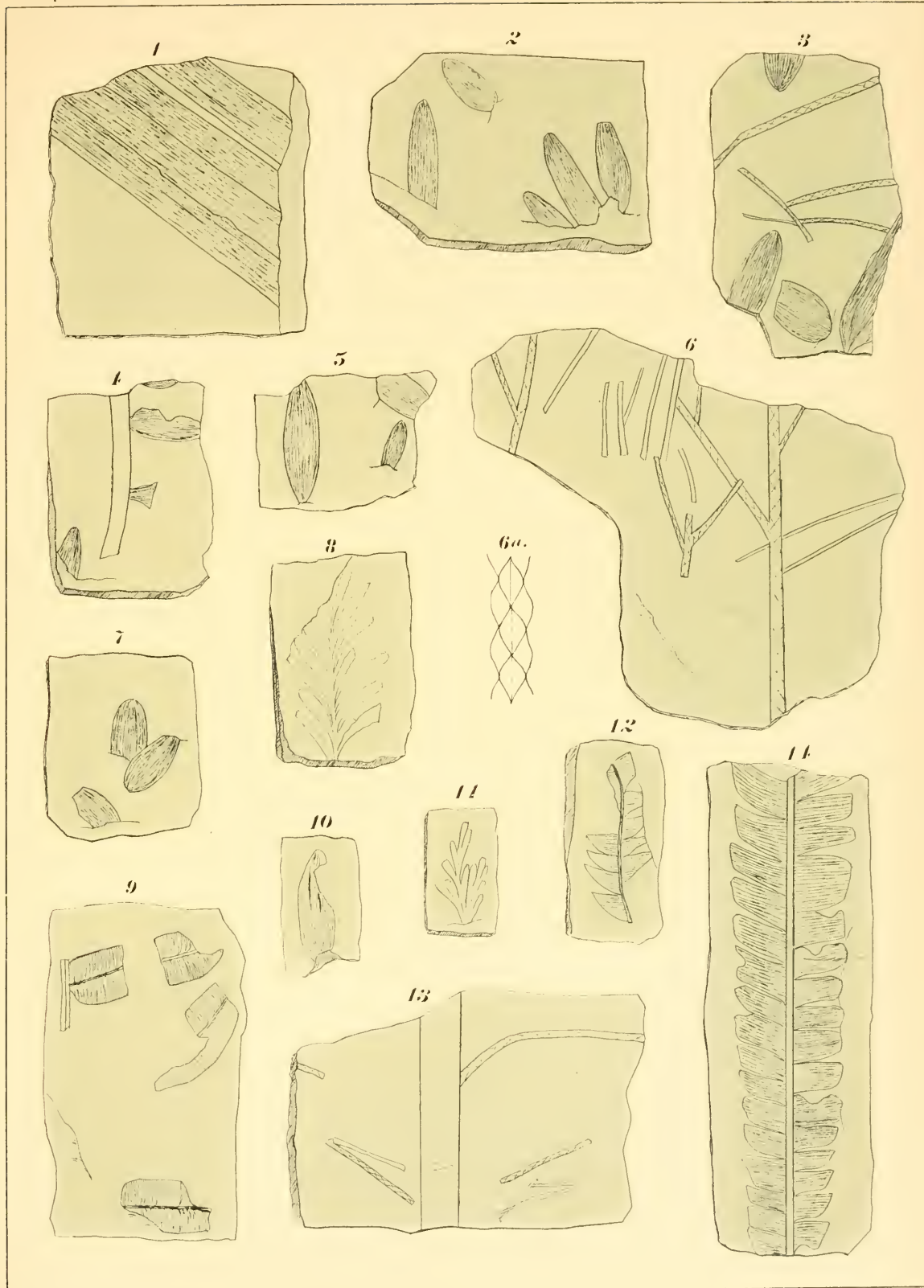


PLATE XXI.

Plate XXI.

Y U A S A.

- Fig. 1, 4.*—*Onychiopsis elongata* Geyl. sp.
.. *2, 2a, 3.*—*Sphenopteris tenuicula* Yok.
.. *5, 5a.*—*Glossozamites parvifolius* Yok.
,, *6, 7.*—*Pteris* (?) sp.
,, *8-11.*—*Chladophlebis Nathorsti* Yok.
.. *12.*—*Pecopteris Geyleriana* Nath.
,, *13.*—*Estheria rectangula* Yok.
,, *14.*—*Nilssonia schaumburgensis* Dunk. sp.
,, *15.*—*Adiantites yuasensis* Yok.

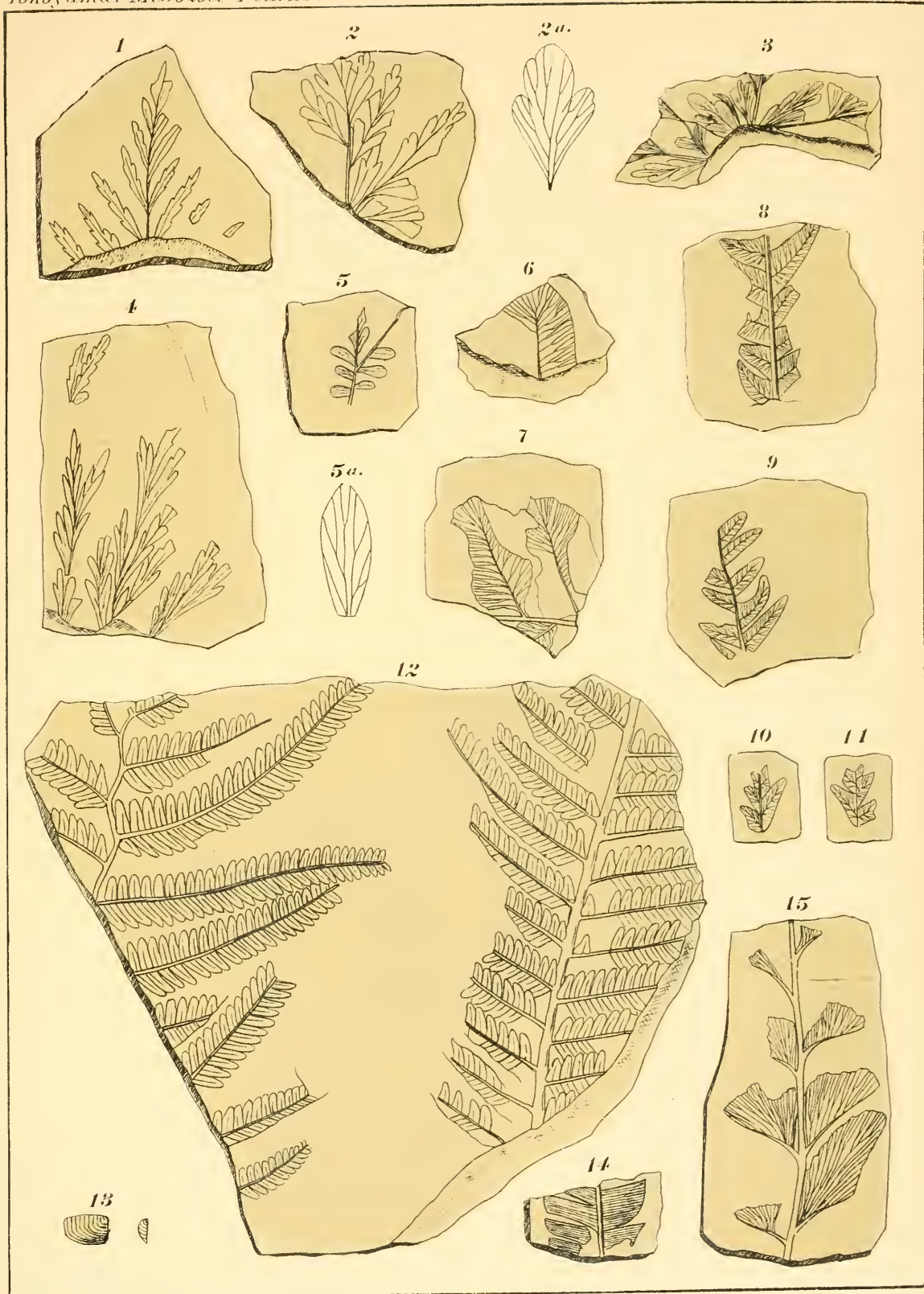


PLATE XXII.

Plate XXII.

Y U A S A.

- Fig. 1, 2.*—*Zamiophyllum Buchianum* Ett. sp.
,, *3.*—*Zamiophyllum Naumannii* Nath.
,, *4.*—*Zamiophyllum Buchianum* var. *angustifolia* Font.
,, *5-7.*—*Nilssonia schauinsburgensis* Dunk. sp.
,, *8-10.*—*Nilssonia pterophylloides* Yok.
,, *11, 12, 12a.*—*Torreya venusta* Yok.

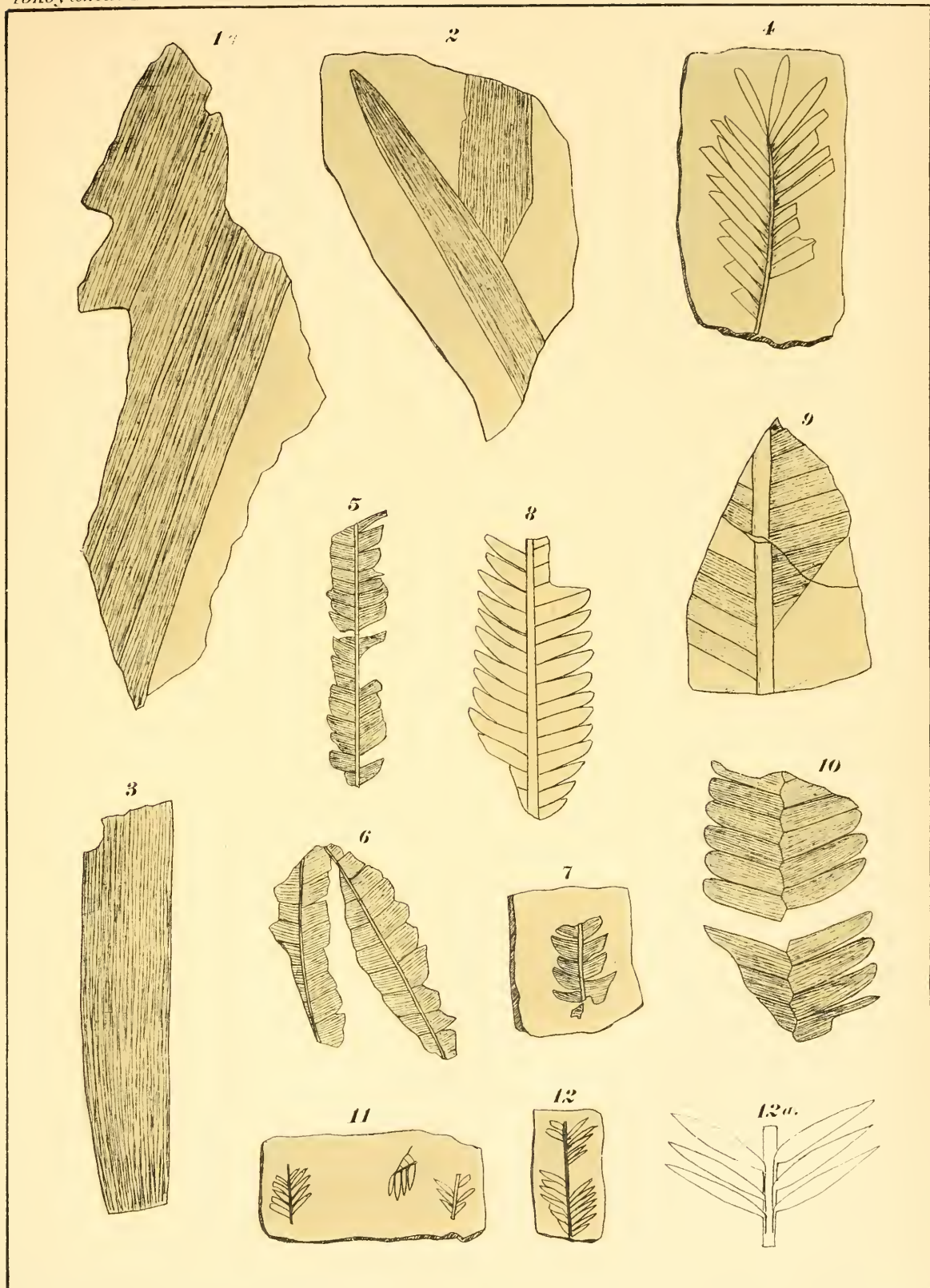


PLATE XXIII.

Plate XXIII.

FUJIKAWA.

Fig. 1, 1a.—*Pecopteris Geyleriana* Nath.

„ 2, 3.—*Thyrsopteris* sp.

TANNŌ.

Fig. 4, 5.—*Podozamites lanceolatus* Lind. et Hutt. sp.

SAKAMOTO.

Fig. 6.—*Zamiophyllum Buchianum* Ett. sp.



PLATE XXIV.

Plate **XXIV.**

FUJIKAWA.

Fig. 1.—*Pecopteris cf. virginiensis* Font.

,, 2, 3.—*Pecopteris Browniana* Dunk.

TANNŌ.

Fig. 4.—*Cyparissidium* (?) *japonicum* Yok.



PLATE XXV.

Plate XXV.

RYŌSEKI.

Fig. 1-4.—*Nilssonia Johnstrupi* Heer.

„ 5.—*Zamiophyllum Buchianum* var. *angustifolia* Font.

„ 6.—*Dicksoniopteris Naumanni* Nath.

„ 7.—*Nilssonia pterophylloides* Yok.

„ 8-12.—*Podozamites* sp.

„ 13, 13a.—*Dicksonia tosana* Yok.

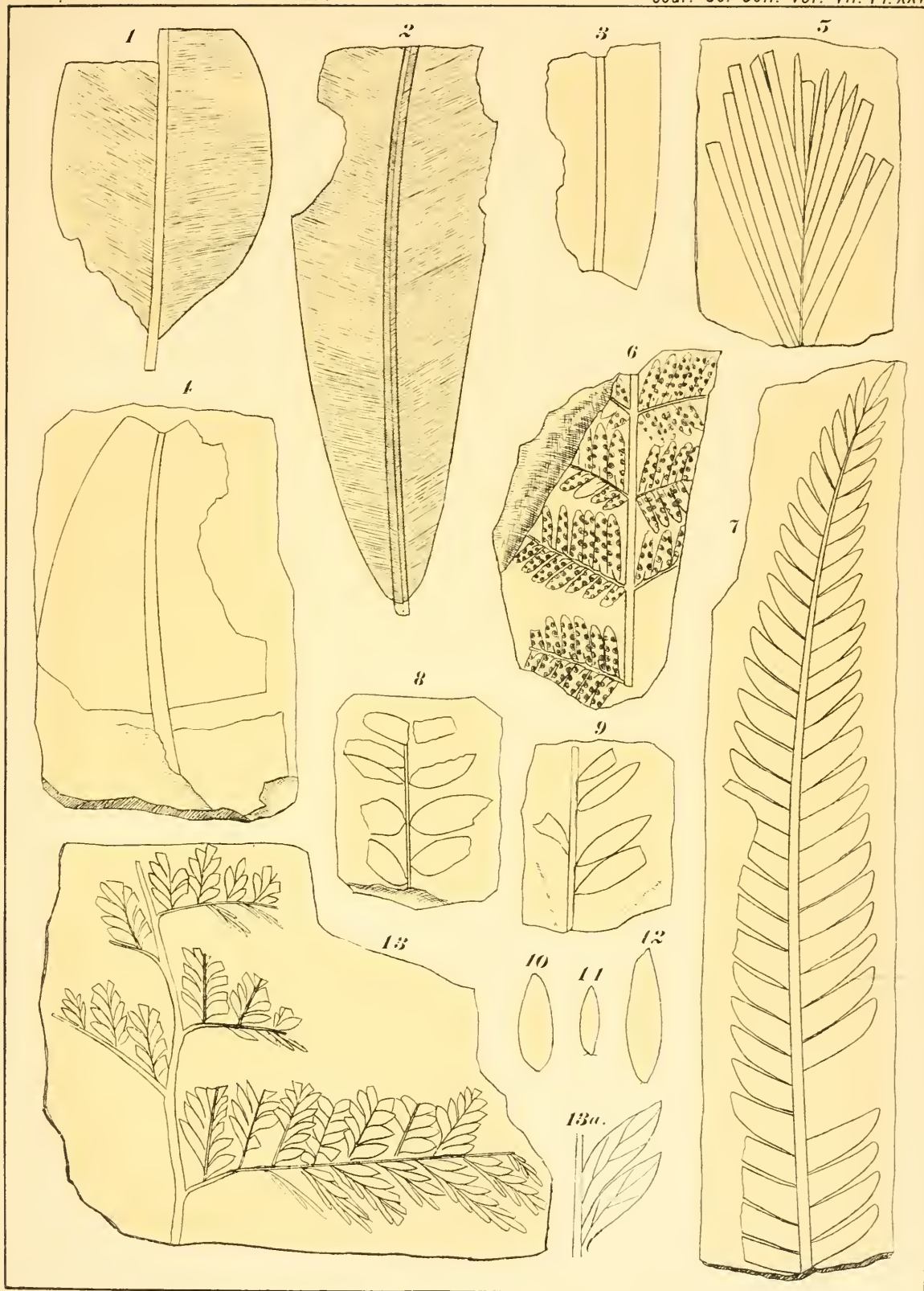


Plate XXVI.

ISHISEKI (RYŌSEKI).

Zamiophyllum Naumannii Nath.

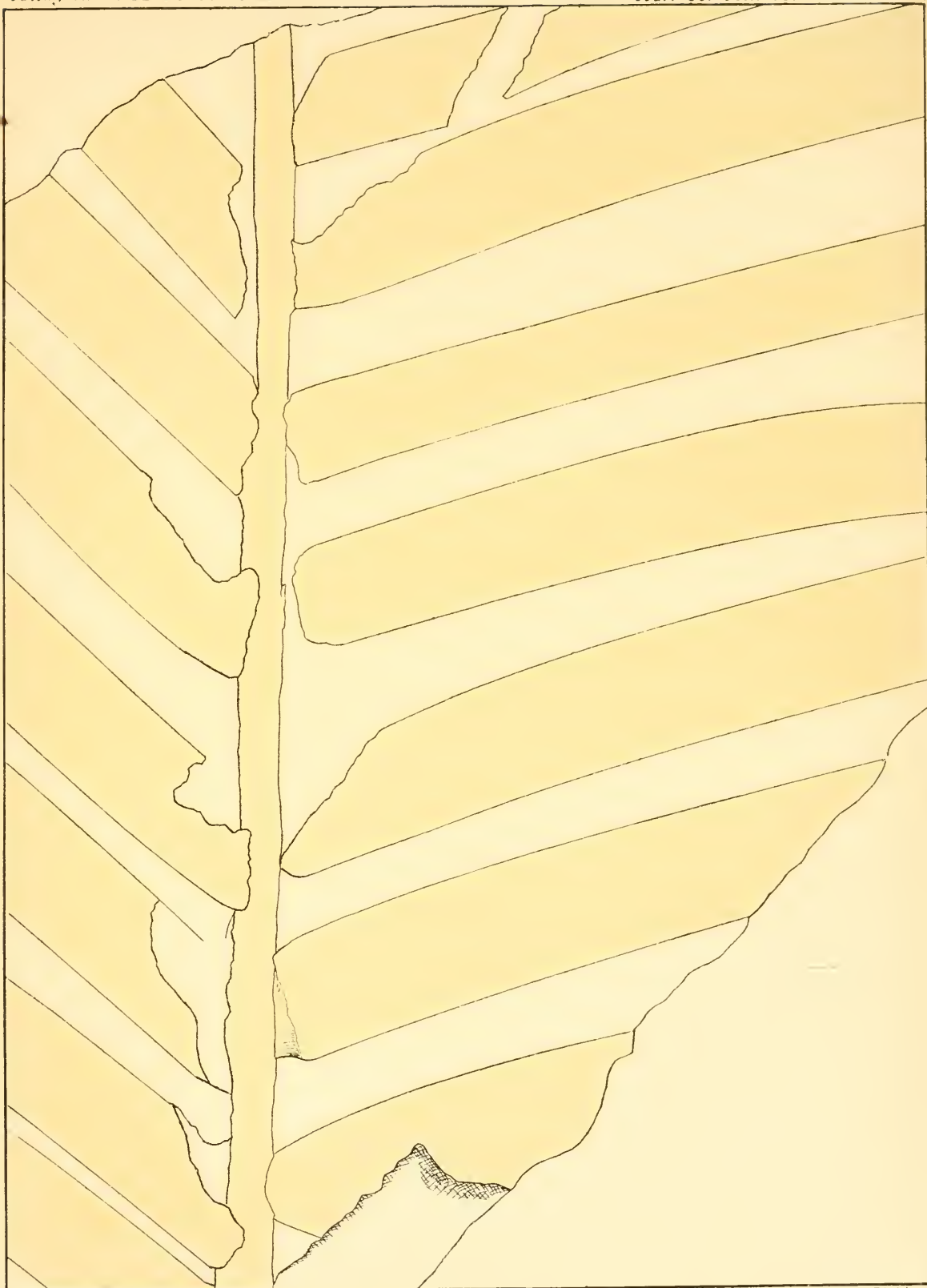




PLATE XXVII.

Plate XXVII.

KAISEKIYAMA.

Fig. 1, 1a, 3, 3a, 4 5cd.—*Pecopteris Browniana* Dunk.

,, *5ab.*—*Zamiophyllum Buchianum* Ett. sp.

SHIRAIHIGAWA.

Fig. 2.—*Pecopteris Browniana* Dunk.

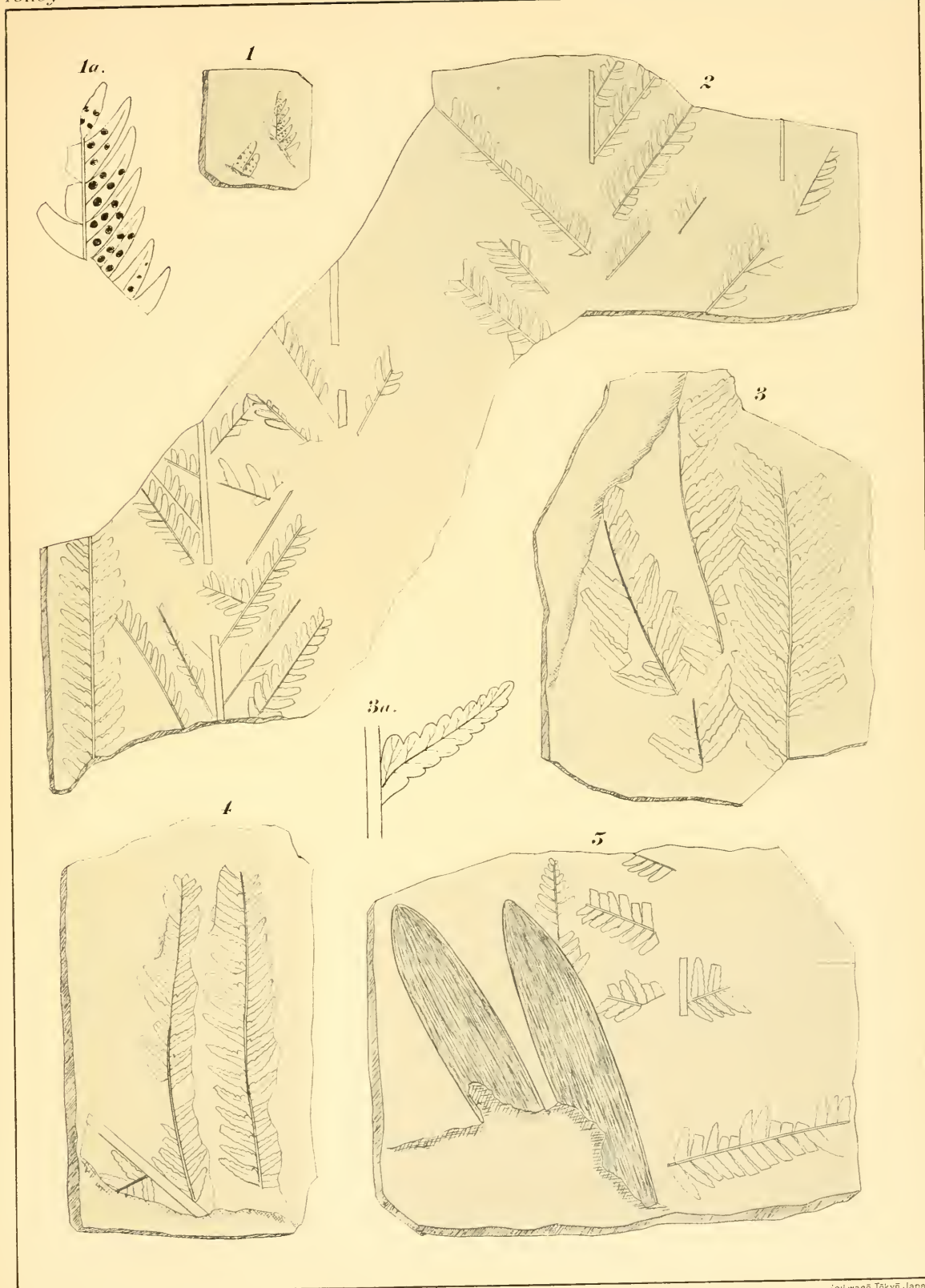
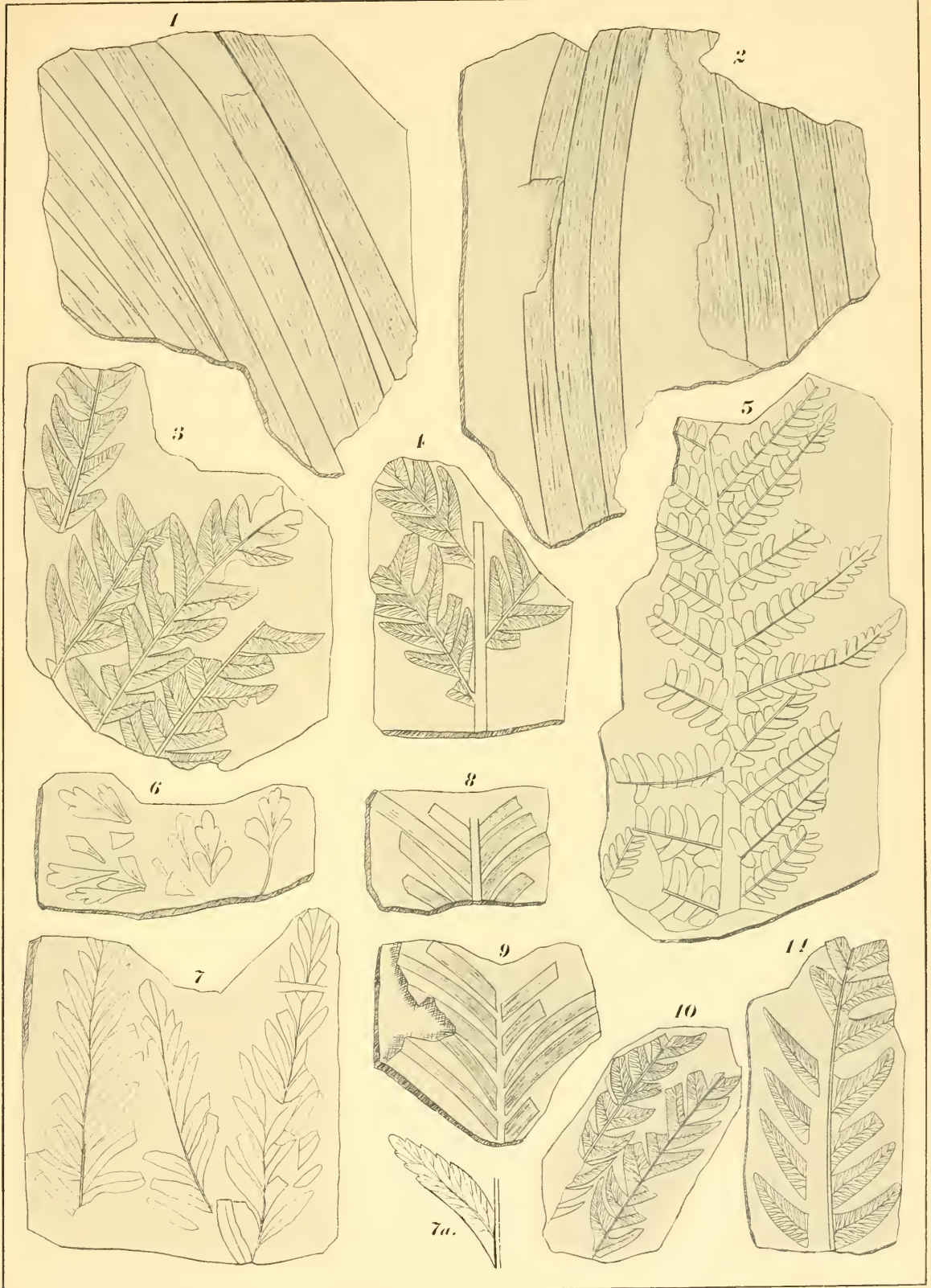


PLATE XXVIII.

Plate XXVIII.

KAISEKIYAMA.

- Fig. 1, 2.*—*Zamiophyllum Buchianum* Ett. sp.
„ *3, 4, 10, 11.*—*Chladophlebis Nathorsti* Yok.
„ *5.*—*Pecopteris Geyleriana* Nath.
„ *6.*—*Sphenopteris tenuicula* Yok.
„ *7, 7a.*—*Onychiopsis elegans* Yok.
„ *8, 9.*—*Zamiophyllum Buchianum* var. *angustifolia* Font.



On some Organic Remains from the Tertiary Limestone near Sagara, Tōtōmi.

By

Kyugaku Nishiwada.

With Plate XXIX.

[In the spring of this year I took advantage of a two days' stay in Sagara, Prov. Tōtōmi, to examine the Tertiary limestone developed near the town and to collect some of the organic remains abounding in it. In this way I came across some nullipore limestone, the occurrence of which in Japan has not yet been recorded. Since then I have made another short visit to this locality and have studied the collected materials, under the supervision of Professor M. Yokoyama, in the laboratory of the Geological Institute, College of Science, Imperial University. Prof. Yokoyama has placed me under great obligations to him for his kind suggestions and for the loan of Gumbel's paper on the fossil *Lithothamnion*.—Hongō, Tōkyō; November, 1894].

So far as I am aware, there are few limestones in the Japanese Tertiary that can give us more geognostic interest than that under consideration.

Upwards of 3 km. north-west of the town of Sagara, there are two limestone hills, lying one on either side of the Haginagawa, a river which, after rising in the environs of Nakanishi, farther north-west of the present site, takes its sinuous course in a south-easterly direction between these hills and empties itself into the sea close to the town. That on the right of the stream is locally known as Mekamiyama and the other on the left of it as Okamiyama or

Okamidai, in allusion to the names of the villages Mekami and Okami. According to the Topographical Survey of the Army Department of the Empire, the former attains an elevation of 111.3 m. above the sea, while the latter is less high, or about 60 m.

The limestone occurring in the form of such isolated hills is of a quite unique character. A glance at the rock brings before us a congeries of the calcareous forms of some organic remains. The colour is cream-white or greyish-white, sometimes grey. It is pretty hard through the crystallisation of some of its organic inclusions. Chemically considered it is almost free from impurities,—ferrie oxide, magnesia, etc., being present in only very small quantities.

As regards structure, its stratification is for the most part indistinct. The Okamiyama limestone is simply massive, there being no structure suggestive of bedding, except fissure-planes of which it is full and through which much solution has been effected. Much of the Mekamiyama stone, hidden as it is by its covering of soil, may be in the same state; but on the east flank of the hill, where extensive quarries afford an opportunity of studying its structure, it is certainly stratified and strikes N. 20°–30° E. with a very high inclination towards SEE. Mr. Nakashima¹ of the Imperial Geological Survey believes that the limestone of these hills suffers from an anticlinal folding, extending to the overlying strata, and that what is seen on the eastern flank of the Mekamiyama shows that the beds dip away from a central axis very steeply to the south.

During my second visit to the locality, an attempt was made to determine the relation of the limestone to the other sedimentary rocks. The evidence then obtained bearing upon this is, however, not quite decisive, although the rocks may in places be observed in association.

1. Shizuoka Zufuku Chishitsu Setsumeisho. p. 12.

Along a brook, on the north-western side of the Mekamiyama, there is exposed a Tertiary formation of sandstones and shales, to which reference will be made later on, which strikes nearly N-S. and dips to the east at an angle of 35° . On the north-eastern side of the hill is also seen an alternation of shale and sandstone quite similar to the others, the strike of which is nearly NE. and the south-easterly dip very high. Moreover, to the west of the hill, beyond a very narrow rice-field is laid bare along a brook another similar alternation with north-westerly inclination. At the Okamiyama, on the other hand, any such exposures as the above are concealed by the talus of limestone blocks on all sides of it. There is, moreover, but little outcrop of rocks in the neighbourhood, with the exception of a very limited patch laid bare at Oiwa, about 150 m. east of the hill, where sandstone and shale are found in a fragmentary state in association with the limestone. So far as my observation goes, it seems most probable that a series of sandstones and shales rests directly upon the limestone, and that the latter appears sporadically from underneath that series and still younger strata, as in the hills here under consideration.

The accompanying figures will perhaps render the mode of occurrence of the limestone clearer than any description.



Fig. 1. Ideal section of the Mekamiyama from SEE. to NWW. Scale 1 : 7,000.

⊕ limestone quarries.

s. sandstone and shale.

a. rice-field.

p. much younger Tertiary.

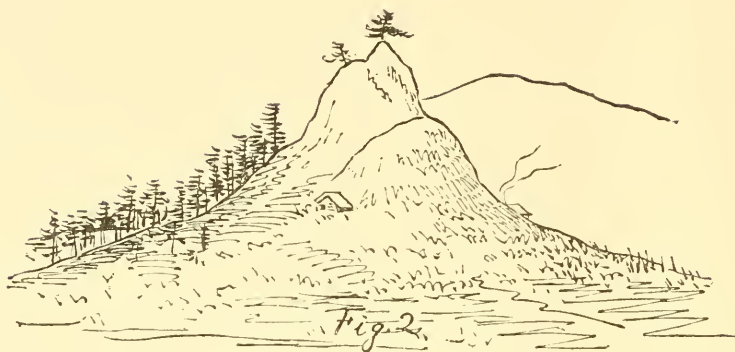


Fig. 2. A southern view of the Okamiyama or Okamidai, wholly made up of limestone. Sketched by the writer, Nov. 5th, 1894.

The organic remains occurring plentifully in the limestone are but few in genera. The following are some of the species which I have been able to recognise:

Lithothamnion.

Corals.

Millepora.

Foraminifera.

Turbo.

Pecten.

The abundance of the remains referable to each of these is generally in the order stated.

Description of the Fossils.

Lithothamnium ramosissimum Reuss.

Pl. XXIX. Fig. 1, 2, 3, and 4.

Nullipora ramosissima—Reuss, Naturw. Abh. v. Haidinger., Bd. II., 1848, p. 29;
T. III., Figs. 10 and 11.

Nullipora ramosissima—F. Unger, Denks. d. k. Akad. d. W. in Wien, Bd. XIV, 1858.
p. 23. T. V. Figs. 18-22.

Lithothamnium ramosissimum—C. W. Gümbel, Abh. der k. bayer. Akad. der W., II. Cl. XI, Bd. I, Abth. München, 1871. p. 24. T. I.

Lithothamnium ramosissimum—A. Rothpletz, Zeits. der deut. Geol. Ges., Bd. 43. 1891. p. 320.

Systematic knowledge of *Lithothamnium* is not as yet in a satisfactory condition. This fossil was formerly relegated to a coral under the various names of *Cellepora*, *Spondites*, *Nullipora*, *Melobesia*, and *Millepora* by Linné, Lamarek, Lamarroux, Cuvier, Ellis, Solander, Reuss, etc., among whom the last author gave the name of *Nullipora ramosissima* to the irregularly ramified, coral-like calcareous form, occurring in the limestone of Leitha near Vienna. Kützing proposed the name of *Spondites stalactica* for the tiny stalactic form of it. Haidinger¹ offered the explanation that *N. ramosissima* is a sedimentary body. In 1858, Franz Unger proved for the first time that *N. ramosissima* is neither an animal nor a stalactic body, but a plant. In 1872, C. W. Gümbel wrote an excellent paper, entitled *Die so-genannten Nulliporen*, etc., in the above-quoted *Abhandlungen*, in which he embodies systematic descriptions of the fossil species and announces that they are to be distinguished from one another only by the relative dimensions of the tissue-cells. Solms-Laubach, on the contrary, said as follows: "It is extremely difficult to distinguish the species in the living representatives of this group, and it may be readily conceived that the difficulty of dealing with the fossil forms is still greater. We shall do well to follow Unger in this matter, and to put them all together as *Lithothamnium ramosissimum*."² Still more recently, A. Rothpletz, in München, accepted Gümbel's

1. *Berichte über die Mittheilungen von Freunden d. Naturw.*, Bd. IV., 1818, p. 442—cited in Unger's paper.

2. *Fossil Botany* (English translation of *Eindeitung in die Palaeophytologie*). Oxford, 1891, p. 45. I am greatly indebted to Mr. Kenjiro Fujii, *Rigakushi*, of the Botanical Institute of the College of Science, for the loan of this work.

opinion as to identification of the fossil species, in conformity with which he described 14 species. In the determination of the present species I also am inclined to follow Gümbel.

The remains of our *Lithothamnion* play such an important rôle in the building up of the limestone, as to warrant the designation of *Lithothamnion-Limestone* (*Nulliporenkalk*). The fractured surfaces have a porcellaneous aspect, and are cream-white in colour.

The thallus is much ramified into tiny stalactic forms or shrubs, which are rounded at the ends, variable in length and breadth, measuring from 1 to 3 mm. across, and having smooth surfaces. (Fig. 1, Pl. XXIX). Generally it resembles either that of *L. byssoides* (Lamarek) Phil., a living species in the Adriatic Sea, which was described and figured by Dr. Hauck in his *Meeresalgen Deutschlands und Oesterreichs*,¹ or the lower one of the two figures given in Prof. Zittel's *Handbuch der Palaeontologie*.²

Examined under the microscope, the cells composing the outer part of a branch of the thallus are 6-8 angled (not unfrequently 5-7 angled according to the imperfection of the slides), as may be seen in a transverse section. A vertical section, on the other hand, shows that the tissue-cells are mainly of rectangular shape and regularly arranged in layers lying one on another as concentric shells. In certain slides, sections of the cells show the walls as either round or sinuous; they can then hardly be distinguished from those seen in a tangential section of *Solenopora*.³

The cell-division is active. In the "hypothallium," so-

1. Leipzig, 1885, p. 275, Taf. II., Fig. 1.

2. II. Abth. *Palaeophytologie*, Leipzig, 1890, p. 38.

3. A. Nicholson and Etheridge, *Geol. Mag.*, Dec. III., Vol. II, p. 529.

—————, *Geol. Mag.* Dec. III., Vol. V., p. 15.

A. Brown, „ „ Dec. IV., Vol. I., p. 145 and 195.

called by Areschong, or "Markstrang" of Solms-Laubach, the cells are divided and multiplied principally by means of dichotomy, or "subdichotomy", according to Bornet, and sometimes trichotomy or "subtrichotomy"; while in the "perithallium," so-called by Rothpletz, the cell-increase takes place mostly by the process of transverse fission or "Quertheilung."

In the slides prepared, traces of the pores suggestive of what are known as "tetraspores" are sometimes seen in the perithallium; no remains of cystocarps have been recognised.

The approximate dimensions of the perithallie-cells are 12-29 μ in length and 12-19 μ in breadth, while the hypothallie-cells are of still larger size, being not unfrequently 25 μ broad and 37-50 μ long. The former dimensions approach much more closely to those of *L. ramosissimum* Reuss than to those of any described by other authors. Relying upon this fact only, and putting aside any point as to form, as Gümbel suggested, it will not be far from the truth to class our species with *L. ramosissimum*.

Stylophora sp.

Pl. XXIX. Fig. 6.

Coral remains are also abundant, but all of them found as casts and consequently indeterminable. One of them, however, may belong to the section of *Madreporaria*, and perhaps to the genus *Stylophora*, in so far as it shows traces of the fully developed six septa, etc.

Millepora sp.

Pl. XXIX. Fig. 7.

Besides the above coral remains, there occurs another coral-like form which may be regarded as belonging to *Millepora* of the

Hydrocorallina. A part of the branches of the cœnosteum entirely converted into crystalline calcite is shown in Pl. XXIX. Its tangential section exhibits traces of the gastropores and dactylopores. A vertical section shows that the cream-white calcified tubes are intersected by transverse partitions or "tabulæ," a structure suggestive of what are called zoöidal tubes, which traversed the calcareous skeleton of the animal, and contained the gastrozoöids and dactylozoöids.

No literature relating to fossil *Millepora* beyond the text-books of Professors Zittel and Nicholson, is accessible to me ; and it is therefore impossible at partition to identify these species or study the details of this doubtful form.

Foraminifera.

The microscope reveals the presence of many of the simpler forms of Foraminifera in the limestone, but few of them are well preserved. On this account no good sections for examination have been got but so far as they can be identified in sections, they appear to belong to *Globigerina*, *Nodosaria*, *Miliola*, *Rotalia* (?), and *Amphistegina*.

Pecten sp.

For this specimen I am indebted to Mrs. Miye Atsumi, of Okami, who was kind enough to submit it to me for examination. Its species proves to be indeterminable through imperfect preservation.

Loc., Okamiyama ; rare.

Turbo mekamiensis n.

Pl. XXIX. Fig. 5 *a* and *b*.

Shell turbinated or ovate-pointed ; composed of 5 whorls convex and separated by subcanaliculated sutures, upper two whors,

nearly smooth, lower three spirally sculptured with liræ, which number 5 on the penultimate and 12 on the last whorl, and are generally wider than their interstitial furrows: among the liræ on the body whorl the subsutural is the largest. Aperture indistinct, but nearly ovate (?)

Height of the shell48mm.

Width43mm.

Spiral angle.....86°

Approximate ratio of body whorl to entire shell 70 : 100.

T. mekamiensis is allied to some of the living species. In the form of the shell, the number of whorls, and the liræ on the body whorl, it resembles *T. artensis* Montrouzier 1860 (Tryon, *Manual of Conchology*, vol. X, p. 196 Pl. 45, Figs. 96, 97), from which, however, it is distinguished in its sculpture, the living one having spiral ribs which are narrower than the interstices. In the last point it coincides with *T. argyrostomus* Linn. 1758 (The same book, p. 197. Pl. 40, Fig. 18 ; Pl. 46, Fig. 8) ; but not in the other characters of the shell of this species.

It is mostly found as casts, of which parts of the limestone bed, common on the eastern flank of the Mekamiyama, are full. The specimen figured was kindly given me by Mr. Sadahe Yagi, of Mekami-

The Tertiary formation, within which the limestone makes its appearance in a local manner, subdivides into an Upper and Lower series.¹

1. Mr. Nakashima has given these divisions the names of Upper Oigawa Tertiary and Lower Oigawa Tertiary, and considers the former to be probably Pliocene and the latter Miocene.

a) The Upper series is of vast extent, covering most of the southern part of the province of Tōtōmi, and is overlaid discordantly by the not-less-widely distributed Quaternary formation. It consists of shale, sandstone, and conglomerate, all of tufaceous nature, and yields a number of fossil shells. Amongst them are species of *Nassa japonica* Adams, *Lampania zonalis* Lamarck, *Cerithium*, *Chemnitzia*, *Rissoa*, *Tellina nasuta* Conrad, *Petricola*, *Corbula*, *Arca granosa* Linné, *Ostea gigas* Thumberg etc.¹

Judging from these shell remains, the Upper series may be considered as contemporaneous with the Pliocene Tertiary developed in the environs of Tōkyō, a detailed account of which is found in Dr. Brauns' *Geology of the Environs of Tōkyō*.

Overlaid by the Pliocene Tertiary just described, there lies b) the Lower series occupying a very limited area. It is mainly made up of dark-greyish shale and brownish or greyish sandstone—just that alternation which is found overlying the limestone, as already described. With the exception of the limestone, this series has yielded so far no harvest of any characteristic fossils for the determination of its geological age. According to the order of superposition, however, there can be no reasonable doubt that this division is older than the Upper one, so that it will be admitted that the limestone may belong to some older epoch than the Pliocene.

Now, putting out of account the other remains in the limestone, we know that *L. ramosissimum* Reuss has hitherto been found in the Miocene Tertiary and in no other formation. Hence, the recurrence of this species in the limestone, together with the fact that the rock is overlaid discordantly by the above-mentioned Pliocene strata,

1. Loc. cit. p. 16.

seems to suggest that it may be assigned to the Miocene, together with the associated sandstones and shales.



PLATE XXIX.

Plate XXIX.

Fig. 1.—A macroscopical view of *L. ramosissimum* in the polished surface of a block of the Limestone.

Fig. 2 and 3.—Vertical sections of its branches $\times 80$.

Fig. 4.—Transverse section of the same $\times 80$.

Fig. 5 a and b.—*Turbo mekamiensis* Nishiwada ; nat. size.

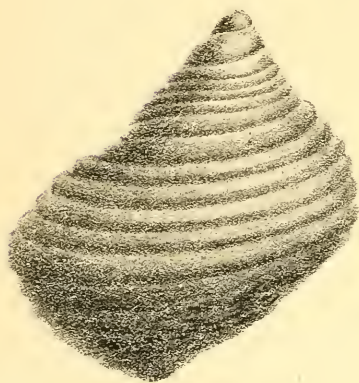
Fig. 6.—Casts of *Stylophora* sp., greatly enlarged.

Fig. 7.—*Millepora* sp. Vertical section of part of a branch of the cœnosteum.

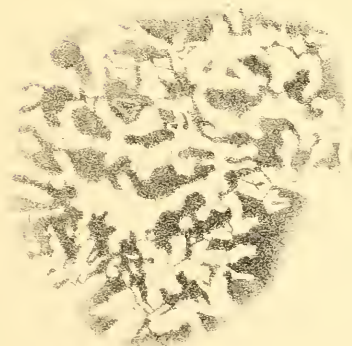
5a



5b



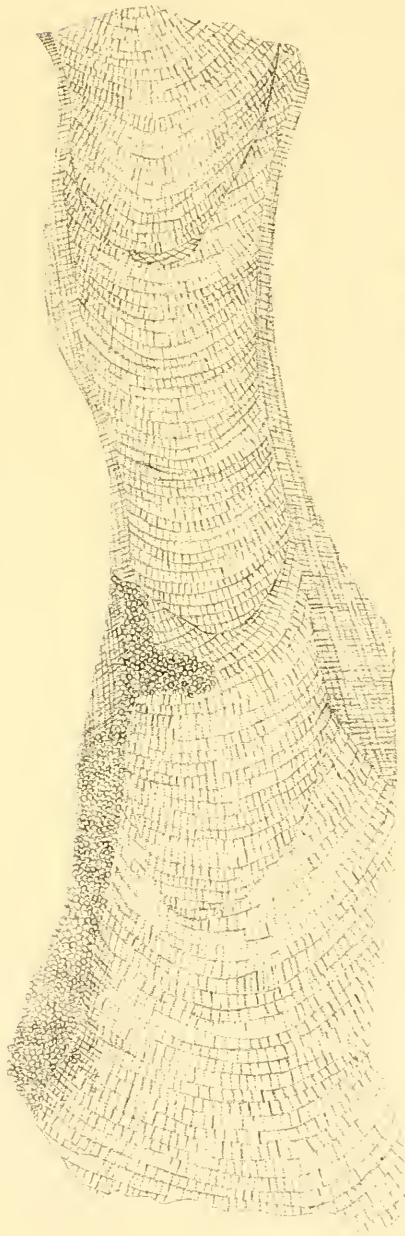
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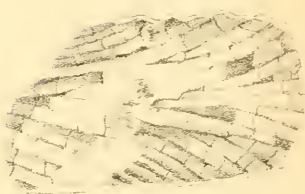
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4



7



3



Mercury and Bismuth Hypophosphites.

by

Seihachi Hada, *Rigakushi*.

College of Science, Imperial University.

Probably because of H. Rose's well-known observation in 1827 of the reduction of mercuric chloride to mercurous chloride and of this to metallic mercury by a solution of hypophosphorous acid, no expectation of success has led to any attempts to prepare a mercury hypophosphite. At the suggestion of Dr. Divers, F.R.S., to whom I am much indebted for advice, I have tried the use of the nitrates of mercury, and have thereby obtained the salt which I shall now describe.

Mercurous nitrate hypophosphite.

This double salt is the only mercury derivative of hypophosphorous acid I have been able to produce. It is precipitated from a solution not too dilute, and almost as free from acid as possible, of either mercuric or mercurous nitrate by a solution of potassium or barium hypophosphite used in quantity small enough to leave some of the mercury nitrate in solution. It can not be got by adding the mercury nitrate to the hypophosphite, or when too much of the latter salt is added to the mercury nitrate, because, in either case, it is at once decomposed.

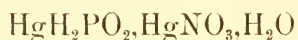
Since the formation of the salt from mercuric nitrate necessarily involves the oxidation and waste of much of the hypophosphite added, and also yields a mother-liquor very active on the precipitate, mercurous nitrate is the proper substance to select in preparing it.

Potassium hypophosphite is also preferable to the barium salt, for when the latter is used the precipitate is liable to contain barium, apparently as nitrate.

Mercurous nitrate, which must be free from nitrous acid, is best prepared by dissolving mercuric oxide to saturation in nitric acid and shaking the solution violently with metallic mercury for a few minutes, for in this way the mercuric salt is quickly and completely changed to mercurous salt.

As the white precipitate obtained by adding the potassium hypophosphite to the excess of mercurous nitrate is slowly decomposed by its mother-liquor, it must be quickly removed and drained on a tile without previous washing.

Mercurous nitrate hypophosphite is unstable, but when dry it only slowly decomposes, becoming grey in the course of some days. It is a white micaceous powder, slightly soluble in water, by which it is soon decomposed with separation of mercury. Its composition is expressed by the formula—



It loses its water in a vacuum desiccator with scarcely any further decomposition for some time. Heated, it turns slightly grey above 90° and explodes a little above 100° yielding mercury and nitrous vapours. Any quantity of it can be exploded at the common temperature by touching it with a hot wire.

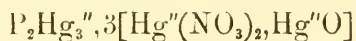
With hydrochloric acid, it first gives mercurous chloride and then metallic mercury. With cold dilute nitric acid it yields metallic mercury, while hot strong nitric acid dissolves it completely with escape of nitrous fumes. Sodium chloride converts it into mercurous chloride and sodium hypophosphite, which only very slowly react to give metallic mercury. Potassium hydroxide blackens it by formation probably of mercurous oxide at first.

The mercury and phosphorus in the salt were determined by dissolving it in nitric acid, evaporating to dryness, dissolving in hydrochloric acid, precipitating mercury by hydrogen sulphide, and phosphoric acid by magnesia mixture. The mercuric sulphide was freed from any co-precipitated sulphur, and dried at 105–110°. The nitric acid was estimated by treating the salt with strong sulphuric acid in Lunge's nitrometer. Loss of weight in the desiccator served for the water determination. The analytical results which follow refer to three separate preparations of the salt.

	Calc.	Found.		
		I.	II.	III.
Mercury	73.39	73.35	73.01	73.04
Nitrogen	2.57	2.78	2.76	2.82
Phosphorus	5.68	5.56	5.25	5.50
Water	3.30	3.01	2.45	—

This salt is of interest as a double salt of univalent or quasi-univalent mercury, since it points to mercurous salts being salts of the radical $(\text{Hg}_2)''$ rather than of $(\text{Hg})'$.

A salt has been described by H. Rose (*Pogg. Ann.*, **40**, 76.), as produced by reaction between mercuric nitrate and phosphine and to which he has given the formula—



This explosive body* would seem to be related to the salt I have described, for it is not remote in composition from $3\text{Hg}'\text{NO}_3, \text{Hg}'\text{H}_2\text{PO}_2$, and is more probably a mercurous than a mercuric salt.

Bismuth Hypophosphite.

How it comes that this salt has hitherto escaped notice it is not

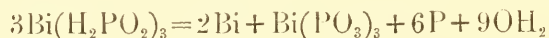
* It was again examined by Aschan in 1885 (*Chem. Zeit.*) but not quantitatively.

easy to understand. It is prepared by mixing a solution of bismuth nitrate, free from any unnecessary excess of nitric acid, with barium or potassium hypophosphite, avoiding excess of the bismuth nitrate, in which it is soluble. The bismuth hypophosphite precipitates as a white crystalline powder, slowly decomposed by its mother-liquor, and soluble in bismuth nitrate solution. Filtered off and dried on a porous tile, it can be preserved for days unchanged.

The analysis of this salt was carried out much in the same way as that of the mercury salt, except that the bismuth sulphide was dissolved in nitric acid and the solution precipitated by ammonium carbonate as usual. Water was determined as loss in the vacuum desiccator. In the following table the calculation is for $\text{Bi}(\text{H}_2\text{PO}_2)_3 \cdot \text{H}_2\text{O}$, which therefore expresses the composition of the salt :—

	Calc.	Found.		
		I.	II.	III.
Bismuth	19.41	49.49	49.40	48.77
Phosphorus	22.09	21.67	21.34	21.19
Water	4.27	3.11	3.26	3.39

Bismuth hypophosphite decomposes very readily by heat, becoming black and giving off phosphine at temperatures only a little above 100° . At a stronger heat metallic globules and bismuth phosphate are obtained. The globules washed with dilute hydrogen chloride to cleanse them from adhering phosphate and then dissolved in nitric acid prove to be metallic bismuth free from phosphorus. As about two-thirds of the bismuth is obtained as metal, the decomposition of the bismuth hypophosphite by heat may be expressed by the equation—



This hypophosphite is noticeable for yielding metal instead of phosphide, but this fact is in accordance with the experience of Berzelius that bismuth phosphide fully decomposes when heated.

The Acid Sulphate of Hydroxylamine.

By

Edward Divers, M, D., F. R. S. *Prof.*

Imperial University.

It is somewhat remarkable that although several hydrochlorides of hydroxylamine have been described by Lossen, the acid sulphate seems never to have been obtained.

It is well known that if more sulphuric acid is present in an aqueous solution of hydroxylamine than is sufficient to constitute the normal salt, the addition of alcohol will cause this and not the acid salt to crystallise out, just as when added to acid ammonium sulphate it will precipitate the normal sulphate. Without this addition of alcohol, a too-acid solution of hydroxylamine sulphate often refuses to deposit anything. By attention, however, to a few details, it can be brought to yield crystals of the acid sulphate.

Solid hydroxylamine hydrochloride is to be treated with, as near as may be, the quantity of sulphuric acid calculated to form the acid salt, $(\text{NH}_3\text{O})\text{HSO}_4$. The mixing is effected in a dish sufficiently large to avoid loss by frothing over, and this is heated for some hours on the water bath until all hydrochloric acid has been expelled. The resulting clear solution becomes viscid when cold. It refuses to yield the normal sulphate when a particle of this salt is dropped on its surface, and slowly dissolves it. But left to stand uncovered in a dry cold atmosphere, and the vessel occasionally moved about, crystallisation suddenly sets in and the solution becomes traversed by long prisms

which almost fill it. Left in a desiccator for a couple of days more, it becomes a translucent cake of damp crystals. The crystals are very deliquescent and, after crushing and pressure between porous tiles, yield results on analysis which prove them to be the acid sulphate of hydroxylamine.

The analysis of the salt was effected by titrating it with sodium hydroxide, with methyl orange as indicator, since to this the normal sulphate is neutral. The sulphuric acid was weighed as barium sulphate.

	Calc.	Found
Hydroxylamine	25.19	24.02
Sulphuric acid	74.81	72.68
	<hr/> 100.00	<hr/> 96.70



Decomposition of Sulphates by Ammonium Chloride in Analysis according to Fresenius.

By

Masumi Chikashigé, *Rigakushi*,

College of Science, Imperial University.

The accuracy in all respects of Fresenius' standard treatises on analysis is usually so unimpeachable, that it seems proper to call attention to a misleading statement contained in a footnote to § 153 A of the seventh and latest English edition of the *Quantitative Analysis*, and also to be found in the earlier editions of this work (4th, 5th, 6th). Having given in the text the method of separating from magnesium and alkali salts the small quantity of barium left in solution by ammonium carbonate, namely, by adding three or four drops of dilute sulphuric acid, Fresenius states in the footnote to it that the gentle ignition there directed to be made in order to expel ammonium salts, will also effect the removal of any small quantity of sulphuric acid which may remain after precipitating the barium. It is this statement which needs correction.

Even in § 68, *a*, and § 74, *a*, of the same work we find enough to cause us to doubt the accuracy of the later statement, for it is there mentioned that magnesium sulphate is not decomposed by igniting it with ammonium chloride, and, on the authority of Rose, that potassium and sodium sulphates, which are decomposable by this treatment, need for it to be effective its repeated application at a red heat inducing effervescence. Obviously, such ignition as this is not that gentle

ignition directed to be used in § 153, and would entail serious loss of alkali chlorides by spirting and volatilisation. But to place the inaccuracy of the statement in the footnote beyond doubt, I have made a few simple trials of the method.

Magnesia, 0.5 gram, was dissolved in a little hydrochloric acid; to the solution were added two drops of dilute sulphuric acid (1 to 10 water by volume), a solution of about two grams of ammonium chloride, and ammonia in small excess; and the whole was then evaporated to dryness and all ammonium salts expelled at a barely red heat. Again, a solution of two grams of ammonium chloride was added, and the evaporation and ignition repeated. The residue was dissolved in dilute hydrochloric acid and mixed with barium chloride, which gave a precipitate. On comparing this with that thrown down by barium chloride from two drops of the same dilute sulphuric acid in about the same volume of water, the ignited salts were found to have lost but very little, if any, of their sulphuric acid. The experiment was repeated three times with fresh magnesium chloride, and the same results obtained.

Similar experiments were made with sodium chloride, and no better removal of the sulphuric acid effected than when magnesium chloride was taken.

It is thus quite evident that other steps must be taken to remove sulphuric acid before it is allowable to calculate the weighed alkali salts as chlorides or to resort to ways of separating magnesia from the alkalis which require the absence of sulphates.

Ewart Johnstone's way to prepare Nitric Oxide.

By

Masumi Chikashigé, *Rigakushi*,

College of Science, Imperial University.

In 1882 D. Ewart Johnstone announced in the *Chemical News*, 45, 159, that cobalt nitrate and potassium thiocyanate heated together readily yield nitric oxide. Except by Schertel, in the *Referate* of the *Berichte* of the *German Chemical Society*, this announcement seems to have been received without criticism, and Michaelis inserted it in his edition of Graham-Otto's *Inorganic Chemistry* among the methods of preparing nitric oxide.

The results of my own testing of the method oblige me to conclude that Johnstone is altogether wrong. We are directed by him to mix four parts of a solution of potassium thiocyanate with one part of a solution of cobalt nitrate, such as are ordinarily in use in the laboratory, and gently heat the mixture, when nitric oxide will be copiously evolved. An equation is given of the action in which four molecules of the potassium salt and one of the cobalt salt appear, and since the quantities to be taken are so indefinitely set down, I started my experiments with these proportions of the salts, both practically in a state of purity. The result proved that, in these proportions, as well as in many others which were tried, whether the solutions are dilute or concentrated, only gently heated or freely boiling, nitric oxide is not formed at all by

them. Beyond the well known fact that the mixed solutions are intensely green, no sign of any action was observed. When the solid salts, a little damp, were heated together, also as recommended by Johnstone, watery fusion occurred and the water boiled off without gas being generated. When, however, the water being gone, the residue got much hotter, there occurred, as might have been anticipated, an explosive reaction in which torrents of gases escaped. These gases, collected over water and gradually mixed with oxygen, proved to consist of nitric oxide to the extent of about two-thirds of their volume, the rest being principally nitrogen. Carbon dioxide and ammonia were also freely given off, and condensed together to form a sublimate, and a cloud, and a solution in the trough-water, of ammonium carbonate or carbamate. The residue smelt strongly of ammonia and was black from the presence of cobalt sulphide, but did not contain the sulphur and the carbon which Johnstone supposed are formed, at least not in quantities which I could detect.

It is a matter of common experience that potassium thiocyanate boiled with dilute nitric acid is decomposed with evolution of nitric oxide and other gases, being partly oxidised and partly converted into the insoluble yellow perthiocyanogen. Now, only a small quantity or, rather, only a weak concentration, of nitric acid is needed for this reaction, and the presence or absence of cobalt nitrate makes, I find, no difference. Probably, therefore, Johnstone's laboratory solution of the latter salt contained nitric acid in some quantity, as Schertel, indeed, has suggested.

The Acidimetry of Hydrogen Fluoride.

By

Tamemasa Haga, F.C.S., *Rigakuhakushi*, Asst. Prof.

and

Yūkichi Ōsaka, *Rigakushi*,

Imperial University.

So far as we can find out, the acidimetry of hydrogen fluoride has not yet been particularly investigated. But it is well known that as an acid it stands apart from hydrogen chloride and other strong mineral acids. For while it even surpasses sulphuric acid in the intensity of its reaction with water and many organic substances, it yet shows such mild acidic characters, that its 'avidity' number places it in this relation among vegetable acids. Further, it not only decomposes the oxides of some metalloids, such as boron, silicon, phosphorus, and sulphur, forming fluorides possessing some degree of stability in presence of water, but also gives with potash, soda, and ammonia, salts which are alkaline to litmus.

To investigate the subject we have experimented with the following common indicators of neutralisation of acids by bases : litmus, lacmoid, methyl orange, phenacetolin, phenolphthaleïn, rosolic acid, cochineal, brazil wood, and turmeric paper. These indicators have been prepared and used in the usual way, for the most part as described in Sutton's *Volumetric Analysis*. In order to ascertain what personal difference there might be in the estimation of the particular shades of colour which indicated neutralisation we worked separately, and upon

materials all prepared by each for his own use. The first and last of the tables appended contain the results obtained by Haga, and the second those obtained by Ōsaka.

For titration in the experiments recorded in Table I and II, decinormal solutions, in the experiments given in Table III twice decinormal solutions of potassium and sodium hydroxides and of ammonia were taken. They were almost pure, containing only the merest traces of silica and alumina, but as these and also carbon dioxide when present in noticeable quantity affect the indications of some of the colour reagents, we were careful to determine for each indicator the exact titre of the alkali solution in terms of a decinormal sulphuric acid which had been standardised gravimetrically by barium chloride, thus rendering ourselves independent of the effects of any impurities present.

The hydrofluoric acid examined was purified in the following way:—Commercial "pure" hydrofluoric acid solution was distilled in a platinum retort after adding a few drops of a saturated solution of potassium permanganate and a little potassium hydroxide. The distillation went on at about 130°C. The product was found to be free from hydrochloric acid and other foreign matters. Silica was sought for by Jörgensen's chloropurpureocobaltic chloride test, and not found. Before purification the acid gave a small quantity of precipitate on standing with this reagent for three days, but the distilled acid gave none after standing for a week.

The purified acid was diluted and preserved for use in a gutta-percha bottle which had for years been holding the acid before its final purification. For each titration, a portion was weighed off in a platinum crucible with well-fitting lid, and was then washed into a large platinum dish in which the neutralisation was effected, except that in some cases the results recorded in Table I were obtained by

transferring the solution, when almost neutral, to a glass vessel, in order to observe the shade of colour more accurately.

The strength of the acid was determined gravimetrically by mixing in a platinum crucible a weighed quantity of the solution with excess of slaked lime made each time from a weighed quantity of precipitated calcium carbonate, letting the mixture stand for a night, drying, and then igniting till the weight became constant. The solution of the acid used for the trials recorded in Table I was also assayed by digesting it with precipitated and finely divided silica, and then igniting with a little sulphuric acid. The results of the two methods agreed well; they are given in the table. That used in trials recorded in Table III was assayed also by gravimetrically estimating the acid as calcium fluoride. The silica method gave in this case a low result and was rejected.

Litmus as indicator. Only historically and because of its universal employment for testing neutrality do we give consideration first to litmus used in titrating hydrogen fluoride. On the addition of potash or soda to dilute solution of hydrofluoric acid coloured by litmus, the red colour rapidly deepens to violet and by the time one molecule of alkali has been added the litmus has become an almost pure blue, that is, blue almost free from any violet tinge. In fact the effect upon litmus in the saturation of hydrogen fluoride with potassium hydroxide is much like that reversed of the saturation of potassium carbonate by sulphuric acid. However, by practice and by keeping before one a second vessel of water coloured by litmus of the right tint,—full blue, as it would be called—it is possible to titrate hydrofluoric acid by means of litmus. But its use is not recommended.

Blue litmus paper reddened with a solution of potassium fluoride containing a small quantity of hydrogen fluoride becomes blue again when left exposed to the air. If the paper be wetted soon after it has

become blue the red colour is generally restored. If, however, it is let dry thoroughly, subsequent moistening with water generally fails to bring back the red colour. The colouring matter seems to be modified by drying up with the fluoride. Dissolved litmus added to such a solution is also coloured red when much water is present, but becomes more and more blue on evaporating the solution, and when it has become sufficiently concentrated the greater part of the colouring matter separates out as a blue powder. This change to blue is not due to loss of hydrofluoric acid, for on addition of water the colouring matter dissolves again giving a red-coloured solution as before. A solution in which litmus paper was turned apparently permanently red and only on drying became neutral or very faintly alkaline, was in one trial found to correspond very nearly to HK_3F_4 . Litmus paper is known to be reddened by monopotassium orthophosphate in solution and to become blue again when dried, from which it may be inferred that hydrogen fluoride, like hydrogen phosphate, is a polybasic acid, H_2F_2 or H_3F_3 or H_4F_4 .

Lacmoid solution behaves essentially like litmus, the difference being that much less alkali is required to produce a bluish violet colour in a solution coloured by lacmoid than in one coloured by litmus. By titrating to a distinct blue, good results may be got with it, but it is not a desirable indicator. Lacmoid paper behaves also like litmus paper.

Phenacetolin changes in colour somewhat gradually when near the neutrality point, but by titrating to pure purple or rose-violet free from any tinge of yellow it may be used successfully.

Methyl orange is quite useless for the ordinary titration of hydrogen fluoride, although it seems to find neutrality in K_2HF_3 . The colour changes are very indefinite.

Phenolphthaleïn is most satisfactory as an indicator for hydrogen

fluoride, giving a very sharp colour change at the point of neutrality. It cannot of course be used with ammonia.

Rosolic acid is almost equal to phenolphthaleïn, but not quite so sharp in its indication. It can be used with ammonia as titrating agent, although ammonia is a little slower than the fixed alkalis in affecting this and other colours. In the case of this indicator and phenolphthaleïn the change of colour is well adapted for being observed in platinum vessels.

Cochineal and *brazil wood* behave alike. Either becomes violet gradually and indefinitely before the acid is neutralised, but finally it turns bluish violet. This change of colour is fairly sharp and is satisfactory for indicating neutrality. Brazil-wood paper becomes bluish violet before all the acid is neutralised and is therefore unsuitable for use.

Turneric paper is satisfactory but not quite as sensitive as some of the other indicators and near the finishing point responds slowly.

It will be seen that, as in the case of the ordinary vegetable acids, the best indicator is phenolphthaleïn or, when ammonia is the titrating agent, rosolic acid.

We have yet to mention that all these indicators give satisfactory results only when the alkali solution is almost pure. The use of an impure alkali solution or of a solution which has been kept long enough in glass vessels to have taken up silica, carbonic acid, and other impurities in appreciable quantity is liable to give ill-defined and generally too-low results.

In conclusion, we have to return our thanks to Dr. Divers for his valuable suggestions in working out this paper.

TABLE I.

Strength of the hydrofluoric acid (*a*) by lime method, 2.79%, (*b*) by silica method, 2.78%, mean 2.785%.

Of this solution from 1.2 to 2.9 grams were taken for each determination, requiring from 16 to 39 ccs. of decinormal alkali.

Titration with Potassium hydroxide.

Indicator.	Colour when neutralised.	Ccs. $\frac{N}{10}$ alkali required by one gram of solution.	Mean.	Per cent. of acid found instead of 2.785.
Rosolic acid.	distinctly red.	14.07 14.06	14.065	2.825
Phenolphthaleïn.	distinctly pink.	14.16 14.05	14.105	2.821
Litmus.	pure litmus blue.	14.11 13.80	13.955	2.791
Brazil wood.	violet.	14.11 14.03	14.07	2.814
Phenacetolin.	purple.	14.22 14.10	14.16	2.832
Lacmoid.	pure lacmoid blue.	13.99 14.10	14.045	2.809

Titration with Sodium hydroxide.

Indicator.	Colour when neutralised.	Ces. $\frac{N}{10}$ alkali required by one gram of solution.	Mean.	Per cent. of acid found instead of 2.785.
Rosolic acid.	distinctly red.	13.90 13.86	13.88	2.776
Phenolphthaleïn.	distinctly pink.	14.00 13.90	13.95	2.790
Litmus.	pure litmus blue.	14.11 13.90	14.01	2.802
Brazil wood.	violet.	13.85 14.01	13.93	2.786
Phenacetolin.	purple.	14.04 13.92	13.98	2.796
Lacmoid.	pure lacmoid blue.	13.99 13.99	13.99	2.798

Titration with Ammonia.

Rosolic acid.	distinctly red.	14.70 13.80 14.10	14.20	2.84
Litmus.	pure litmus blue.	13.62 13.68	13.66	2.732
Brazil wood.	violet.	13.85 14.20	14.025	2.805
Phenacetolin.	purple.	13.95 13.70	13.825	2.765
Lacmoid	pure lacmoid blue.	13.60 13.50	13.55	2.710

TABLE II.

Strength of the hydrofluoric acid by the line method. (1) 6.29%, (2) 6.34%, (3) 6.30%, mean 6.32%. Of this solution from 0.7457 gram to 2.1622 grams were taken for each experiment, requiring from 23.3 to 68.7 ccs. of decinormal alkali.

Titration with Potassium hydroxide.

Indicator.	Colour when neutralised.	Ccs. $\frac{N}{10}$ alkali required by one gram of solution.	Mean.	Per cent. of acid found instead of 6.32.
Litmus.	distinctly blue.	32.28 32.42	32.35	6.47
Litmus.	faintly blue.	31.34 31.47	31.41	6.28
Rosolic acid.	distinctly red.	31.65 31.49	31.57	6.31
Phenacetolin.	purple.	32.31 31.73	32.02	6.40
Phenacetolin.	faintly violet.	31.94 30.98	31.46	6.29
Phenolphthaleïn.	just pink.	31.39 31.57	31.48	6.30
Cochineal.	violet.	31.14 31.06	31.10	6.22
Cochineal.	faintly violet.	30.79 30.79	30.79	6.16
Lacmoid.	distinctly blue.	31.01 30.84	30.93	6.19

Titration with Sodium hydroxide.

Indicator.	Colour when neutralised.	Ces. $\frac{N}{10}$ alkali required by one gram of solution.	Mean.	Per cent. of acid found instead of 6·32.
Litmus.	distinctly blue.	31·93 32·24	32·09	6·42
Rosolic acid.	distinctly red.	31·87 31·93	31·90	6·38
Phenolphthaleïn.	just pink	31·92 31·86	31·89	6·37
Cochineal.	violet.	30·07 29·95	30·01	6·00
Phenacetolin.	purple.	30·52 30·66	30·59	6·11
Phenacetolin.	faintly violet.	29·71 29·64	29·68	5·94
Lacmoid.	distinctly blue.	30·07 29·88	29·98	6·00

Titration with Ammonia.

Indicator.	Colour when neutralised.	Ccs. $\frac{N}{10}$ alkali required by one gram of solution.	Mean.	Per cent. of acid found instead of 6.32.
Litmus.	distinctly blue.	32.37 32.86	32.62	6.52
Litmus.	violet slightly blue.	31.16 31.71	31.44	6.29
Rosolic acid.	distinctly red.	32.00 32.05	32.03	6.41
Rosolic acid.	faintly red.	31.74 31.82	31.78	6.35
Phenacetolin.	purple.	31.54 31.73	31.63	6.33
Phenacetolin.	faintly violet.	31.17 31.01	31.09	6.22
Cochineal.	violet.	31.26 31.69	31.48	6.30
Cochineal.	faintly violet.	30.77 30.85	30.81	6.16
Lacmoid.	distinctly blue.	31.13 31.12	31.125	6.22

Table III.

Strength of hydrofluoric acid (*a*) by lime method, 24.98%, (*b*) by silica method, 23.98%, (*c*) by calcium fluoride method, 25.11%; mean of (*a*) and (*c*), 25.02%.

Of this solution from 1.488 to .7457 grams were taken for each determination, requiring from 93.23 to 45.76 ccs. of twice-decinormal alkali.

Titration with Potassium hydroxide.

Indicator.	Colour when neutralised.	Ccs. $\frac{N}{5}$ alkali required by one gram of solution.	Mean.	Per cent. of acid found instead of 25.02.
Rosolic acid.	distinctly red.	62.20 61.90	62.05	24.82
Phenolphthaleïn.	distinctly pink.	62.66 62.32	62.49	25.00
Litmus.	pure litmus blue.	61.23 61.54	61.385	24.55
Brazil wood.	violet.	61.80 61.76	61.78	24.71
Phenacetolin.	purple.	62.66 61.19	61.925	24.77
Cochineal.	violet.	62.80 61.49	62.145	24.86
Lacmoid.	pure lacmoid blue.	61.30 61.19	61.245	24.50
Turmeric paper.	reddish brown.	61.83 62.01	61.92	24.77

Titration with Sodium hydroxide.

Indicator.	Colour when neutralised.	Ccs. $\frac{N}{5}$ alkali required by one gram of solution.	Mean.	Per cent. of acid found instead of 25.02.
Rosolic acid.	distinctly red.	61.81 62.03	61.92	24.77
Phenolphthaleïn.	distinctly pink.	62.52 62.48	62.50	25.00
Litmus.	pure litmus blue.	61.36 61.64	61.50	24.60
Brazil wood.	violet.	62.73 62.23	62.48	24.99
Phenacetolin.	purple.	62.72 61.52	62.12	24.85
Cochineal.	violet.	61.63 61.58	61.605	24.64
Lacmoid.	pure lacmoid blue.	61.10 61.32	61.21	24.48
Turmeric paper.	reddish brown.	62.70 62.80	62.75	25.10

Titration with Ammonia.

Indicator.	Colour when neutralised.	Ccs. $\frac{N}{5}$ alkali required by one gram of solution.	Mean.	Per cent. of acid found instead of 25.02.
Rosolic Acid.	distinctly red.	62.10 62.70	62.40	24.96
Litmus.	pure litmus blue.	61.23 61.20	61.215	24.49
Brazil wood.	violet.	61.23 61.33	61.28	24.51
Phenacetolin.	purple.	62.66 61.19	61.925	24.77
Cochineal.	violet.	61.10 61.52	61.31	24.52
Lacmoid.	pure lacmoid blue.	61.03 60.99	61.01	24.40
Turmeric paper.	reddish brown.	62.89 62.34	61.615	25.05

On the Poisonous Action of Alcohols upon Different Organisms.

By

M. Tsukamoto. *Nōgakushi.*

The physiological action of different alcohols has been investigated by various authors, but the conclusions reached have not been always concordant. According to Dogiel,¹ for instance, methylic alcohol is less poisonous than ethylic alcohol, while Dujardin-Beaumetz and Andigé² state just the reverse as the result of their experiments. A. Schneegans and J. v. Mering³ conclude that the primary alcohols are less narcotic than the secondary ones, and the latter less so than the tertiary, but this does not agree with the results of the experiments made by W. Gibbs and E. J. Reichert.⁴ Nevertheless it may be concluded from many different experiments that on the whole the toxic action of alcohols in the methylic series runs parallel with the increase in the number of carbon atoms contained in their molecule. However, the subjects serving for these experiments were chiefly warm-blooded animals, and very rarely the lower forms of life. It seemed, therefore, of some interest to compare in this respect *some representatives of all kinds of living organisms.* Of special interest also it

1. *Pflüg. Arch.*, **8**, 605; or *Americ. Chem. Jour.*, **13**, 370.

2. *Jour. Chem. Soc. Lond.*, **30**, 539, from *Compt. rend.*, **83**, 80-82. In the *Americ. Chem. Jour.*, **13**, 370, and *Jour. Chem. Soc. Lond.*, **60**, 1393, they are said to have found that "methylic alcohol was less poisonous than ethylic alcohol, but it is evident from the abstract in *J. Ch. Soc. Lond.*, **30**, 539 that they have been misquoted.

3. *Chem. Centralb.*, 1892, II., 367.

4. *Americ. Chem. Jour.*, **13**, 370.

seemed to be to compare together the effects of methylic and ethylic alcohols. I have examined altogether the action of nine alcohols : methylic, ethylic, normal and iso-propylic, normal, iso-, and tertiary butylic, normal amylic alcohol, and, finally, allylic alcohol ($\text{CH}:\text{CH}.\text{CH}_2.\text{OH}$). These alcohols were diluted with distilled water¹ to the extent stated in connection with each experiment, the percentage given being that by volume.

Action of the Alcohols upon Lower Vertebrate Animals.

For the experiments were chosen tadpoles of *Bufo vulgaris*, Laur. in such a stage of development that the hind legs had made their appearance. Three individuals were put into 50 c.c. of the alcoholic solution of a certain strength. Control experiments with plain water were made in every case. In 0.1% solutions they became motionless after 1½-2 hours in allylic alcohol ; after 10-25 minutes in amylic alcohol ; and after one hour in butylic alcohol ; while those in the other 6 alcohols of the same dilution were apparently not injured. After 24 hours those in the butylic alcohol had almost entirely recovered, but those in the amylic and allylic alcohols were dead. The tadpoles in the other 6 alcohols were alive after 10 days.²

With methylic and ethylic alcohol the experiments were still further extended. In 0.3%, 0.5%, 0.7%, and 1% solutions the intensity of the narcotic action increased with the degree of the concentration, as was to be expected, but in all cases the action of ethylic alcohol was stronger than that of methylic alcohol. Further, in 1.5% ethylic alcohol all the tadpoles showed great stupor, while in methylic alcohol of the same strength one only of the three was

1. Only in those cases where infusoria served for the experiment, was ordinary well water used.

2. In all cases some *Spirogyra* threads were added as food for the animals.

rendered insensible, the other two were still moving after one hour. After 24 hours, however, all of them recovered, those in ethylic alcohol being much weakened, while those in methylic alcohol were almost normal. Tadpoles in 2% solution of ethylic alcohol ceased to move in 10 minutes and after 24 hours two of them died, while a third, although apparently recovered, died 40 hours later. A 2% solution of methylic alcohol proved to be much weaker in action upon them than ethylic alcohol. All were apparently insensible in from 20–50 minutes, but only one of them died after 24 hours, while the other two recovered, and were still alive 40 hours after. In 2.5% solutions, however, they were insensible in from 4–18 minutes, and died soon afterwards.

In further experiments with propylic alcohols (normal and iso) we found that:—in 0.5% solutions the tadpoles stopped their motion within 40 minutes in the normal and within 30 minutes in the iso-propylic alcohol, and after 24 hours two of them recovered in each case, while the others died. In 0.7% solutions motion ceased after 17–30 minutes immersion in the normal and after 9–20 minutes in the iso-alcohol. After 24 hours, however, although those in the normal alcohol were dead, two in the iso alcohol partly recovered, only one out of the three dying. In 1% solutions of these alcohols, the animals became insensible within 10 minutes, and died soon afterwards. In the case of the butylic alcohols insensibility set in within 5 minutes in 0.3% normal butylic, after 3–9 minutes in 0.5% isobutylic, and after 3–15 minutes in 0.7% tertiary butylic alcohol, death following in all these cases within 24 hours. Insensibility was also established after 9–14 minutes in 0.3% isobutylic alcohol and after 9–20 minutes in 0.5% trimethylcarbinol, but in both cases the animals recovered, though those in the former alcohol seemed much more prostrated than those in the latter.

It would seem from the preceding experiments with 0.1% amylic and allylic alcohols that the toxicity of amylic alcohol upon the tadpole is stronger than that of allylic alcohol, contrary to what has been found in experiments upon other forms of life which will be described later on. I made, therefore, further experiments with the two alcohols, of the same dilution (0.1%), as well as of higher dilutions. In 0.1% solutions tadpoles again died sooner in amylic than in allylic alcohol. When the animals however appeared to be insensible they were this time removed into fresh water; whereupon those that had been in the amylic alcohol recovered, while those from the allylic alcohol did not; the latter also had convulsions, but not the former. In 0.05% amylic alcohol solution most of the tadpoles were paralysed in $1\frac{1}{2}$ hours, but after 18 hours they recovered, and were alive after 5 days. In 0.01% solution of that alcohol hardly any action could be noticed. In other experiments, the animals were killed in—

0.05	%	allylic alcohol	after 2	hours
0.01	%	„	„	„ 4-5 $\frac{1}{2}$ „
0.005	%	„	„	„ 5-6 $\frac{1}{2}$ „
0.002	%	„	„	„ 6-6 $\frac{1}{2}$ „
0.001	%	„	„	„ 6 $\frac{2}{3}$ -7 „
0.0005	%	„	„	„ 8 „
0.0001	%	„	„	„ 12-15 „

These observations show that allylic alcohol is a *very strong poison*, far stronger than amylic alcohol.

Action upon Lower Aquatic Animals.

Experiments were carried out upon Ostracodes (*Cypris* and *Cypridina*) and Infusoria (principally *Paramecium*) in the same manner as those described above. Allylic alcohol was applied in solutions of

0.1–0.005 % and even in the latter case all life was extinct within 24 hours. Amylic alcohol in 0.1 % killed all ostracodes and many infusoria within one day, though some infusoria were found alive even after 5 days, but by a solution of 0.5 % all animals without exception were killed after one day. In butylic alcohol of the same strength most of the animals died after two days, and in isobutylic alcohol after 3 days; the few individuals that still showed signs of life were evidently more or less paralysed.¹

In 1 % solutions of normal butylic, isobutylic, and tertiary butylic alcohol (trimethylcarbinol), and of isopropylic and propylic alcohols, infusoria were found dead after 18 hours, ostracodes after 18 hours in isopropylic and butylic alcohol, after two days in isobutylic alcohol, and after three days in propylic alcohol. In tertiary butylic alcohol most ostracodes were dead after two days, but some individuals still showed convulsions two days later.

The observations made concerning the activity of methylic and ethylic alcohol are contained in the following table :—

Alcohol.	1 %	3 %
Methylic $\text{CH}_3 \cdot \text{OH}$.	Infusoria and Ostracodes were seen alive after 4 days.	Strong narcotic action observed after 20 hours; no life after 4 days.
Ethylic $\text{CH}_3 \cdot \text{CH}_2 \cdot \text{OH}$.	Ditto ²	Strong narcotic action after 4 hours; all life ended after 20 hours.

1. In another experiment the animals died sooner, the temperature, which, of course, has great influence, being higher. In the experiments with solutions of normal and isopropylic alcohols of the same dilution, the animals were found alive after four days.

2. Compare O Liew, *Natürl. System der Giftwirkungen*, S. 26.

Action upon Phænogams.

The first series of experiments was carried out with seeds which were just beginning to germinate after having been soaked in water. In each experiment 12 seeds were left in 50 c.c. of the diluted alcohol for 24 hours in a closed vessel; after pouring off the liquid the covered vessel was left at the ordinary temperature and the further development of the germ observed. The seeds treated were those of barley, *soya* bean, Swedish turnip, Japanese onion (*Allium fistulosum*, L.), and *shungiku* (*Chrysanthemum coronarium*, L.). In one set of experiments solutions of 0.1% of the alcohols were used, while in another set barley seeds were treated with 1% and turnip seeds with 0.5% and 1% solutions. The results are shown in the following table:—

Alcohol.	0.1 %	0.5 %	1 %
Methylic $\text{CH}_3 \cdot \text{OH}$.	Alive.	Alive.	Alive.
Ethylic $\text{CH}_3 \cdot \text{CH}_2 \cdot \text{OH}$.	Alive.	Alive.	Alive.
Propylic $\text{CH}_3 \cdot \text{CH}_2 \cdot \text{CH}_2 \cdot \text{OH}$.	Alive.	Alive.	Barley mostly, turnip all killed.
Isopropylic $\text{CH}(\text{CH}_3)_2 \cdot \text{OH}$.	Alive.	Alive.	Barley mostly, turnip all killed.
Butylic $\text{CH}_3 \cdot (\text{CH}_2)_2 \cdot \text{CH}_2 \cdot \text{OH}$.	Alive.	Dead.	Dead.
Isobutylic $\text{CH}_3 \cdot (\text{CH}_3)_2 \cdot \text{CH} \cdot \text{OH}$.	Alive.	Dead.	Dead.

Alcohol.	0.1 %	0.5 %	1 %
Tertiary butylic $C(CH_3)_3 \cdot OH$.	Alive.	Alive.	Barley mostly, turnip all killed.
Amylic ¹ $CH_3 \cdot (CH_2)_3 \cdot CH_2 \cdot OH$.	Only turnip mostly injured.	Dead.	————
Allylic $CH_2 : CH \cdot CH_2 \cdot OH$.	All seeds killed. ²	————	————

In experiments with methylic and ethylic alcohols of higher concentrations, of 2% and 3%. I found that while 2% ethylic alcohol had killed in 24 hours the turnip germs, methylic alcohol had not killed them even when of the strength of 3%.

In other experiments the action of propylic alcohol was compared with that of allylic alcohol on young *soja* beans which had reached the height of 15 cm. in water-culture. One plant was placed in 500 c.c. of 0.1% solutions of each alcohol. That in the allylic alcohol died on the 3rd day, while that in the propylic alcohol was not injured. A similar experiment was carried out upon pea plants about 35 cm. long. The lower three leaves of the plant placed in allylic alcohol (0.1%) turned yellow and dried up in 3 days, and its upper leaves in 5 days; and the plant itself was found dead in 7 days. In a control experiment, also no injurious effect was observed in the case of propylic alcohol.

1. In the case of amylic alcohol turnip seeds were also killed in the 0.3 % solution.

2. In a few turnip seeds the cotyledons showed some development, but in none did the rootlets of the embryo show any.

Action upon Algæ.

In 0.1% solution of allylic alcohol *Spirogyra communis* was killed in 24 hours, while in the solutions of the other alcohols of the same dilution it was found healthy even after 10 days. In other experiments, allylic alcohol of 0.05% also killed *Spirogyra* in 24 hours. Allylic alcohol of 0.01% killed it in three days, but 0.005% had no effect even in 10 days. In 0.5% solutions, this alga was killed by amylic alcohol in one day, by butylic in 3 days, by isobutylic in 4 days, still later by tertiary butylic, propylic, and isopropylic alcohols. In dilutions of 1%, methylic and ethylic alcohols had no injurious effect, but propylic alcohol killed the cells within 3 days, and isopropylic, butylic, isobutylic, and tertiary butylic alcohols killed it within 2 days.¹ In 2% solution, ethylic alcohol proved much more injurious than methylic, for while most of the cells were dead after 3 days in the former, only a few were so in the latter, but in either alcohol all the cells were dead after 5 days. In a 3% solution of ethylic alcohol all were killed within 3 days, while in that of methylic alcohol they lived 4 days. In a 4% solution of methylic alcohol, however, all the cells were killed within two days.

Action upon Microbes.

For these experiments principally methylic, ethylic, amylic, and allylic alcohols were used. In one series of experiments their toxical actions were tested and in other series their nutritive qualities. One drop of putrefying broth was introduced into the alcoholic solution of a certain dilution, and then a sterilised solution of meat-extract was infected from the alcoholic solution after this had stood 24 hours.² The results are shown in the following tables :

-
1. In another case 1% isopropylic alcohol solution killed *Spirogyra* cells within one day.
 2. Of course, control experiments with plain water were also made.

I.

Alcohol.	0.1 %	0.5 %	1 %
Amylic	Strong development at the same time as in the control experiment.	The same as with 0.1 %	Development two ² days later than in the control expt.
Allylic	Development one day later than in the control experiment.	Weak development ¹ 4 days later than in the control expt.	Weak development 4 days later than in the control expt.

II.

Alcohol.	5 %	15 %	20 %
Methylic and Ethylic	No difference from the control experiment.	Much less development than in the control expt.	No development after 10 days' standing.

In a 2% solution of amylic alcohol bacterial life is evidently greatly depressed, for the infection of meat-extract solution with this alcoholic solution did not induce any development within 7 days.

In a second series of experiments 0.1% potassium phosphate, 0.01% magnesium sulphate, and the alcohol to be tested in a certain dilution, were added to a 0.5% meat-extract solution. The infection was made from putrid meat. The results are as follows :—

0.1% allylic alcohol : after 6 days, a weak bacterial development.

0.5% „ „ : after 8 days, very slight development.

1. The bacterial vegetation consisted principally of one kind of thin long bacilli.

2. Here principally micrococci were noticed.

- 1% amylic alcohol : after 4 days, considerable development.
 10% ethylic .. : after 14 days, large development.
 15% : after 14 days, quite clear. free from
 bacteria.
 20% : no development.
 10% methylic .. : strong development.
 15% : weak development.
 20% : no development.

In a third series of experiments no source of carbon for the growth of bacteria was contained in the solution except the alcohol itself, the other constituents being only 0.5% urea, 0.1% potassium phosphate, and 0.01% magnesium sulphate. The infection was made from putrid meat and the flask left to stand at the ordinary temperature. The results were as follows :—

- 0.1% allylic alcohol : after 9 days, slightly turbid ; a few
 bacteria.
 0.5% : after 24 days, quite clear, no bacteria.
 1% amylic .. : after 14 days, no bacteria.
 10% ethylic .. : after 10 days, slight turbidity ; small
 oval *yeast cells*¹ were seen but *no* bacteria.
 10% methylic .. : after 5 days, slight turbidity ; small oval
 shaped bacteria only were seen.²

In a further series of experiments I compared the nutritive effect of the nine alcohols in dilutions of 0.1% with only the addition of 0.5% ammonium phosphate, 0.1% monopotassium phosphate, and 0.01% magnesium sulphate. These solutions were all infected from the same source, viz : a Pasteur's solution that had been exposed in

1. Compare O Löw, *Natürl. System der Giftwirkungen*, S. 26.

2. According to R. Brown (*Chem. Soc. Jour.*, 1886.) *Bacterium aceti* utilises methylic but not amylic alcohol.

the open air and contained bacteria, yeast cells, and mould fungi. The results are as follows :—

Methylic	alcohol :	some development observed in one day, but increase moderate even after 21 days ; small and large yeast cells, and bacilli observed.
Ethylic	„	like the preceding, but the increase greater ; besides yeast cells and bacilli some mucor-like mycelium was present.
Propylic	„	like the preceding, but a mould fungus with white spores was observed in this case.
Isopropylic	„	like the preceding (very little mycelium but numerous bacilli) ; no spore-bearing mould fungus.
Butylic	„	after 2 days turbidity had set in, but the increase was very small even in 21 days ; bacilli and a trace of mycelium.
Isobutylic	„	the quantity of fungi seemed a little larger than in the previous case.
Tertiary butylic	„	the fungoid growth was here much larger than in the last two cases, many bacilli, small yeast cells, and <i>mycoderma</i> -like yeast, but no mycelium observed.
Amylic	„	after 2 days only very little development, but considerable in 21 days ; here more mycelium was observed than in any of the other cases.
Allylic	„	after 21 days, the liquid was perfectly clear and free from fungi. ¹

1. Perhaps a development might have been noticed if the ordinary phosphate had been used here instead of the monophosphate.

These observations of the action of the alcohols upon microbes lead to the conclusion that the common microbes of putrefaction are killed by 0.5% allylic, 2% amylic, or 20% methylic or ethylic alcohol (but probably not the spores¹), if these substances are permitted to act for 24 hours in absence of any nourishing materials; further, that in high dilution the higher alcohols are better food for the microbes than methylic alcohol, but as the dilution lessens, methylic alcohol proves a better nutrient than the higher alcohol, the latter then showing poisonous action.

Furthermore, all my experiments on different organisms go to prove that ethylic alcohol is a stronger poison than methylic alcohol, although Dujardin-Beaumetz and Andigé (*l. c.*) have asserted the contrary, at least for higher animals. My results, moreover, agree well with those of Gibbs and Reichert (*l. c.*), who compared the action of these alcohols upon the higher animals. It is further to be concluded that the normal and iso-propylic alcohols behave nearly alike, and that of the three butylic alcohols the normal is the most poisonous, and the tertiary the least. Allylic alcohol is not only much more poisonous than the corresponding propylic alcohol but also more so than amylic alcohol. The toxic action of the allylic alcohol is evidently somewhat different from that of the saturated alcohols, whose toxical power generally increases with the number of carbon atoms in their molecule. According to Meissner² allylic alcohol acts 50 times more strongly than propylic. According to

1. Koch has found that even a much higher concentration would not kill certain spores.

2. Meissner observes: "Allylic alcohol damages the circulatory system, enlarging the blood vessels and paralysing heart-action. Allylic alcohol has none of the narcotic action of the alcohols in the saturated series.—Other important differences between allylic and other alcohols are that, when inhaled, it attacks the mucous membrane, causes great loss of protein matters, and acts 50 times stronger than propylic alcohol." (Chem. Centralb. 1891, II 715).

my observations, however, the difference is much greater. Allylic alcohol seems to attack protoplasm directly, by chemical affinities arising from the double linking of two carbon atoms, while the saturated alcohols act merely catalytically by transferring certain kinds of motions.¹

Living protoplasm seems a very delicate indicator of differences in chemical constitution, and when we consider how indifferent dead protoplasm is towards most of those poisons that react easily upon living protoplasm, we cannot doubt for a moment but that a great chemical change must have taken place at the moment of death in the proteids of the living protoplasm.

In conclusion, I tender hearty thanks to Prof. Dr. O. Loew for the interest he has taken in my investigation.

1. Therefore those alcohols have here been compared only in equal weights and not in equivalent quantities.



Formulæ for $\operatorname{sn} 9u$.

By

O. Sudo.

The following calculations of $\operatorname{sn} 9u$ were made to show the advantage of the method of finding the multiplication-formulæ of elliptic functions, given by Prof. Fujisawa in the second part of his paper *Researches on the Multiplication of Elliptic Functions* (this journal, vol. VI, pp. 151–226). For the notation adopted, as well as for a full account of the method of calculation, reference is to be made to that paper.

Considering the numerator and denominator of $\operatorname{sn} 9u$ as expressed in terms of $a \left(= k + \frac{1}{k} \right)$ and $\hat{z} (= \sqrt{k} \operatorname{sn} u)$, and putting $n=9$, $q=10$ in the general formulæ for $H_q, H_{q-1}, H_{q-2}, H_{q-3}$, the values of H_{10}, H_9, H_8, H_7 , were found without much difficulty, and, thence, by integrating differential equations (117) (*loc. cit.* p. 202), H_6, H_5, H_4, H_3 , were successively obtained. On the other hand, H_0 which corresponds to H for $k^2=-1$, was derived from the expressions of $\operatorname{sn} 4(u, i)$ and $\operatorname{sn} 5(u, i)$ by addition, and, then, by again making use of the same differential equations, but this time in the reverse order, H_1, H_2, H_3 , were got by successive differentiations. The agreement of the values of H_3 deduced in two different ways verified the results.

The value of E is at once obtained from that of H in virtue of equation (111) (*loc. cit.* p. 200).

Finally, the denominator of $\operatorname{sn} 9u$ was transformed into its usual form, where the variables are taken to be k and x ($=\operatorname{sn} u$). The result was verified by comparing it with that derived from formulæ (146) and (150) (*loc. cit.* p. 215 and pp. 217–218).

In every case the calculation was performed in duplicate, by myself and Mr. Fujii, to whom I owe my best thanks. The results are given in the accompanying two tables, in which the mode of arrangement is obvious.



Table of H and E for $\text{sn}\Theta u$.

	Π_0	Π_1	Π_2	Π_3	Π_4	Π_5	Π_6	Π_7	Π_8	Π_9	Π_{10}			
0	+	1												
2	—	540												
4		+	5544											
6	—	7722		28512										
8		+	118800	+	82368									
10	—	275724	—	570960	—	139776								
12		+	2075976	+	1322496	+	138240							
14	—	214731	—	6262272	—	1624320	—	73728						
16			2747520	+	9300480	+	1019904	+	16384					
18	—	7035984	—	28003968	—	6773760	—	258048						
20		+	73907424	+	94938624	+	1935360							
22	—	40109256	—	300769920	—	173209344	—	202051584						
24		+	295009344	+	957960000	+	976181760	+	165703680					
26	—	83514960	—	562384608	—	1884824064	—	988793856	—	97517568				
28		+	135340722	+	1807764480	+	2501100288	+	703217664	+	38928384			
30	—	999105408	—	3498582528	—	4179875840	—	2240372736	—	331776000	—	9437184		
32		+	1679225040	+	5240563200	+	2992066560	+	1282842624	+	93192192	+	1048576	
34	—	261189624	—	3161982336	—	4765582080	—	2992066560	—	424673280	—	11796480		
36		+	366014340	+	4101762240	+	2287411200	+	1187020800	+	61931520			
38	—	1674487200	—	4811760000	—	2716070400	—	201523200	—	201523200				
40		+	256239000	+	2694880800	+	503331840	+	455270400					
42	—	680919120	—	1653696000	—	56851200	—	90316800						
44		+	46878210	+	276687360	+	409522176	+	22560768	+	22560768			
46	—	158433408	—	721073664	—	766983168	—	116895744	—	3538944	—	3538944		
48		+	305542048	+	758865408	+	256198656	+	20971520	+	20971520	+	262144	
50	—	49155768	—	417280896	—	306789120	—	53968896	—	1769472				
52		+	110517120	+	215205120	+	78852096	+	5308416					
54	—	13546224	—	87023808	—	71723520	—	9326592						
56		+	18343584	+	41900544	+	10644480							
58	—	1537011	—	15759360	—	8273664								
60		+	3688320	+	4465152									
62	—	432828	—	1634688	—	4608								
64		+	357480	+	576	+	256							
66	—	32100	—	9120	—									
68		+	5616	+										
70	—	1044	—	432										
72		+	120											
74	—													
76		+												
78	—													
80		+												
82	—													
84		+												
86	—													
88		+												
90	—													
92		+												
94	—													
96		+												
98	—													
100		+												
	E_0	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}			

The Common Denominator of $\text{sn } \vartheta u$, $\text{cn } \vartheta u$, $\text{dn } \vartheta u$ =

x^0	+	k^0							x^0
x^2		k^2							x^2
x^4	—	540	k^2						x^4
x^6	+	5544	$k^2 + k^4$						x^6
x^8	—	28512	$k^2 + k^6$						x^8
x^{10}	+	82368	$k^2 + k^8$	+	64746	k^4			x^{10}
x^{12}	—	139776	$k^2 + k^{10}$	—	1130064	$k^4 + k^8$			x^{12}
x^{14}	+	138240	$k^2 + k^{12}$	+	2013696	$k^4 + k^{10}$	+	2256300	x^{14}
x^{16}	—	73728	$k^2 + k^{14}$	—	2066688	$k^4 + k^{12}$	—	7425864	x^{16}
x^{18}	+	16384	$k^2 + k^{16}$	+	1134592	$k^4 + k^{14}$	+	13865472	x^{18}
x^{20}	—	258048	$k^2 + k^{18}$	—	8322048	$k^4 + k^{16}$	—	14744064	x^{20}
x^{22}	+	1935360	$k^2 + k^{20}$	+	85261824	$k^4 + k^{18}$	+	2961792	x^{22}
x^{24}	—	173209344	$k^2 + k^{22}$	—	993607296	$k^4 + k^{20}$	—	339369696	x^{24}
x^{26}	+	202951584	$k^2 + k^{24}$	+	1684230912	$k^4 + k^{22}$	+	1680905160	x^{26}
x^{28}	—	165703680	$k^2 + k^{26}$	—	1970493840	$k^4 + k^{24}$	—	4337444160	x^{28}
x^{30}	+	67517568	$k^2 + k^{28}$	+	1671416832	$k^4 + k^{26}$	+	7348242240	x^{30}
x^{32}	—	38928384	$k^2 + k^{30}$	—	1014644736	$k^4 + k^{28}$	—	8876662272	x^{32}
x^{34}	+	9437184	$k^2 + k^{32}$	+	410719056	$k^4 + k^{30}$	+	7810401024	x^{34}
x^{36}	—	1048576	$k^2 + k^{34}$	—	103677952	$k^4 + k^{32}$	—	4902543360	x^{36}
x^{38}	+	11796480	$k^2 + k^{36}$	+	530841600	$k^4 + k^{34}$	+	2075566080	x^{38}
x^{40}	—	61931520	$k^2 + k^{38}$	—	1682472960	$k^4 + k^{36}$	—	6389452800	x^{40}
x^{42}	+	201523200	$k^2 + k^{40}$	+	3698973600	$k^4 + k^{38}$	+	13621789440	x^{42}
x^{44}	—	455270400	$k^2 + k^{42}$	—	5147692800	$k^4 + k^{40}$	—	20480803200	x^{44}
x^{46}	+	503331840	$k^2 + k^{44}$	+	4170353200	$k^4 + k^{42}$	+	20388218400	x^{46}
x^{48}	—	90316800	$k^2 + k^{46}$	—	598752000	$k^4 + k^{44}$	—	10684325520	x^{48}
x^{50}	+	22560768	$k^2 + k^{48}$	+	567447552	$k^4 + k^{46}$	+	1305460440	x^{50}
x^{52}	—	3538944	$k^2 + k^{50}$	—	145207296	$k^4 + k^{48}$	—	3242460672	x^{52}
x^{54}	+	262144	$k^2 + k^{52}$	+	23332816	$k^4 + k^{50}$	+	1567448064	x^{54}
x^{56}	—	1769472	$k^2 + k^{54}$	—	68124672	$k^4 + k^{52}$	—	412436480	x^{56}
x^{58}	+	5368416	$k^2 + k^{56}$	+	116011568	$k^4 + k^{54}$	+	680147712	x^{58}
x^{60}	—	9326592	$k^2 + k^{58}$	—	127683072	$k^4 + k^{56}$	—	720642336	x^{60}
x^{62}	+	10644480	$k^2 + k^{60}$	+	95122044	$k^4 + k^{58}$	+	513816768	x^{62}
x^{64}	—	8273664	$k^2 + k^{62}$	—	48854016	$k^4 + k^{60}$	—	250490016	x^{64}
x^{66}	+	4465152	$k^2 + k^{64}$	+	17083776	$k^4 + k^{62}$	+	82697715	x^{66}
x^{68}	—	1634688	$k^2 + k^{66}$	—	37022204	$k^4 + k^{64}$	—		x^{68}
x^{70}	+	4608	$k^2 + k^{68}$	+	343056	$k^4 + k^{66}$	+		x^{70}
x^{72}	—	256	$k^2 + k^{70}$	—	10144	$k^4 + k^{68}$	—	12330	x^{72}
x^{74}	+	576	$k^2 + k^{72}$	+	7344	$k^4 + k^{70}$	+		x^{74}
x^{76}	—	432	$k^2 + k^{74}$	—	1908	$k^4 + k^{72}$	—		x^{76}
x^{78}	+	120	$k^2 + k^{76}$	+			+		x^{78}
x^{80}	+	0	k^2						x^{80}

Formulæ for $\operatorname{sn} 10u$, $\operatorname{cn} 10u$, $\operatorname{dn} 10u$ in terms of $\operatorname{sn} u$.

By

E. Sakai, *Student*,

College of Science, Imperial University.

Formulæ for $\operatorname{sn} 10u$, $\operatorname{cn} 10u$, $\operatorname{dn} 10u$ in terms of $x = \operatorname{sn} u$ are given in the following four tables, in which the mode of arrangement will be apparent on inspection. They were calculated by two entirely different methods.

Firstly, they were deduced from the well-known equations :

$$\begin{aligned}\operatorname{sn} 10u &= \frac{x \sqrt{1-x^2} \sqrt{1-k^2 x^2} A(x^2)}{D(x^2)} = \frac{2 P Q R S}{S^4 - k^2 P^4}, \\ \operatorname{cn} 10u &= \frac{B(x^2)}{D(x^2)} = \frac{S^4 - 2 S^2 P^2 + k^2 P^4}{S^4 - k^2 P^4}, \\ \operatorname{dn} 10u &= \frac{C(x^2)}{D(x^2)} = \frac{S^4 - 2 k^2 S^2 P^2 + k^2 P^4}{S^4 - k^2 P^4},\end{aligned}$$

where A, B, C, D denote algebraic rational integral functions of x^2 and

$$\begin{aligned}P = x \{ &5 - (20 + 20 k^2) x^2 + (16 + 94 k^2 + 16 k^4) x^4 - (80 k^2 + 80 k^4) x^6 - 105 k^4 x^8 \\ &+ (360 k^4 + 360 k^6) x^{10} - (240 k^4 + 780 k^6 + 240 k^8) x^{12} + (64 k^4 + 560 k^6 \\ &+ 560 k^8 + 64 k^{10}) x^{14} - (160 k^6 + 445 k^8 + 160 k^{10}) x^{16} + (140 k^8 + 140 k^{10}) x^{18} \\ &- 50 k^{10} x^{20} + k^{12} x^{24} \},\end{aligned}$$

$$\begin{aligned}Q = \sqrt{1-x^2} \{ &1 - 12 x^2 + (16 + 50 k^2) x^4 - (80 k^2 + 140 k^4) x^6 + (335 k^4 + 160 k^6) x^8 \\ &- (264 k^4 + 464 k^6 + 64 k^8) x^{10} + (208 k^4 + 508 k^6 + 208 k^8) x^{12} - (64 k^4 \\ &+ 464 k^6 + 264 k^8) x^{14} + (160 k^6 + 335 k^8) x^{16} - (140 k^8 + 80 k^{10}) x^{18} + (50 k^{10} \\ &+ 16 k^{12}) x^{20} - 12 k^{12} x^{22} + k^{12} x^{24} \},\end{aligned}$$

$$R = \sqrt{1-k^2} x^2 \{1 - 12 k^2 x^2 + (50 k^2 + 16 k^4) x^4 - (140 k^2 + 80 k^4) x^6 + (160 k^2 + 335 k^4) x^8 - (64 k^2 + 464 k^4 + 264 k^6) x^{10} + (208 k^4 + 508 k^6 + 208 k^8) x^{12} - (264 k^6 + 464 k^8 + 64 k^{10}) x^{14} + (335 k^8 + 160 k^{10}) x^{16} - (80 k^8 + 140 k^{10}) x^{18} + (16 k^8 + 50 k^{10}) x^{20} - 12 k^{10} x^{22} + k^{12} x^{24}\},$$

$$S = 1 - 50 k^2 x^4 + (140 k^2 + 140 k^4) x^6 - (160 k^2 + 445 k^4 + 160 k^6) x^8 + (64 k^2 + 560 k^4 + 560 k^6 + 64 k^8) x^{10} - (240 k^4 + 780 k^6 + 240 k^8) x^{12} + (360 k^6 + 360 k^8) x^{14} - 105 k^8 x^{16} - (80 k^8 + 80 k^{10}) x^{18} + (16 k^8 + 94 k^{10} + 16 k^{12}) x^{20} - (20 k^{10} + 20 k^{12}) x^{22} + 5 k^{12} x^{24}.$$

To obtain PQR , PS was first formed and the result multiplied by Q and R successively. P^4 was got by multiplying P^2 by P twice in succession. S^4 was found by squaring S^2 , and, as a verification, they were substituted in the well-known relation—

$$S(x, k) = P\left(\frac{1}{kx}, k\right) k^{2p+1} x^{4p+1}$$

where $4p = n^2 - 1$, n being odd. $P^2 S^2$ was obtained by multiplying P^2 and S^2 and also by squaring PS . In every case, the result was further verified by putting $k^2 = 1$ and $k^2 = i$.

Secondly, A, B, C, D were calculated by a totally independent method given by Professor Fujisawa in his paper entitled “Researches on the Multiplication of Elliptic Functions” (this Journal, vol. VI. pp. 151–226).

In the course of calculation, these functions were considered to be arranged according to the powers of x^2 as well as according to the powers of k^2 . The formulæ to be used in the former case are given in the paper just alluded to, and the corresponding formulæ in the latter case are as follows :

$$A(x^2) = \sum_{r=0}^{r=p} \left\{ \sum_{m=r} A_{2m, 2r} x^{2m} \right\} k^{2r}, \quad n^2 - 4 = 4p,$$

where

$$A_{0,0} = n, \quad A_{2m,0} = (-1)^m \frac{n(n^2-4)(n^2-16)\dots(n^2-4m^2)}{(2m+1)!},$$

$$A_{2,2} = -\frac{n(n^2-4)}{3!}, \quad A_{2m,2} = (-1)^m \frac{n(n^2-4)(n^2-16)\dots\{n^2-4(m-1)^2\}}{(2m+1)!} \\ \times m\{n^2-(3m+1)\},$$

and, generally,

$$2m(2m+1)A_{2m,2r} + \{n^2(4r+1)-4m^2\}A_{2m-2,2r} \\ = \{n^2(4r-3)-2n^2(2m-1)+4m^2\}A_{2m-2,2r-2} - (n^2-2m+1)(n^2-2m)A_{2m-4,2r-2}.$$

$$D(x^2) = \sum_{r=0}^{r=p} \left\{ \sum_{m=r}^n D_{2m,2r} x^{2m} \right\} k^{2r}, \quad n^2=4p,$$

where

$$D_{0,0} = 1, \quad D_{2m,0} = 0,$$

$$D_{2,2} = 0, \quad D_{2m,2} = (-1)^{m-1} 2.4^{m-2} \frac{n^2(n^2-1)(n^2-4)\dots\{n^2-(m-1)^2\}}{2m!},$$

$$D_{4,4} = 0, \quad D_{2m,4} = (-1)^{m-1} 4^{m-3} \frac{n^2(n^2-1)(n^2-4)\dots\{n^2-(m-2)^2\}}{2m!} \\ \times \{[2m^2-5(m-1)]n^2-(m-1)(7m-5)\},$$

and, generally,

$$2m(2m-1)D_{2m,2r} + \{4rn^2-4(m-1)^2\}D_{2m-2,2r} \\ = \{n^2(4r-4)-2n^2(2m-2)+4(m-1)^2\}D_{2m-2,2r-2} \\ - (n^2-2m+4)(n^2-2m+3)D_{2m-4,2r-2}.$$

$$B(x^2) = \sum_{r=0}^{r=p} \left\{ \sum_{m=r}^n B_{2m,2r} x^{2m} \right\} k^{2r}, \quad n^2=4p,$$

where

$$B_{0,0} = 1, \quad B_{2m,0} = (-1)^m \frac{n^2(n^2-4)(n^2-16)\dots\{n^2-4(m-1)^2\}}{2m!}.$$

$$B_{2,2} = 0, \quad B_{2m,2} = (-1)^m \frac{n^2(n^2-4)(n^2-16)\dots\{n^2-4(m-2)^2\}}{2m!} \\ \times m(m-1)(n^2-1),$$

and, generally,

$$2m(2m-1) B_{2m,2r} + \{n^2(4r+1) - 4(m-1)^2\} B_{2m-2,2r} \\ = \{n^2(4r-4) - 2n^2(2m-2) + 4(m-1)^2\} B_{2m-2,2r-2} \\ - (n^2-2m+4)(n^2-2m+3) B_{2m-4,2r-2}.$$

The Common Denominator of $\operatorname{sn} 10u$, $\operatorname{cn} 10u$, $\operatorname{dn} 10u =$

The Diagram of the Semi-destructive Earthquake of June 20th, 1894 (Tōkyō).

By

S. Sekiya, *Rigakuhakushi, Professor of Seismology,*

and

F. Ōmori, *Rigakushi,*

Imperial University, Japan.

(Pl. XXX.)

The earthquake of June 20th, 1894, was the most violent that has shaken Tōkyō since the well-known great catastrophe of the 2nd year of Ansei (1855). The mean radius of the disturbed area was about 80 ri or 200 miles, and the total land area was 7,100 square ri, or 42,000 square miles. The meizoseismal tract was a zone lying to the east of Tōkyō, and extending in a N-S direction from the vicinity of the town of Iwatsuki to Tōkyō Bay. No house was absolutely destroyed, but in the lower parts of Tōkyō, many brick buildings received severe damage, and chimneys in particular were mostly thrown down; some *dozō* (godowns) had their plastered mud walls very much cracked and shaken down, tomb-stones and *ishidōrō* (stone lanterns in gardens) were overturned, small cracks were formed in the ground, and, in a few cases, ejection of water took place. The number of casualties in the three Prefectures of Tōkyō, Kanagawa, and Saitama were 26 persons killed and 171 wounded. In fact it was the severest shock that the younger generation has felt in this metropolis.

The diagram of the earthquake (Pl. XXX.) was taken by a Large Motion Seismograph, set up in the Seismological Institute of

the University, which was specially designed for recording strong earthquakes. The instrument is in principle the same as those often described in the papers in the earlier numbers of this Journal treating of seismological subjects. The main differences are, (a) the working parts are made stouter to withstand severe shakings, (b) the writing pointers are made longer so as to record large ranges of motion, and (c) the pointers have no multiplication ratio so that the actual magnitude of the motion is given. This is the first time that a clear instrumental record of a destructive earthquake was ever taken in this country: probably no such has ever been obtained in any other country.

The Seismograph records the motion decomposed into three components. The wave lines on the two inner circles indicate one the NE-SW, and the other the SE-NW components, and that on the outermost circle the vertical component. By compounding the component motions we can find the resultant. The recording plate revolved once in 118 seconds, and the short radial lines mark successive seconds of time counted from the start. We can thus determine the magnitude and direction of motion at any instant during the earthquake, as well as the duration, the intensity, and other elements of the shock. Below we give results deduced from the diagram.

Time of Occurrence. 1894, June 20th, 2^h 4^m 10^s p.m.

Horizontal Motion. The earthquake began as usual with tremors. The duration of tremors, as indicated by ordinary seismographs, lasted about 10 seconds. The Large Motion Seismograph does not record minute tremors, and the motion was already a few millimetres in range at the beginning of this diagram. However, we shall take this latter point as the beginning of the earthquake. The motion, already strong in the 1st and 2nd seconds, became suddenly violent and the ground moved 37 mm. during the time interval between the 3rd

and 4th seconds. This was followed by a counter-movement of 73 mm., which was the maximum horizontal motion during the earthquake, and was again followed by a motion of 42 mm. The motion during about 1 minute succeeding the above three most prominent shocks was very much weaker, though still great in range. Some large undulations occurred between the 40th and 53rd seconds and again between the 70th and 78th seconds. But the intensity of motion was not so strong as before. The comparatively little damage occasioned by the present earthquake notwithstanding such great horizontal movements is no doubt due to the small number of violent oscillations.

Period of Horizontal Motion. The above maximum horizontal motion was executed in 0.9 second, so that the complete period of oscillation would be 1.8 second.

Direction of Motion. The direction of the motion changed as usual during the earthquake. But the maximum horizontal motion was directed toward S. 70° W., and the chief movements before and after were also directed nearly towards the same point, or else the opposite direction. We have also examined the directions of overturning of 245 stone-lanterns (*ishidōrō*) in different parts of Tōkyō. Their mean direction of overturning was toward S. 71° W. Thus the direction of overturning of columns is seen to be identical with that of the maximum horizontal motion.

Vertical Motion. The maximum vertical motion of 10 mm. occurred in the 3rd second nearly simultaneously with the first prominent horizontal motion. Vertical motions occurred more or less during the next 30 seconds.

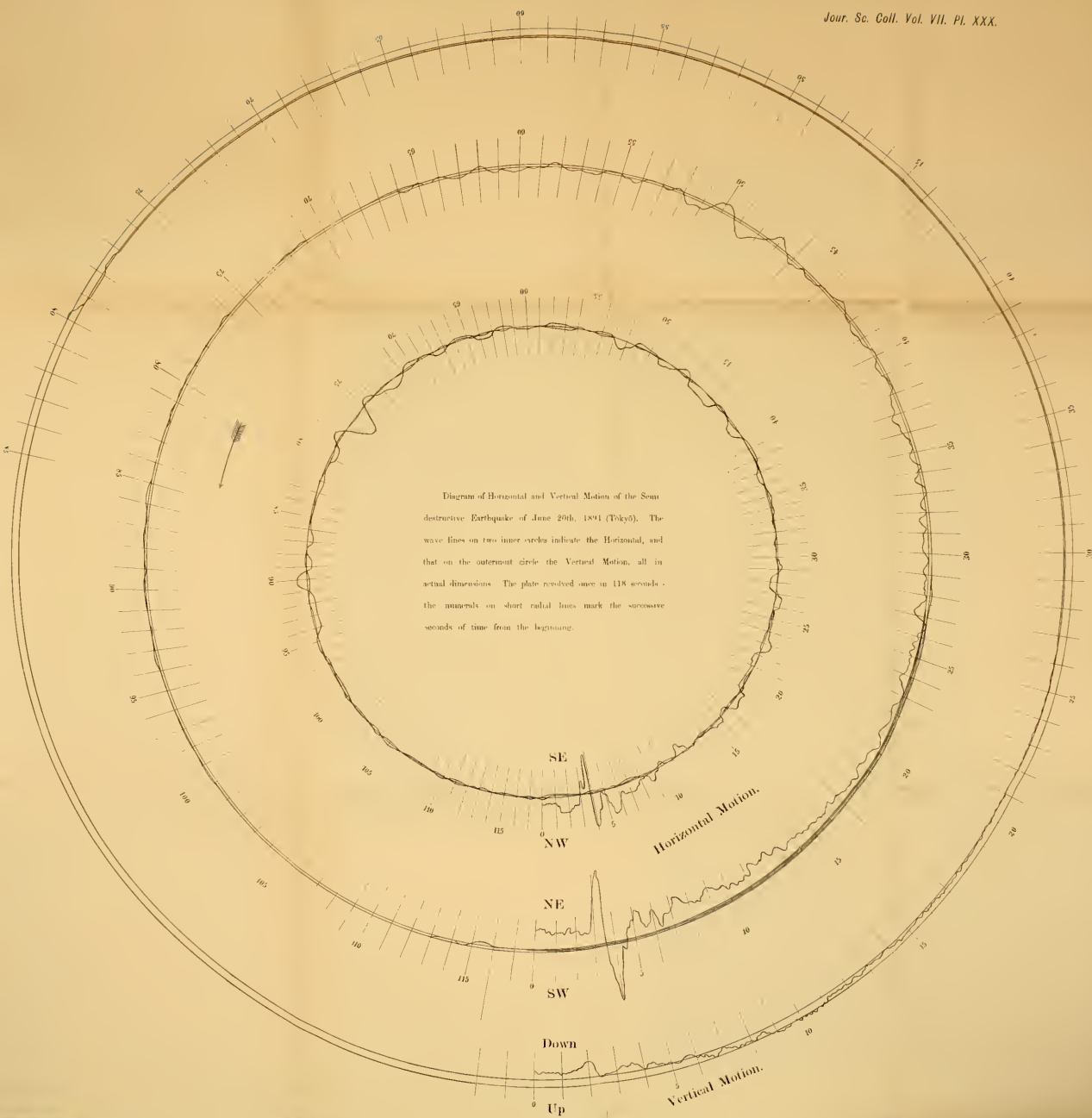
Maximum Acceleration. The maximum acceleration of the motion of an earth particle, calculated from the above values of the maximum horizontal motion and its period, is 444 mm. per sec. per sec.

This was the maximum acceleration in the upper districts of Tōkyō where the ground is composed of hard loamy soil; in the lower districts, where the ground is soft and marshy, it nearly reached the value of 1,000 mm. per sec. per sec. This latter has been calculated from a similar record given by another Large Motion Seismograph set up in the lower part of the city. Now the maximum acceleration is the quantity which measures the destructive power of earthquake motion, and it may therefore be inferred that whenever it reaches the above values, chimneys will be greatly damaged and buildings affected as on this occasion. For the Mino-Owari Great Earthquake of 1891, one of us has calculated, from observations of numerous overturned and fractured bodies, the maximum acceleration of earthquake motion in the meizoseismal district to have been from 3,000 mm. up to nearly 10,000 mm. per sec. per sec.¹ These results are probably the first numerical estimation that has been made of the destructive power of great earthquakes.

Duration of the Earthquake. The shaking lasted about 4 minutes and 30 seconds.

1. See F. Ōmori: *Table of the overturning acceleration, etc.* Seismological Jour. Japan, Vol. I, p. 113.





Beiträge zur Theorie der Bewegung der Erdatmosphäre und der Wirbelstürme,

(dritte Abhandlung)

Von

Dr. Phil. Diro Kitao.

Professor für Physik und Mathematik an der landwirthschaftlichen Facultät
der Kaiserlichen Universität zu Tokyo.

§ XII.—Bedingung für einen vom beweglichen Wirbelgebiete erreichbaren Ort bei zweifacher Wirbelbildung. Veränderung der Windstärke, der Windrichtung, und des Luftdrucks in einem solchen Ort.

Es lassen sich auch leicht Windstärke, Windrichtung und Luftdruck in einem gegebenen Ort als Functionen von der Zeit (t) darstellen, wenn ein kreisförmiges Wirbelgebiet unter dem Einfluss eines anderen unendlich fernen Wirbelgebietes über den Ort hinwegschreitet. Ehe wir dazu schreiten, haben wir die Bedingung

Nils Ekholm (Svenska vet. akad. Handlingar V. 15, N° 14) hat einige Gleichungen, die ich früher in diesem Journal (Vol. I pag. 137-149) entwickelt habe, als unrichtig bezeichnet, weil ich die horizontale d. i. zur Normale der Erdoberfläche senkrechte Componente der Geschwindigkeit mit der resultirenden Geschwindigkeit verwechselt habe. Ich habe dagegen Folgendes zu erinnern.

Da die Erde durch Einführung der mit der Erde beweglichen Coordinaten, und der Reibungskräfte als ein unbewegter reibungsloser Körper betrachtet werden kann, haben die Componenten der Luftgeschwindigkeit v, u, w der Bedingung zu genügen

$$0 = u \cos (nx) + v \cos (ny) + w \cos (nz)$$

und zwar für jedes System x, y, z , welches der Gleichung der Erdoberfläche genügt. Diese Bedingungsgleichung, der die Componenten der Geschwindigkeit einer bewegten Flüssigkeit an einer starren reibungslosen Fläche *durchaus* genügen müssen, sagt nun nichts anderes aus,

näher festzustellen, unter der ein gegebener Ort von einem der kreisförmigen Wirbelgebiete erfasst werden kann. Es seien R_1 und R_2 der Halbmesser der beiden Wirbelgebiete (1) und (2) x_1, y_1 und x_2, y_2 die Coordinaten des Mittelpunktes derselben, und x, y die des gegebenen Ortes in Bezug auf den Schwerpunkt der beiden Wirbelgebiete. Bezeichnen ferner ρ_1 und ρ_2 die Abstände der Wirbel-

als dass die zur starren Körperfläche senkrechte Componente der Geschwindigkeit verschwindet für jedes System x, y, z , welches die Oberfläche des starren Körpers darstellt, d, h , dass die Geschwindigkeit der bewegten Flüssigkeit entlang der starren Körperfläche keine andere Resultirende hat, als die zur Normal der Körperfläche senkrechte. Ich habe daher bei der Ableitung der allgemeinen Beziehung zwischen Isodynamen und Windbahn durch die Definition der isodynamischen Curven darauf hingewiesen, dass sie nur für Lufttheilchen gilt, welche sich an der Erdoberfläche bewegen. Sie gilt darum nicht für die Coordinaten, welche die Gleichung der Erdoberfläche nicht befriedigen, d. h. für Lufttheilchen, deren Coordinaten nicht die obenstehende Grenzbedingung erfüllen.

Das sogenannte Hadley'sche Princip hat sich in der Wirklichkeit nirgends bewährt. Wenn aber Ekholm wegen des Umstandes dass die Gleichung 21, [pag 148 Vol. I dieses Journals] zu demselben führen kann, glaubt, diese Gleichung selbst als unrichtig, und jede Schlussfolgerung daraus als falsch bezeichnen zu können, so erscheint mir dieses etwas zu voreilig, denn die Folgerung jener Gleichung (19d) [W. Ferrel (Recent advances in the Meteorology Washington 1886 pag 205.) hat übrigens für den Winkel zwischen der resultirenden Geschwindigkeit und dem Meridiankreis eine ähnliche Beziehung abgeleitet] setzt ja, wie ich pag. 149 ausdrücklich betont habe, voraus; dass die Windbahn die Isodynamen, oder unter Umständen Isobaren überall unter dem nämlichen Winkel durchsetze: d. h. das sog. Hadley'sche Princip setzt eine besondere Vertheilung des Luftdrucks und der Reibungskräfte voraus. Wenn man daher auch durchaus berechtigt ist, dem Hadley'schen Princip jede schlechthinige Giltigkeit abzuspreehen, so ist man keineswegs berechtigt, demselben jede Giltigkeit schlechthin abzuspreehen.

Der ganze Einwurf Ekholm's rührt daher, weil er nach dem Vorgang des Dr. Sprungs und A. die Bewegung der Luft unter dem Einfluss der Erdrotation, als eines discreten Massentheilchens betrachtet. Da auf ein solches, das sich entlang der Erdoberfläche bewegt, die Ablenkungskraft der rotirenden Erde

$$2 \lambda \cos \theta. w. \quad , \quad -2 \lambda \sin \theta. v.$$

die Bewegungsbahn sowohl in horizontaler als verticaler Richtung ablenkt, glaubt Ekholm zu der Schlussfolgerung berechtigt, dass auch die Lufttheilchen, welche die unterste Schichte der Erdatmosphäre bilden, nur eine Bewegungsform annehmen können, bei der die Resultirende der Geschwindigkeit an der starren Erdoberfläche gegen die Verticale nicht senkrecht steht und die Richtung ihrer Projection auf die Erdoberfläche, von der ursprünglichen Richtung, welche es auch sei, abgelenkt wird. Ich halte es indess für unzulässig, die Gesetze, gemäss denen ein discretes Massentheilchen sich bewegt, so ohne Weiteres, wie Ekholm es thut, auf die Bewegung eines Flüssigkeitstheilchens zu übertragen: da die Bewegung einer auch incompressiblen Flüssigkeit in ihrem ganzen Verlauf *wesentlich* durch die Vertheilung des Drucks, und die Grenzbedingung bestimmt ist, nicht aber diejenige eines discreten Massentheilchens, welche lediglich durch den *anfänglichen Bewegungszustand* bestimmt ist.

gebiete von ihrem Schwerpunkt zur Zeit t , so ist, wie wir in dem vorhergehenden Paragraphen gefunden haben (Gleichungen 105 und 106 pag 374 Vol. II dieses Journals).

$$\rho_1 = \sqrt{\rho_{01}^2 + \frac{\mu_2^2}{\pi(\rho_1 + \rho_2)}} \cdot t \quad \rho_2 = -\sqrt{\rho_{02}^2 + \frac{\mu_1^2}{\pi(\rho_1 + \rho_2)}} \cdot t$$

wo ρ_{01} ρ_{02} den Werth von ρ_1 und ρ_2 zur Zeit $t=0$ bedeuten und

$$\mu_1 = \mp \int r_1 d\omega_1 \quad \mu_2 = \mp \int r_2 d\omega_2$$

Die Integration ist über den ganzen Querschnitt des Gebietes der verticalen Strömung auszudehnen und das obere Vorzeichen gilt für die aufsteigende verticale Strömung, und das untere für die niedersteigende. Es ist, wie wir gefunden haben

$$\frac{\mu_2^2}{\pi(\rho_{01})^2(\rho_1 + \rho_2)} = \frac{\mu_1^2}{\pi(\rho_{02})^2(\rho_1 + \rho_2)}$$

Bezeichnet man diese Grösse mit ε , so hat man einfacher

$$\rho_1 = \rho_{01} \sqrt{1 + \varepsilon t} \quad \rho_2 = -\rho_{01} \sqrt{1 + \varepsilon t}. \quad (1)$$

Ist ferner z_1 der Winkel, welchen die Richtung ρ_1 zur Zeit t mit einer gegen Süd gerichteten Linie schliesst, so ist

$$z_1 = z_0 - \frac{K}{2} \log(1 + \varepsilon t) = z_2 \quad (2)$$

wo $K = \frac{2k \sin \theta}{\pi}$ ist, und z_2 den Winkel zwischen der Richtung ρ und einer nach Nord gerichteten Linie bedeutet. Die Grössen

$$\sqrt{(x_1 - x)^2 + (y_1 - y)^2} \quad , \quad \sqrt{(x_2 - x)^2 + (y_2 - y)^2}$$

geben die Entfernung des gegebenen Ortes vom Mittelpunkt des

Wirbelgebietes (1). resp. (2). Die Bedingung, dass der Ort x, y von dem Wirbelgebiete (1) oder (2) zur Zeit t erreicht wird, ist

$$R_1 = \sqrt{(x_1 - x)^2 + (y_1 - y)^2}$$

oder

$$R_2 = \sqrt{(x_2 - x)^2 + (y_2 - y)^2}$$

Indem wir die Polarcoordinaten einführen und setzen

$$x_1 = \rho_1 \cos \chi_1 \quad x_2 = \rho_2 \cos \chi_2 \quad x = \rho \cos \chi$$

$$y_1 = \rho_1 \sin \chi_1 \quad y_2 = \rho_2 \sin \chi_2 \quad y = \rho \sin \chi$$

so erhalten wir

$$R_1^2 = \rho_1^2 + \rho^2 - 2 \rho \rho_1 \cos (\chi_1 - \chi)$$

$$R_2^2 = \rho_2^2 + \rho^2 - 2 \rho \rho_2 \cos (\chi_2 - \chi)$$

oder, indem wir die Ausdrücke (1) und (2) einführen,

$$(3) \quad R_1^2 = \rho^2 + \rho_{01}^2 (1 + \varepsilon t) - 2 \rho \rho_{01} \sqrt{1 + \varepsilon t} \cos \left[\chi_0 - \chi - \frac{K}{2} \log (1 + \varepsilon t) \right]$$

$$R_2^2 = \rho^2 + \rho_{02}^2 (1 + \varepsilon t) - 2 \rho \rho_{02} \sqrt{1 + \varepsilon t} \cos \left[\chi_0 - \chi - \frac{K}{2} \log (1 + \varepsilon t) \right]$$

Ob der gegebene Ort ρ, χ von einem der Wirbelgebiete, oder von keinem erreicht wird, hängt davon ab, ob diese transcendenten Gleichungen reelle Werthe für t geben, oder nicht. Es ist leicht dieses in jedem gegebenen Fall mittelst der graphischen Methode zu entscheiden. Zu dem Ende denken wir uns den Anfang der Zeit so verlegt, dass $\chi_0 = \chi$ wird, d. h. wir zählen t von dem Augenblick an, wo der gegebene Ort auf der Richtung der Gerade liegt, welche die Mittelpunkte der Wirbelgebiete mit einander verbindet, und setzen

$$\sqrt{1 + \varepsilon t} = e^{\vartheta}$$

wobei ϑ negativ genommen werden muss, wenn ε negativ ist d. h. wenn die beiden Wirbelgebiete cyclonal sind, oder die Masse des

cyclonalen Gebietes grösser ist, als diejenige des anticyclonalen Gebietes. Wir erhalten dann für das Wirbelgebiet (1)

$$R_1^2 = \rho^2 + \rho_{01}^2 e^{2\vartheta} - 2 \rho \rho_{01} e^{\vartheta} \cos K\vartheta$$

oder indem wir dies nach e^{ϑ} auflösen.

$$\rho_{01} e^{\vartheta} = \rho \cos K\vartheta \pm \sqrt{(R_1^2 - \rho^2 \sin^2 K\vartheta)} \quad (4)$$

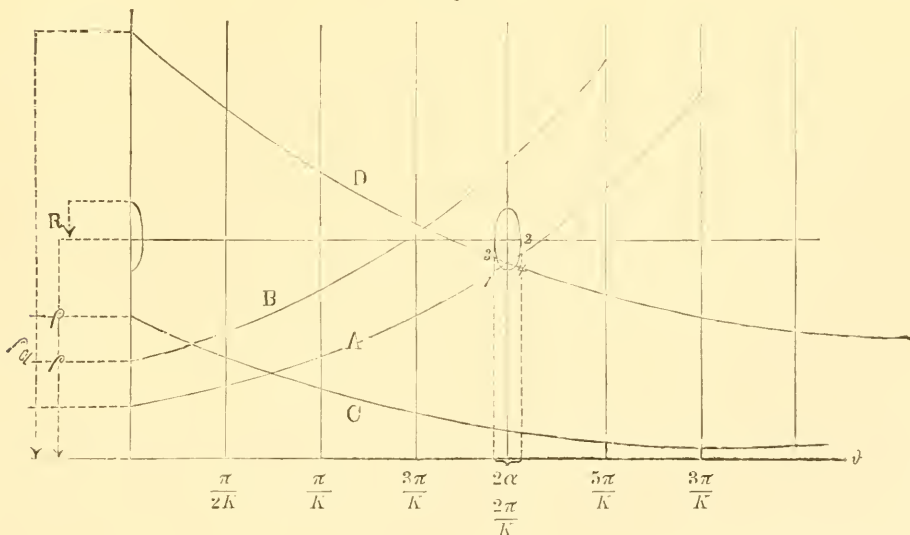
Wenn wir die Gleichungen

$$y = \rho_{01} e^{\vartheta} \quad y = \rho \cos K\vartheta \pm \sqrt{(R_1^2 - \rho^2 \sin^2 K\vartheta)}$$

als Curven construirt denken, so geben die Durchschnittspunkte dieser Curven die reellen Wurzeln der Gleichung (4).

Die erste Curve ist eine exponentiale und die zweite Gleichung stellt eine Reihe elliptisch geschlossener Curven dar mit der Periode $\frac{m\pi}{K}$, wo m eine ganze Zahl ist. Die Figur (1) veranschaulicht den ungefähren Verlauf dieser Curve, indem $\frac{R_1}{\rho} = \frac{1}{6}$ angenommen, und die Breite der elliptischen Curven grösser gezeichnet wurde, als sie bei diesem Verhältniss von $\frac{R_1}{\rho}$ hätte sein müssen.

Fig. 1



Die geschlossenen Curven sind congruent und symmetrisch um die Ordinate für $\frac{m\pi}{K}$, und haben dabei den Durchmesser $2R_1$, während der Durchmesser $(2a)$, welcher die Curvenpunkte $\frac{dy}{d\vartheta} = \pm \infty$ mit einander verbindet

$$= \frac{2}{K} \operatorname{arc sin} \left(\frac{R_1}{\rho} \right)$$

ist.

Wenn die beiden Wirbelgebiete anticyclonal sind, oder die Masse des anticyclonalen Gebietes überwiegt, so trifft die Curve $\rho_{01} e^{\vartheta}$ keine der ovalen Curven, wenn $\rho + R_1 < \rho_{01}$ ist. Für relativ sehr kleinen Werth von ρ_{01} kann die Exponentialcurve wohl eine der ovalen Curven durchsetzen, wie es bei A der Fall ist, und die Werthe von ϑ , denen die Schnittpunkte 1, 2 entsprechen, geben die Zeitpunkte wo das Wirbelgebiet (1) den gegebenen Ort erreicht und dann verlässt, und zwar so, dass der dem Schwerpunkte zugewandte oder von demselben abgewandte Theil des Wirbelgebietes durch den gegebenen Ort passirt, je nach dem die Schnittpunkte unter-oder oberhalb der Punkte liegen, in denen $\frac{dy}{d\vartheta} = \pm \infty$ d. h. das Radical $\sqrt{R^2 - \rho^2 \sin^2 K\vartheta}$ verschwindet.

Sind dagegen die beiden Wirbelgebiete cyclonal oder überwiegt die Masse des cyclonalen Gebietes, so schneidet die Curve $\rho_{01} e^{-\vartheta}$ keine der ovalen Curven, wenn $\rho_{01} < \rho - R_1$ ist, wie bei der Curve C der Fall ist. Ist hingegen $\rho_{01} > \rho + R_1$, was bei der Curve D der Fall ist, so kann die Exponentialcurve wohl die ovale Curve treffen und zwar, wenn ρ_{01} gross ist, mehr als ein Mal; d. i. die Gleichung (4) kann reelle Wurzeln haben, wenn $\rho_{01} > \rho + R_1$ ist.

Die Dimension der ovalen Curven vergrößert sich mit wachsendem Verhältniss $\frac{R_1}{\rho}$ und die Curve verwandelt sich schliesslich in die Sinuslinie, wenn $\frac{R_1}{\rho} = 1$ wird. Je grösser demnach R_1 ist gegen

den Abstand des gegebenen Ortes von dem Schwerpunkt, desto öfter werden die ovalen Curven durch die Linie $\rho_{01} r^{\pm \vartheta}$ geschnitten d. h. desto öfter wird der Ort von dem Wirbelgebiet erreicht.

Die Zeitpunkte, wo der gegebene Ort in ein Wirbelgebiet gerüth und wieder daraus hervortritt, sind demnach bestimmt, wenn die reellen Wurzeln der Gleichung (4) bestimmt sind. Es seien diese ϑ_1 und ϑ_2 , wo $\vartheta_1 < \vartheta_2$ sein möge. Bezeichnen ferner t_1 und t_2 die Zeiten, wo der Ort ρ z in das Wirbelgebiet ein, — und daraus hervortritt, so ist

$$\sqrt{1 + \varepsilon} t_1 = r^{\vartheta_1} \quad \sqrt{1 + \varepsilon} t_2 = r^{\vartheta_2}$$

Mithin

$$t_1 = \frac{r^{2\vartheta_1} - 1}{\varepsilon} \quad t_2 = \frac{r^{2\vartheta_2} - 1}{\varepsilon}$$

Die Differenz

$$t_2 - t_1 = \frac{r^{2\vartheta_1} - r^{2\vartheta_2}}{\varepsilon}$$

gibt dann die Dauer, während deren der Ort in dem Wirbelgebiet verweilt. Die Grössen

$$\rho_{01} r^{\vartheta_1} \quad , \quad K \vartheta_1$$

gibt den Punkt der kreisförmigen Begrenzung des Wirbelgebietes, wo der gegebene Ort eintritt, und

$$\rho_{01} r^{\vartheta_2} \quad , \quad K \vartheta_2$$

den Punkt, wo derselbe austritt, und zwar in Bezug auf den gemeinsamen Schwerpunkt der beiden Wirbelgebiete.

So sind Zeitpunkte bestimmt, wo die in dem vorhergehenden Paragraphen entwickelten Formeln für einen gegebenen Ort ungiltig und dann wieder giltig werden. Innerhalb dieser Zeitpunkte verändern sich Windstärke, Windrichtung und Luftdruck nach anderen Gesetzen, die wir jetzt ableiten wollen.

Wir nehmen zunächst an, dass die beiden in Rede stehenden Wirbelgebiete cyclonal seien, und setzen

$$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\frac{1}{2\pi} \int (z_1 - 2\lambda \sin \theta) d\omega_1 = \frac{1}{2\pi} \int W_1 d\omega_1 = P_1$$

$$\frac{1}{2\pi} \int (z_2 - 2\lambda \sin \theta) d\omega_2 = \frac{1}{2\pi} \int W_2 d\omega_2 = P_2$$

$$\frac{1}{2\pi} \gamma_1 \int d\omega_1' = \mu_1 \quad \frac{1}{2\pi} \gamma_2 \int d\omega_2' = \mu_2$$

wo $d\omega$ ein Querschnittselement des Wirbelgebietes, und $d\omega'$ dasjenige des Gebietes der verticalen Strömung bedeutet. Da bei aufsteigender Strömung vom kreisförmigen Querschnitt das Gebiet derselben mit dem Wirbelgebiete zusammenfällt, hat man, wenn die beiden Wirbelgebiete cyclonal sind, $d\omega = d\omega'$ zusetzen. Wir setzen ferner

$$R_1 = P_2 \log r \quad R_1' = -\mu_2 \log r$$

$$R_2 = P_1 \log r \quad R_2' = -\mu_1 \log r$$

und bilden

$$\frac{\partial R_1}{\partial x_1} = A_1 \quad \frac{\partial R_1'}{\partial x_1} = A_1'$$

$$\frac{\partial R_1}{\partial y_1} = B_1 \quad \frac{\partial R_1'}{\partial y_1} = B_1'$$

$$\frac{\partial R_2}{\partial x_2} = A_2 \quad \frac{\partial R_2'}{\partial x_2} = A_2'$$

$$\frac{\partial R_2}{\partial y_2} = B_2 \quad \frac{\partial R_2'}{\partial y_2} = B_2'$$

so dass wir erhalten

$$A_1 = \frac{P_2}{r^2} (x_1 - x_2) \quad B_1 = \frac{P_2}{r^2} (y_1 - y_2) \quad A_1' = -\mu_2 \frac{(x_1 - x_2)}{r^2} \quad B_1' = -\mu_2 \frac{(y_1 - y_2)}{r^2}$$

$$A_2 = -\frac{P_1}{r^2} (x_1 - x_2) \quad B_2 = -\frac{P_1}{r^2} (y_1 - y_2) \quad A_2' = \mu_1 \frac{(x_1 - x_2)}{r^2} \quad B_2' = \mu_1 \frac{(y_1 - y_2)}{r^2}$$

Es ist früher (pag. 351 Vol. II dieses Journals) nachgewiesen worden, dass der Einfluss der unendlich fernen Wirbelgebiete auf ein Wirbelgebiet berücksichtigt ist, wenn wir der Function W eine lineare Function hinzufügen, deren Coefficienten auf gewisse Weise von den Coordinaten der Mittelpunkte der Wirbelgebiete abhängen. Bei zweifacher Wirbelbildung ist diese lineare Function für das Wirbelgebiet (1)

$$(B_1' - A_1) x - (A_1' + B_1) y^\dagger$$

und für das Wirbelgebiet (2)

$$(B_2' - A_2) x - (A_2' + B_2) y$$

so dass wir für das Wirbelgebiet (1) erhalten

$$\zeta_1 = -\frac{\gamma_1}{2\alpha} \int \log \rho' d\omega'$$

$$W_1 = W_1' - \mu_2 (y_1 - y_2) + P_2 (x_1 - x_2) \frac{x}{r^2} - [P_2 (y_1 - y_2) - \mu_2 (x_1 - x_2)] \frac{y}{r^2}$$

wo

$$\rho' = \sqrt{(x_1 - x)^2 + (y_1 - y)^2}$$

und W_1' der Differentialgleichung genügt

$$\begin{aligned} \frac{\partial J W_1'}{\partial t} + \frac{\partial J W_1'}{\partial x} \left(\frac{\partial W_1'}{\partial y} + \frac{\partial \zeta_1}{\partial x} \right) + \frac{\partial J W_1'}{\partial y} - \left(\frac{\partial \zeta_1}{\partial y} - \frac{\partial W_1'}{\partial x} \right) \\ + (\kappa - \gamma_1) J W_1' + 2\lambda \sin \theta \gamma_1 = 0 \end{aligned}$$

† Pag. 351 II Vol. dieses Journals ist ein Druckfehler unbemerkt geblieben. Das Vorzeichen des Gliedes $(B_1' - A_1) x$ ist positiv, wie hier angegeben ist.

und für das Wirbelgebiet (2)

$$\varphi_2 = -\frac{\gamma_2}{2\pi} \int \log \rho' d\omega'$$

$$W_2 = W_2' - \left[\mu_1 (y_1 - y_2) + P_1 (x_1 - x_2) \right] \frac{x}{r^2} - \left[\mu_1 (x_1 - x_2) - P_1 (y_1 - y_2) \right] \frac{y}{r^2}$$

$$\rho' = \sqrt{(x_2 - x)^2 + (y_2 - y)^2}$$

wo W_2' der Gleichung genügt

$$\begin{aligned} \frac{\partial W_2'}{\partial t} + \frac{\partial W_2'}{\partial y} \left(\frac{\partial W_2'}{\partial y} + \frac{\partial \varphi_2}{\partial x} \right) + \frac{\partial W_2'}{\partial x} \left(\frac{\partial \varphi_2}{\partial y} - \frac{\partial W_2'}{\partial x} \right) \\ + (x - \gamma_2) \Delta W_2' + 2\lambda \sin \theta, \gamma_2 = 0 \end{aligned}$$

Wir haben somit als Componenten der Geschwindigkeit in dem Wirbelgebiete (1)

$$u_1 = \frac{\partial W_1'}{\partial y} + \frac{\partial \varphi_1}{\partial x} - \frac{1}{r^2} \left[P_2 (y_1 - y_2) - \mu_2 (x_1 - x_2) \right]$$

$$v_1 = -\frac{\partial W_1'}{\partial x} + \frac{\partial \varphi_1}{\partial y} + \frac{1}{r^2} \left[P_2 (x_1 - x_2) + \mu_2 (y_1 - y_2) \right]$$

$$w_1 = \gamma_1 z,$$

und in dem Wirbelgebiete (2)

$$u_2 = \frac{\partial W_2'}{\partial y} + \frac{\partial \varphi_2}{\partial x} - \frac{1}{r^2} \left[\mu_1 (x_1 - x_2) - P_1 (y_1 - y_2) \right]$$

$$v_2 = -\frac{\partial W_2'}{\partial x} + \frac{\partial \varphi_2}{\partial y} + \frac{1}{r^2} \left[\mu_1 (y_1 - y_2) + P_1 (x_1 - x_2) \right]$$

$$w_2 = \gamma_2 z.$$

Für einen kreisförmigen Querschnitt ist, wie wir bereits (pag 188 I dieses Journals) entwickelt haben

$$\frac{\partial W_1'}{\partial y} + \frac{\partial \varphi_1}{\partial x} = -\frac{\gamma_1}{2} \left[(x-x_1) + f(\rho')(y-y_1) \right]$$

$$-\frac{\partial W_1'}{\partial x} + \frac{\partial \varphi_1}{\partial y} = -\frac{\gamma_1}{2} \left[(y-y_1) - f(\rho')(x-x_1) \right]$$

wo zur Abkürzung gesetzt ist

$$\frac{2\lambda \sin \theta}{(\kappa - \gamma_1)} \left[1 - \frac{\gamma_1}{\kappa} \left(\frac{\rho'}{R_1} \right)^2 \left(\frac{\kappa}{\gamma_1} - 1 \right) \right] = f(\rho')$$

Entsprechende Ausdrücke für das zweite Wirbelgebiet sind durch Vertauschung des Index abzuleiten.

Es ist nun

$$r = \rho_1 + \rho_2$$

wo ρ_1 ρ_2 die absolut genommenen Abstände der Wirbelgebiete von ihrem Schwerpunkte bedeuten, so dass

$$\rho_1 = \rho_{01} \sqrt{1 - \varepsilon t} \quad \rho_2 = \rho_{02} \sqrt{1 - \varepsilon t},$$

und

$$x_1 = \rho_1 \cos \chi_1 = \rho_{01} \sqrt{1 - \varepsilon t} \cos \left[\chi_0 - \frac{K}{2} \log (1 - \varepsilon t) \right]$$

$$y_1 = \rho_1 \sin \chi_1 = \rho_{01} \sqrt{1 - \varepsilon t} \sin \left[\chi_0 - \frac{K}{2} \log (1 - \varepsilon t) \right]$$

$$x_2 = \rho_2 \cos \chi_2 = \rho_{02} \sqrt{1 - \varepsilon t} \cos \left[\chi_0 - \frac{K}{2} \log (1 - \varepsilon t) \right]$$

$$y_2 = \rho_2 \sin \chi_2 = \rho_{02} \sqrt{1 - \varepsilon t} \sin \left[\chi_0 - \frac{K}{2} \log (1 - \varepsilon t) \right]$$

Es ist ferner

$$\begin{aligned}
& \frac{P_2 (y_1 - y_2) - \mu_2 (x_1 - x_2)}{r^2} = \frac{P_2 \sin \chi_1 - \mu_2 \cos \chi_1}{r} \\
& = \frac{P_2 \sin \left[\chi_0 - \frac{k}{2} \log (1 - \varepsilon t) \right] - \mu_2 \cos \left[\chi_2 - \frac{k}{2} \log (1 - \varepsilon t) \right]}{(\rho_{01} + \rho_{02}) \sqrt{1 - \varepsilon t}} \\
& \frac{P_2 (x_1 - x_2) + \mu_2 (y_1 - y_2)}{r^2} = \frac{P_2 \cos \chi_1 + \mu_2 \sin \chi_1}{r} \\
& = \frac{P_2 \cos \left[\chi - \frac{k}{2} \log (1 - \varepsilon t) \right] + \mu_2 \sin \left[\chi_2 + \frac{k}{2} \log (1 - \varepsilon t) \right]}{(\rho_{01} + \rho_{02}) \sqrt{1 - \varepsilon t}} \\
& \rho' = \sqrt{\rho_1^2 + \rho^2 - 2\rho_1 \rho_2 \cos (\chi - \chi_1)} \\
& = \sqrt{\rho^2 + \rho_{01}^2 (1 - \varepsilon t) - 2\rho_1 \rho_{01} \sqrt{1 - \varepsilon t} \cdot \cos \left[\chi - \chi_0 + \frac{k}{2} \log (1 - \varepsilon t) \right]}
\end{aligned}$$

Als Componenten der Windgeschwindigkeit in dem kreisförmigen Wirbelgebiete (1) hat man

$$\begin{aligned}
(5) \quad u_1 &= -\frac{\tilde{r}_2}{2} \left[\rho \cos \chi - \rho_1 \cos \chi_1 + f(\rho') (\rho \sin \chi - \rho_1 \sin \chi_1) \right] - \frac{(P_2 \sin \chi_1 - \mu_2 \cos \chi_1)}{r} \\
v_1 &= -\frac{\tilde{r}_1}{2} \left[\rho \sin \chi - \rho_1 \sin \chi_1 - f(\rho') (\rho \cos \chi - \rho_1 \cos \chi_1) \right] + \frac{P_2 \cos \chi_1 + \mu_2 \sin \chi_1}{r} \\
W_1 &= \tilde{r}_1 \varepsilon,
\end{aligned}$$

womit die Componenten der Windgeschwindigkeit in dem gegebenen Ort $\rho_1 \chi$ zur Zeit t bestimmt sind. Wir wollen zurest die Grössen P_2 und μ_2 bestimmen. Es ist vermöge der Gleichung für $W = -\zeta + 2k \sin \theta$ (pag 194. Vol. I dieses Journals)

$$P_2 = -\frac{1}{2\pi} \int_0^{2\pi} \int_0^{\rho_2} \left[\frac{2k \sin \theta}{(m_2 - 1)} \left(\frac{\rho'}{P} \right)^{2(m_2 - 1)} - \frac{2k \sin \theta}{(m_2 - 1)} \right] \rho' d\rho' d\chi$$

wo $\frac{\rho}{\tilde{r}_2} = m_2$ gesetzt ist. Die Ausführung der Integration ergibt

$$P_2 = \frac{2\lambda \sin \theta}{\kappa} \frac{\tilde{\gamma}_2 R_2^2}{2}$$

Es ist ferner

$$\mu_2 = \frac{\tilde{\gamma}_2}{2\kappa} \int_0^{2\pi} \int_0^{R_2} \rho' d\rho' d\chi = \frac{\tilde{\gamma}_2}{2} R_2^2$$

Mithin ist

$$\frac{P_2}{\mu_2} = \frac{2\lambda \sin \theta}{\kappa} \quad (6)$$

Nun ist $\frac{2\lambda \sin \theta}{\kappa}$ die Tangente des Deviationswinkels im wirbel-freien Gebiet. Da derselbe im Allgemeinen sehr gross und $\frac{2\lambda \sin \theta}{\kappa}$ vermuthlich > 2 ist,* so folgt, dass im Allgemeinen

$$P_2 > \mu_2$$

sein muss. Als die Windstärke für den gegebenen Ort zur Zeit t erhält man aus (5)

$$\begin{aligned} E^2 = & \gamma_1^2 z^2 + \frac{\tilde{\gamma}_1^2}{4} \left(1 + [f(\rho')]^2 \right) \rho'^2 + \frac{P_2^2 + \mu_2^2}{r_2^2} \\ & + \frac{\tilde{\gamma}_1}{r} \left(P_2 \rho' \sin(\chi_1 - \chi) + \mu_2 [\rho_1 - \rho' \cos(\chi_1 - \chi)] \right) \\ & + \frac{\tilde{\gamma}_1 f(\rho')}{r} \left(\mu_2 \rho' \sin(\chi_1 - \chi) - P_2 [\rho_1 - \rho' \cos(\chi_1 - \chi)] \right) \end{aligned} \quad (7)$$

Die Vertheilung der Windstärke in dem Wirbelgebiete ist dem-nach nicht symmetrisch in Bezug auf das Centrum ($\rho = \rho_1$, $\chi_1 = \chi$, $\rho' = 0$), wo, wie es sein sollte

$$E^2 = \gamma_1^2 z^2 + \frac{P_2^2 + \mu_2^2}{r_2^2}$$

ist, vorausgesetzt, dass $2\kappa < \gamma_1$ oder nicht $\kappa = \gamma_1$ ist.

* Vergleiche Anmerkung pag. 148 I dieses Journals.

Wir wollen die Gleichung (7) etwas vereinfachen. Wenn der gegebene Ort ρ z unendlich weit von dem Wirbelgebiete (2) entfernt ist, und sich innerhalb des Wirbelgebietes (1) befindet, so ist $z'-z$ unendlich klein, da die Entfernung des Wirbelgebietes von dem Schwerpunkt, welcher gleichzeitig der Pol des Polarsystems ist, unendlich gross gegen den Querschnitt des Wirbelgebietes sein soll. Diese Bemerkung ergibt

$$E^2 = \dot{z}_1^2 z^2 + \frac{\dot{z}_1^2}{2} \left(1 + [f(\rho')]^2 \right) \rho_1^2 + \frac{P_2^2 + \mu_2^2}{\rho^2} + Q \quad (8)$$

wo

$$Q = \frac{\dot{z}_1}{r} \left[P_2 \rho (z_1 - z) + \mu_2 (\rho_1 - \rho) \right] + \frac{\dot{z}_1}{r} f(\rho') \left[\mu_2 \rho (z_1 - z) - P_2 (\rho_1 - \rho) \right]$$

ist, oder indem wir die Beziehung (6) berücksichtigen

$$Q = \frac{\dot{z}_1 \mu_2}{r} \left[\rho (z_1 - z) \left(f(\rho') + \frac{2\lambda \sin \theta}{\kappa} \right) - (\rho_1 - \rho) \left(\frac{2\lambda \sin \theta}{\kappa} f(\rho') - 1 \right) \right]$$

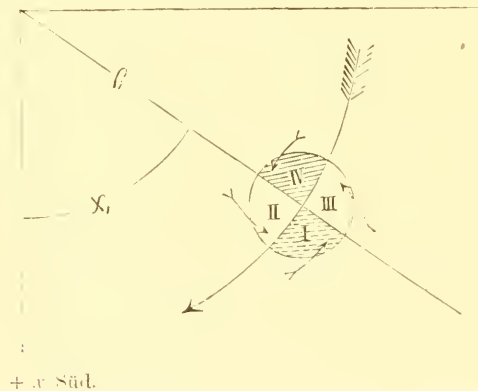
Diese Grösse ist verschieden je nach der Lage des gegebenen Ortes zu dem Centrum des Wirbelgebietes. Wenn in dem Augenblick wo $z_1 = z$ ist, der Ort so liegt, dass $\rho_1 < \rho$ ist, so ist Q positiv. Wenn der Ort aber so liegt, dass $\rho_1 > \rho$, so ist Q negativ.

Im ersteren Ort ist die Windstärke grösser, als in dem letzteren Ort. Man denke sich das kreisförmige Wirbelgebiet in 4 Quadranten getheilt, theils durch die Verlängerung der Linie ρ_1 , theils durch den dazu senkrecht mit dem Radius ρ_1 beschriebenen Kreisbogen. Indem wir den nach der Bewegungsrichtung gelegenen Theil des Wirbelgebietes den vorderen, und den dem Schwerpunkt zugekehrten Theil desselben den inneren Theil nennen, so ist, da bei den Cyclonen z_1 mit t wächst

III positiv, weil $f(\rho')$ von dem Umfang des Grenzkreises des Wirbelgebietes nach dem Centrum zu von $\frac{2\lambda \sin \theta}{\kappa}$ bis $\frac{2\lambda \sin \theta}{\kappa - \gamma_1}$ wächst, und $\frac{2\lambda \sin \theta}{\kappa}$ immer > 1 ist. Es folgt hieraus, dass die Windstärke P' in dem hinteren äusseren Quadranten den grössten, und in dem vorderen inneren Quadranten den kleinsten Werth besitzt, wenn das cyclonale Wirbelgebiet durch ein zweites cyclonales Wirbelgebiet bewegt wird. Wenn das zweite Wirbelgebiet anticyclonal ist, so ist μ_2 negativ und $K = \frac{P_2}{\mu_2}$ hat [wie weiter unten nachgewiesen wird] denselben Werth $\frac{2\lambda \sin \theta}{\kappa}$. Es wird

- (I) in dem vorderen inneren Quadranten $\rho_1 - \rho < 0$ $\chi_1 - \chi > 0$
- (II) in dem vorderen inneren Quadranten $\rho_1 - \rho > 0$ $\chi_1 - \chi > 0$
- (III) in dem hinteren äusseren Quadranten $\rho_1 - \rho < 0$ $\chi_1 - \chi < 0$
- (IV) in dem hinteren inneren Quadranten $\rho_1 - \rho > 0$ $\chi_1 - \chi > 0$

Fig. 3



In dem Quadranten (I) ist demnach

$$Q_I = -\frac{\tilde{\gamma}_1 \mu_2}{r} \left[\rho (\chi_1 - \chi) (f(\rho') + K) + (\rho - \rho_1) (K f(\rho') - 1) \right]$$

In dem Quadranten (II)

$$Q_{II} = -\frac{\tilde{\gamma}_1 \mu_2}{r} \left[\rho (\chi_1 - \chi) (f(\rho') + K) - (\rho_1 - \rho) (K f(\rho') - 1) \right]$$

In dem Quadrante (III)

$$Q_{\text{III}} = -\frac{\tilde{\gamma}_1''_2}{r} \left[\rho (\chi - \chi_1) (f(\rho') + K) - (\rho - \rho_1) (K f(\rho') - 1) \right]$$

und in dem Quadrante (IV)

$$Q_{\text{IV}} = \frac{\tilde{\gamma}_1''_2}{r} \left[\rho (\chi - \chi_1) (f(\rho') + K) + (\rho_1 - \rho) (K f(\rho') - 1) \right]$$

Wieder ist die Windstärke in dem vorderen äusseren Quadrant am kleinsten, und in dem hinteren inneren Quadrante am grössten. Diese Asymmetrie gemahnt uns lebhaft an die sogenannte gefährliche Seite der Seefahrer im Wirbelsturm, und sie ist auch durch die fortschreitende Bewegung des Wirbelsturms erklärt worden.[†]

Als Windazimuth für den gegebenen Ort zur Zeit t erhält man aus der Gleichung

$$\tan \sigma = \frac{dy}{dx} = \frac{dy}{dt} : \frac{dx}{dt} = \frac{v_1}{u_1}$$

indem man für u v_1 ihre Ausdrücke setzt

$$\tan \sigma = \frac{\left(\frac{\tilde{\gamma}_1}{2} \left[\rho \sin \chi - \rho_1 \sin \chi_1 - f(\rho') (\rho \cos \chi - \rho_1 \cos \chi_1) \right] - \frac{P_2 \cos \chi_1 + \mu_2 \sin \chi_1}{r} \right)}{\left(\frac{\tilde{\gamma}_1}{2} \left[\rho \cos \chi - \rho_1 \cos \chi_1 + f(\rho') (\rho \sin \chi - \rho_1 \sin \chi_1) \right] + \frac{P_2 \sin \chi_1 - \mu_2 \cos \chi_1}{r} \right)}$$

Da χ_1, ρ' , wie r , als Functionen von der Zeit bekannt sind, so ist auch $\tan \sigma$ als eine Function von der Zeit dargestellt. Das Gesetz, gemäss dem das Windazimuth sich in dem gegebenen Ort mit der Zeit ändert, ist gleichfalls sehr verwickelt. Indessen: da das von dem zweiten Wirbelgebiete herrührende Glied im Zähler und Nenner wegen des Factors $\frac{1}{r}$ verhältnissmässig klein ist, so dürfte die Veränderung des Windazimuthes wenig abweichen von derjenigen,

[†] Vergleiche Ferrel: Annual report of the chief signal officer of the army for the year 1885. part (2) Recent advances in meteorology. pag 267.

welche eintreten würde, wenn der gegebene Punkt in einem sonst in Ruhe befindlichen Wirbelgebiete seine Lage zum Centrum desselben in der Richtung der Sehne ändern würde. Für einen Ort, über den das Centrum des Wirbelgebietes hinwegschreiten kann, kann man angenähert setzen

$$\rho = \rho_1$$

Befindet sich ferner der Ort nahezu südlich von den Schwerpunkt, so ist χ klein, und χ_1 unterscheidet sich von dem geraden Vielfachen von π um eine kleine Grösse. Man kann daher für $\sin \chi - \sin \chi_1$, $\chi - \chi_1$ und $\cos \chi - \cos \chi_1 = 0$ setzen. Da ferner $\frac{P_2}{\rho_2} = \frac{2\lambda \sin \theta}{\pi} = \text{tag } i$ gesetzt werden kann, so folgt für einen solchen Ort

$$\text{tag } \sigma = \frac{\frac{\gamma_1}{2} \rho (\chi - \chi_1) - \frac{\mu_2}{r \cos i} \sin (i + \chi_1)}{\frac{\gamma_1}{2} f(\rho') \rho (\chi - \chi_1) - \frac{\mu_2}{r \cos i} \cos (i + \chi_1)}$$

So lange der gegebene Ort sich auf der Vorderseite des Wirbelgebietes befindet, ist $\chi - \chi_1 > 0$. In der Nähe des Centrums wird

$$\text{tg } \sigma = \text{tg } (i + \chi_1)$$

d. h.

$$\sigma = i + \chi_1$$

die Windrichtung macht mit der Richtung χ_1 d. i. des ρ_1 den Winkel i . Geräth nun der Ort in die Hinterseite des Wirbelgebietes, so ist $\chi - \chi_1 < 0$. Mithin folgt

$$\text{tag } \sigma' = \frac{\frac{\gamma_1}{2} \rho (\chi_1 - \chi) + \frac{\mu_2}{r \cos i} \sin (i + \chi_1)}{\frac{\gamma_1}{2} f(\rho') \rho (\chi_1 - \chi) + \frac{\mu_2}{r \cos i} \cos (i + \chi_1)}$$

$\text{tag } \sigma'$ ist demnach grösser als $\text{tag } \sigma$; denn die Differenz $\text{tag } \sigma' - \text{tag } \sigma$ ist positiv für denselben Werth von ρ' und $\rho (\chi - \chi_1)$

$$= \frac{\frac{\gamma_1}{2} f(\rho') \rho (\zeta - \zeta_1) \frac{\mu_2}{r \cos i} \sin(i + \zeta_1)}{\left[\frac{\gamma_1}{2} \rho f(\rho') (\zeta - \zeta_1) \right]^2 - \left[\frac{\mu_2}{r \cos i} \cos(i + \zeta_1) \right]^2}$$

da der Nenner durchaus positiv ist. Es folgt hieraus, dass das Windazimuth auf der hinteren Seite des Wirbelgebietes grösser ist, als auf der Vorderseite desselben, eine Thatsache, die man in der That bei manchen Wirbelstürmen beobachtet hat. †

Es ist auch leicht, den Luftdruck in einem gegebenen Ort als Function von der Zeit darzustellen. Es sei ϕ_1 die Gleichung für die Isodynamen im Fall, wo das Wirbelgebiet (1) allein existirt. Sie ist bestimmt durch die Gleichung (Gleichung (14) pag. 170 Vol I. dieses Journals).

$$0 = \Delta \phi_1 - \frac{\partial \mathcal{W}_1}{\partial y} \left(\frac{\partial \mathcal{W}_1}{\partial y} + \frac{\partial \zeta_1}{\partial x} \right) + \frac{\partial \Delta \mathcal{W}_1}{\partial x} \left(\frac{\partial \zeta_1}{\partial y} - \frac{\partial \mathcal{W}_1}{\partial x} \right) - (\mathcal{W}_1 - 2\lambda \sin \theta) \mathcal{W}_1 - \kappa \gamma_1$$

Es lässt sich hieraus die Gleichung der Isodynamen für den Fall ableiten, wo zwei Wirbelgebiete vorhanden sind. Indem wir für \mathcal{W}_1 setzen

$$\mathcal{W}_1 = \left(\mu_2(y_1 - y_2) + P_2(x_1 - x_2) \right) \frac{x}{r^2} - \left(P_2(y_1 - y_2) - \mu_2(x_1 - x_2) \right) \frac{y}{r^2}$$

oder kürzer

$$\mathcal{W}_1 = A x - B y, \quad A = \frac{\mu_2(y_1 - y_2) + P_2(x_1 - x_2)}{r^2}, \quad B = \frac{P_2(y_1 - y_2) - \mu_2(x_1 - x_2)}{r^2}$$

erhalten wir

$$(9) \quad \begin{aligned} & \mathcal{W}_1 - \frac{\partial \mathcal{W}_1}{\partial x} \left(\frac{\partial \zeta_1}{\partial y} + \frac{\partial \mathcal{W}_1}{\partial y} - B \right) + \frac{\partial \mathcal{W}_1}{\partial y} \left(\frac{\partial \zeta_1}{\partial y} - \frac{\partial \mathcal{W}_1}{\partial x} + A \right) \\ & - (\mathcal{W}_1 - 2\lambda \sin \theta) \mathcal{W}_1 - \kappa \gamma_1 = 0 \end{aligned}$$

† Vergleiche. R. P. Benito Viñes: Apuntes relativos à los huracanes de los antillas en setiembre y octubre de 1875-1876.

Wir denken uns eine Function ϕ_1' , welche diese Gleichung in dem Fall befriedigt, wo $A+B=0$ ist, und setzen

$$\phi_i = \phi_1' + \psi_1 + \psi_2$$

wo ψ_1, ψ_2 gewisse Functionen sind, die noch näher zu bestimmen sind. Die Substitution dieses Ausdrucks in die Gleichung (9) ergibt

$$\begin{aligned} J\phi_1' + J\psi_1 + J\psi_2 - \frac{\partial JW_i}{\partial y} \left(\frac{\partial W_i}{\partial y} + \frac{\partial \varphi_i}{\partial x} - B \right) + \frac{\partial JW_i}{\partial x} \left(\frac{\partial \varphi_i}{\partial y} - \frac{\partial W_i}{\partial x} + A \right) \\ = (JW_i - 2\lambda \sin \theta) JW_i - \kappa \gamma_1 = 0 \end{aligned}$$

Hieraus kommt vermöge der Bedeutung der Function ϕ_1'

$$J\psi_1 + J\psi_2 + B \frac{\partial JW_i}{\partial y} + A \frac{\partial JW_i}{\partial x} = 0$$

was befriedigt wird durch die Annahme

$$J\psi_1 = -B \frac{\partial JW_1}{\partial y} \quad J\psi_2 = -A \frac{\partial JW_1}{\partial x}$$

Im Fall eines kreisförmig begrenzten Wirbelgebietes lassen sich diese Gleichungen leicht integrieren. Man setze

$$\begin{aligned} \rho &= \sqrt{(x-x_1)^2 + (y-y_1)^2} \\ \cos \varphi &= \frac{x-x_1}{\rho} \quad \sin \varphi = \frac{y-y_1}{\rho} \end{aligned}$$

Da in diesem Fall JW_1 eine Function von ρ allein ist, so kann man schreiben

$$J\psi_1 = -B \frac{dJW_1}{d\rho} \sin \varphi \quad J\psi_2 = -A \frac{dJW_1}{d\rho} \cos \varphi$$

Nun ist

$$J\psi_1 = \frac{\partial^2 \psi_1}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \psi_1}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \psi_1}{\partial \varphi^2}$$

Wenn wir setzen

$$\psi_1 = I \sin \varphi$$

wo $I = I(\rho)$ allein enthalten soll, so haben wir

$$\frac{d^2 I}{d\rho^2} + \frac{1}{\rho} \frac{dI}{d\rho} - \frac{I}{\rho^2} = -B \frac{dW_1}{d\rho}$$

Die beiden Integrale der reduirten Gleichung

$$\frac{d^2 I}{d\rho^2} + \frac{1}{\rho} \frac{dI}{d\rho} - \frac{I}{\rho^2} = 0$$

sind

$$I = C \rho^2, \quad I = \frac{1}{\rho}$$

Mithin erhalten wir

$$I = \rho \left(C - \frac{B}{2} \int \frac{dW_1}{d\rho} d\rho \right) + \frac{1}{\rho} \left(C' + \frac{B}{2} \int \rho^2 \frac{dW_1}{d\rho} d\rho \right)$$

wo C, C' die beiden Integrationsconstanten sind.

Weil nun W_1 von der Form $a - b \rho^n$ ist, wenn nicht $n = \gamma_1$ ist, so haben wir

$$I = C\rho + \frac{Bb}{2} \rho^{n+1} - \frac{Bnb}{2(n+2)} \rho^{n+1} + \frac{C'}{\rho}$$

Da ψ_1 in dem Centrum des Wirbelgebietes bei jedem Verhältniss $\frac{n}{\gamma_1}$ nicht unendlich gross werden darf, so muss $C' = 0$ sein.

Wir erhalten somit

$$\psi_1 = \sin \varphi \cdot \rho \left(C + \frac{bB\rho^n}{n+2} \right)$$

Der Ausdruck für ψ_2 lässt sich hieraus ableiten, wenn wir hierin für B, A und für $\sin \varphi, \cos \varphi$ einsetzen.

$$\psi_2 = \cos \varphi \cdot \rho \left(C' + \frac{b A \rho^n}{n+2} \right)$$

Da nun

$$\mathcal{J} W_1 = \frac{2\lambda \sin \theta}{(m-1)} \left(\frac{\rho}{R_1} \right)^{2(m-1)} - \frac{2\lambda \sin \theta}{(m-1)} \quad m = \frac{\kappa}{\gamma_1}$$

ist, daher

$$a = -\frac{2\lambda \sin \theta}{(m-1)} \quad b = -\frac{2\lambda \sin \theta}{(m-1)} \left(\frac{1}{R_1} \right)^{2(m-1)} \quad n = 2(m-1)$$

da ferner für ϕ_1' die Function (69) pag. 193 Vol. I dieses Journals zu setzen ist, so haben wir als Gleichung der Isodynamen., indem wir ρ' für ρ setzen

$$\begin{aligned} \phi_1 = \text{const} + \frac{\gamma_1^2 \rho'^2}{4} + \frac{\lambda^2 \sin^2 \theta \rho'^2}{m(m-1)^2} \left[m^2 - 2 \left(\frac{\rho'}{R_1} \right)^{2(m-2)} + \frac{1}{(2m-1)} \left(\frac{\rho'}{R_1} \right)^{4(m-1)} \right] \\ + \sin \varphi \rho' \left[C - \frac{2\lambda \sin \theta A}{2m(m-1)} \left(\frac{\rho'}{R_1} \right)^{2(m-1)} \right] \\ + \cos \varphi \rho' \left[C' - \frac{2\lambda \sin \theta B}{2m(m-1)} \left(\frac{\rho'}{R_1} \right)^{2(m-1)} \right] \end{aligned}$$

Die Gleichung der Isodynamen für das äussere wirbelfreie Gebiet ist (Gleichung (50) pag. 172 Vol. I dieses Journals)

$$\phi_a = - \left(\kappa + \frac{4\lambda^2 \sin^2 \theta}{\kappa} \right) \varphi_a + \text{Const.}$$

wo φ_a eine particuläre Lösung der Gleichung $\mathcal{J}\varphi_a = 0$ ist. Bezeichnet man die Abstände des Punktes $x y$ von den Centren der beiden Wirbelgebiete mit ρ_1' und ρ_2' , so ist für unseren Fall

$$\varphi_a = -\mu_1 \log \rho_1' - \mu_2 \log \rho_2'$$

Nennt man ferner den Werth von ϕ_a für eine selbst gegen ρ_1' und ρ_2' unendlich grosse Entfernung ϕ_{ao} , so ist

$$\phi_{ao} = \left(\kappa + \frac{4k^2 \sin^2 \theta}{\kappa} \right) (\mu_1 \log \rho_0 + \mu_2 \log \rho_0) + \text{const.}$$

wo ρ_0 eine Länge ist, die so gross gedacht werden kann, dass die durch zweifache Wirbelbildung verursachte Störung der Atmosphäre in der Entfernung ρ_0 unmerklich, d. h., $\frac{F^2}{2}$ unendlich klein wird, also dass man für ρ_0 setzen kann

$$\phi_{ao} = \frac{P_0}{\mu} + G$$

wo P_0 den Druck, und G das Potential der Schwere und μ die Dichte der Luft bedeutet. Wir erhalten somit

$$\phi_a = \left(\kappa + \frac{4k^2 \sin^2 \theta}{\kappa} \right) \left(\mu_1 \log \frac{\rho_1'}{\rho_0} + \mu_2 \log \frac{\rho_2'}{\rho_0} \right) + \frac{P_0}{\mu} + G$$

An dem Grenzkreis des Wirbelgebietes (1) ist $\rho_1' = R_1$, und ρ_2' kann $= r$ gesetzt werden. Soll daher an dem Grenzkreis $\rho' = R_1$ $\phi_a = \phi_1$ sein, d. h.,

$$\begin{aligned} & \left(\kappa + \frac{4k^2 \sin^2 \theta}{\kappa} \right) \left[\mu_1 \log \frac{R_1}{\rho_0} + \mu_2 \log \left(\frac{r}{\rho_0} \right) \right] + \frac{P_0}{\mu} + G \\ &= \text{const.} + \frac{\kappa \Gamma_1 R_1^2}{4} + \frac{k^2 \sin^2 \theta R_1^2}{m(1-m)^2} \left(m^2 - 2 + \frac{1}{2m-1} \right) \\ &+ \sin \varphi R_1 \left(C - \frac{2k \sin \theta B}{2m(m-1)} \right) + \cos \varphi R_1 \left(C' - \frac{2k \sin \theta A}{2m(m-1)} \right) \end{aligned}$$

so wird dieses erfüllt, wenn wir setzen

$$C = \frac{2k \sin \theta B}{2m(m-1)} \quad C' = \frac{2k \sin \theta A}{2m(m-1)}$$

$$Const = \frac{P_0}{\mu} + G + \left(\kappa + \frac{4k^2 \sin^2 \theta}{\kappa} \right) \left[\mu_1 \log \left(\frac{R_1}{\rho_0} \right) + \mu_2 \log \left(\frac{r}{\rho_0} \right) \right] \\ - \frac{\kappa \gamma_1 R_1^2}{4} - \frac{k^2 \sin^2 \theta R_1^2}{m(m-1)^2} \left(m^2 - 2 + \frac{1}{2m-1} \right)$$

Mithin erhalten wir

$$\phi_1 = \frac{P_0}{\mu} + G + \left(\kappa + \frac{4k^2 \sin^2 \theta}{\kappa} \right) \left[\mu_1 \log \left(\frac{R_1}{\rho_0} \right) + \mu_2 \log \left(\frac{r}{\rho_0} \right) \right] \\ - \frac{\kappa \gamma_1}{4} \left(R_1^2 - \rho'^2 \right) + \frac{k^2 \sin^2 \theta \rho'^2}{m(m-1)^2} \left[m^2 - 2 \left(\frac{\rho'}{R_1} \right)^{2(m-1)} + \frac{1}{2m-1} \left(\frac{\rho'}{R_1} \right)^{4(m-1)} \right] \\ - \frac{k^2 \sin^2 \theta R_1^2}{m(m-1)^2} \left(m^2 - 2 + \frac{1}{2m-1} \right) \\ + \frac{k \sin \theta \rho'}{m(m-1)} \left[1 - \left(\frac{\rho'}{R_1} \right)^{2(m-1)} \right] \left(B \sin \varphi + A \cos \varphi \right)$$

Da

$$\rho' \sin \varphi = \rho \sin \chi - \rho_1 \sin \chi_1 \\ \rho' \cos \varphi = \rho \cos \chi - \rho_1 \cos \chi_1$$

ist, so hat man, indem man für B und A ihre Werthe einsetzt

$$\rho' \left(B \sin \varphi + A \cos \varphi \right) = \frac{1}{r} \left(P_2 \rho \cos (\chi_1 - \chi) - \rho_1 \right] + \mu_1 \rho \sin (\chi_1 - \chi) \right)$$

da ferner $\phi_1 = \frac{p_i}{\mu'} - G + \frac{1}{2} F_1^2$

ist, wo p_i μ' den Druck und die Luftdichte in dem Wirbelgebiete bezeichnet, und F_1 die Resultante der Geschwindigkeit, so erhält man

$$\frac{p_i}{\mu'} = \frac{p_0}{\mu} + \left(\kappa + \frac{4k^2 \sin^2 \theta}{\kappa} \right) \left[\mu_1 \log \left(\frac{R_1}{\rho_0} \right) + \mu_2 \log \left(\frac{r}{\rho_0} \right) \right]$$

$$\begin{aligned}
 & -\frac{\kappa \gamma_1}{4} (R_1^2 - \rho'^2) + \frac{\lambda^2 \sin^2 \theta \rho'^2}{m(m-1)^2} \left[m^2 - 2 \left(\frac{\rho'}{R_1} \right)^{2(m-1)} + \frac{1}{2m-1} \left(\frac{\rho'}{R_1} \right)^{4(m-1)} \right] \\
 & - \frac{\lambda^2 \sin^2 \theta R_1^2}{m(m-1)^2} \left(m^2 - 2 + \frac{1}{2m-1} \right) \\
 & + \frac{\lambda \sin \theta}{m(m-1)} \frac{1}{r} \left[1 - \left(\frac{\rho'}{R_1} \right)^{2(m-1)} \right] \left(P_2 \rho \cos(\chi_1 - \chi) - \rho_1 + \mu_2 \rho \sin(\chi_1 - \chi) \right) \\
 & - \frac{1}{2} \left[\gamma_1^2 z^2 + \frac{\gamma_1^2}{4} \left(1 + f(\rho')^2 \right) \rho'^2 + \frac{P_2^2 + \mu_2^2}{\rho'^2} + Q \right] \quad (10)
 \end{aligned}$$

womit der Luftdruck in dem Wirbelgebiete als eine Function von der Zeit dargestellt ist.

Die Vertheilung des Drucks in dem Wirbelgebiete ist nicht symmetrisch in Bezug auf das Centrum. Bezeichnet man die Glieder, die nur ρ' enthalten, mit H , und setzt man wieder $\cos(\chi_1 - \chi) = 1$ und $\sin(\chi_1 - \chi) = \chi_1 - \chi$, so erhält man

$$\begin{aligned}
 \frac{P_1}{\rho'} &= H + \frac{\lambda \sin \theta}{m(m-1)} \frac{1}{r} \left[1 - \left(\frac{\rho'}{R_1} \right)^{2(m-1)} \right] \left(P_2 (\rho - \rho_1) + \mu_2 \rho (\chi_1 - \chi) \right) \\
 & - \frac{1}{2} \frac{\gamma_1 \mu_2}{r} \left[\rho (\chi_1 - \chi) \left(f(\rho') + \frac{2\lambda \sin \theta}{\kappa} \right) - (\rho_1 - \rho) \left(\frac{2\lambda \sin \theta}{\kappa} f(\rho') - 1 \right) \right]
 \end{aligned}$$

oder indem wir die Beziehung (6) berücksichtigen, für $f(\rho')$ ihren Ausdruck einführen, und $\frac{2\lambda \sin \theta}{\kappa} = K$ setzen

$$\begin{aligned}
 \frac{P_1}{\rho'} &= H + \frac{\kappa \mu_2}{2mr} (\rho - \rho_1) \left[\frac{K^2}{m-1} \left\{ 1 - \left(\frac{\rho'}{R_1} \right)^{2(m-1)} \right\} \right. \\
 & \quad \left. + \frac{K^2 m}{(m-1)} \left\{ 1 - \frac{1}{m} \left(\frac{\rho'}{R_1} \right)^{2(m-1)} - 1 \right\} \right] \\
 & + \frac{\kappa \mu_2}{2m} \rho (\chi_1 - \chi) \left[\frac{K}{(m-1)} \left\{ 1 - \left(\frac{\rho'}{R_1} \right)^{2(m-1)} \right\} \right. \\
 & \quad \left. - \frac{K m}{(m-1)} \left\{ 1 - \frac{1}{m} \left(\frac{\rho'}{R_1} \right)^{2(m-1)} + K \right\} \right]
 \end{aligned}$$

$$= H + \frac{\kappa \mu_2}{2mr} \frac{(\rho - \rho_1)}{(m-1)} \left[K^2 \left\{ 1+m - 2 \left(\frac{\rho'}{R_1} \right)^{2(m-1)} \right\} - m + 1 \right] \\ + \frac{\kappa \mu_2}{2mr} \rho (\chi_1 - \chi) K \left(\frac{1}{m-1} - \frac{m}{m-1} + 1 \right)$$

d. i.

$$\frac{P_i}{\rho'} = H + \frac{\kappa \mu_2}{2mr} \frac{(\rho - \rho_1)}{(m-1)} \left[K^2 \left\{ 1+m - 2 \left(\frac{\rho'}{R_1} \right)^{2(m-1)} \right\} - m + 1 \right]$$

Der Factor $K^2 \left[1+m - 2 \left(\frac{\rho'}{R_1} \right)^{2(m-1)} \right] - m + 1$ ist positiv, so lange $m > 1$ ist, da $K^2 > 1$ ist. Ist aber $m < 1$, so wird der Factor für $\rho' = R_1 = (K^2 - 1)(m-1)$, also negativ, und für $\rho' = 0$, negativ unendlich gross. Die Grösse

$$\frac{1}{(m-1)} \left[K^2 \left\{ 1+m - 2 \left(\frac{\rho'}{R} \right)^{2(m-1)} \right\} - m + 1 \right]$$

bleibt darum durchaus positiv, welchen Werth m auch haben mag. Es folgt hieraus, dass der Druck P_1 auf der äusseren Hälfte der Cyclone ($\rho > \rho_1$) bei derselben Entfernung des Centrums grösser sein muss, als auf der inneren Hälfte ($\rho < \rho_1$), wenn μ_2 positiv ist, d. h. das andere Wirbelgebiet auch cyclonal ist. Das Umgekehrte findet statt, falls μ_2 negativ, ist, d. h. das andere Gebiet anticyclonal ist.

Beobachtungen, an welchen man diese Asymmetrie der Druckvertheilung in einer unter dem Einfluss eines anderen Wirbelgebietes wandernden Cyclone wenigstens qualitativ prüfen könnte, liegen, so viel ich weiss, noch nicht vor. Ich bemerke hier nur noch, dass nach Thom der Ort des tiefsten Barometerstandes in einem Wirbelsturm sich gelegentlich etwas vor dem Centrum befinden soll.*

Vom Interess ist, wie ich glaube, ein System von Curven, welche durch die folgende Gleichung definirt ist

$$H^2 = \text{Const.}$$

* Wie Thom zu diesem Schluss gelangt war, ist mir unbekannt. Reye hat diese Bemerkung nur gelegentlich in seinem "Wirbelstürme etc." (pag 96) angeführt.

d. h. die Curven gleicher Windstärke, weil man damit eine Gleichung für die Gestalt des sogenannten Sturmgebietes erhält; d. h. eines Gebietes, innerhalb dessen der Wind als "Sturm" charakterisirt werden kann. Wir denken uns zu dem Ende eine Richtung parallel dem Meridian durch das Centrum des Wirbelgebietes gelegt, und bezeichnen den Winkel, den der Radius ρ' mit dieser Richtung schliesst mit ψ , und zwar so, dass ψ in dem Sinne von S. O. N. W. wachsen soll.

Wir haben dann die Beziehungen

$$\rho \cos \chi = \rho' \cos \psi + x_1$$

$$\rho \sin \chi = \rho' \sin \psi + y_1$$

Da ferner

$$\frac{y_1}{\rho_1} = \sin \chi_1 \quad \frac{x_1}{\rho_1} = \cos \chi_1$$

ist, so folgt

$$\rho \sin (\chi_1 - \chi) = \frac{\rho'}{\rho_1} (\cos \psi \cdot y_1 - \sin \psi \cdot x_1)$$

$$\rho \cos (\chi_1 - \chi) = \frac{\rho'}{\rho_1} (\cos \psi \cdot x_1 + \sin \psi \cdot y_1) + \sqrt{x_1^2 + y_1^2}$$

da $\sqrt{x_1^2 + y_1^2} = \rho_1$ ist, so folgt aus der zweiten Gleichung

$$\rho_1 - \rho \cos (\chi_1 - \chi) = \frac{\rho'}{\rho_1} (\cos \psi \cdot x_1 + \sin \psi \cdot y_1)$$

Wenn wir diese Ausdrücke in (7) substituiren, und für ρ', ρ für x_1 und y_1 α und β setzen, so erhält man nach einiger Umformung unter der Berücksichtigung der Beziehung (6)

$$F^2 = \varphi(\rho) + \frac{\mu_2 \gamma_1}{r} \frac{\rho}{\rho_1} \left(\cos \psi [\alpha + \beta K + f(\rho)(\beta - \alpha K)] \right. \\ \left. + \sin \psi [\beta - \alpha K - f(\rho)(\alpha + \beta K)] \right)$$

wo zur Abkürzung gesetzt worden ist

$$K = \frac{2\lambda \sin \theta}{\pi} \quad \varphi(\rho) = \gamma_1^2 z^2 + \frac{\gamma_1^2}{4} \left(1 + [f(\rho)]^2\right) \rho^2 + \frac{P_2^2 + \mu_2^2}{\gamma_2^2}$$

Die Grössen $\alpha + \beta K$, und $\beta - \alpha K$, —Functionen der Zeit t —hängen ab von der Lage des Centrums des Wirbelgebietes in Bezug auf die durch den Schwerpunkt gelegten Coordinatenachsen. Setzt man zur Abkürzung

$$A = \frac{\rho_2 \gamma_1}{r \rho_1} (a + \beta K) \quad B = \frac{\mu_2 \gamma_1}{r \rho_1} (\beta - \alpha K)$$

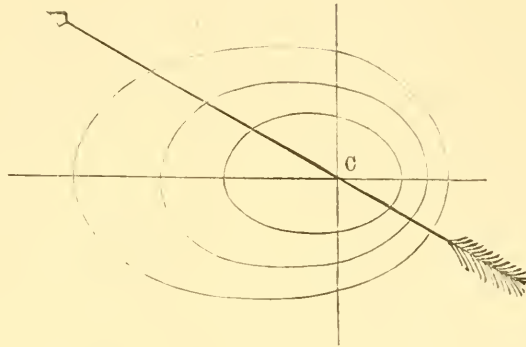
so hat man

$$\begin{aligned} F^2 = \varphi(\rho) + \rho \cos \psi [A + B f(\rho)] \\ + \rho \sin \psi [B - A f(\rho)] \end{aligned} \quad (10^a)$$

Die Curve $F^2 = \text{const}$ für $z=0$ ist demnach kein Kreis und kann einen Kreis $\rho = \text{const}$ in vier reellen Punkten schneiden, da diese Gleichung für $\rho = \text{const}$. vier reelle Wurzeln für ψ geben kann. Da ρ für keinen Werth von ψ unendlich werden kann, so ist die in Rede stehende Curve eine algebraische geschlossene Curve, wenn nicht $m=1$ ist. Sie ändert ihre Gestalt, wie ihre Lage fortwährend, das erstere, weil die Grössen A und B durch Verlegung der Coordinatenachsen allein sich nicht wegschaffen lässt.

Es ist ein System ovaler um das Centrum des Wirbelgebietes excentrischer Curven. Da die Function $\varphi(\rho)$, und $\rho [A + B f(\rho)]$ mit abnehmendem ρ im allgemeinen abnimmt, wenn $m > 1$ oder $m > \frac{1}{2}$ ist, so wird die Gleichung (10a) in dem Quadrante, wo die Windstärke am grössten ist, durch kleineres ρ erfüllt als in dem Quadrante, wo die Windstärke am kleinsten ist; d. h. in dem Quadrante III [Fig. pag. (29)] ist der Werth von ρ in der Gleichung (10a) am kleinsten und am grössten in dem Quadranten II. Die Curve der gleichen Windstärke liegt in dem Quadrante III näher gegen das Centrum des Wirbelgebietes hin als in dem Quadrante II;

das Sturmgebiet erscheint demnach nach der linken Seite der Fortschreitungsrichtung verbreitert, wie es die Figur ungefähr veranschaulichen mag.



Die Curven sind in der Ellipticität übertrieben gezeichnet, denn sie kann man in erster Annäherung als Kreise betrachten, deren Mittelpunkte verschiedene Lage in Bezug auf das Wirbelcentrum C haben. Da nämlich $f(\rho)$ sich von $\frac{2\lambda \sin \theta}{\alpha}$ bis zu $\frac{2\lambda \sin \theta}{\alpha - \gamma_1}$ ändert, während ρ von R bis θ abnimmt, so kann man wohl annehmen, dass $f(\rho)$ entlang der Curve $F^2 = \text{const.}$, für die ρ überall doch einen endlichen Werth hat, nicht all zu viel variire, dass man daher der Function $f(\rho)$ in erster Annäherung einen gewissen mittleren Werth M geben könne. Unter dieser Annahme wird die Gleichung (10a)

$$(10) \quad F^2 = \frac{\gamma_1^2}{4} \left((1 + M^2) \rho^2 + \rho \cos \psi (A + B M) + \rho \sin \psi (B - A M) \right) + \frac{(P_2^2 + \mu_2^2)}{\gamma^2}$$

was einen Kreis bedeutet, dessen Mittelpunkt durch

$$-\frac{2(A + B M)}{\gamma_1^2(1 + M^2)}, \quad -\frac{2(B - A M)}{\gamma_1^2(1 + M^2)}$$

und dessen Radius durch

$$\left[\frac{4 \left(F^2 - \frac{P_2^2 + \mu_2^2}{\gamma^2} \right)}{\gamma_1^2(1 + M^2)} + \frac{4(A^2 + B^2)}{\gamma_1^2(1 + M^2)} \right]^{\frac{1}{2}}$$

bestimmt wird. Wenn das Wirbelgebiet sich süd-östlich von dem Schwerpunkt befindet, so ist A wie B positiv, vorausgesetzt, dass das

andere Wirbelgebiet auch cyclonal sei. Das in Rede stehende Wirbelgebiet hat dann die Fortschreitungsrichtung nach N , NO , O . Ist dabei $B > AM$, so liegt der Mittelpunkt des Kreises (10b) nordwestlich vom Centrum des Wirbelgebietes, in dem $(-x, -y)$ Gebiete der Coordinaten. Ist aber $B < AM$, so liegt derselbe nordöstlich vom Centrum des Wirbelgebietes, —also ebenfalls auf der linken Seite der Fortschreitungsrichtung.

Die Gleichung (10a) gilt indessen nur innerhalb des mit R_1 beschriebenen Kreises. Da aber F an dieser Grenze der verticalen Strömung durchaus stetig bleibt und jenseits dieser Grenze wieder wachsen kann, in Folge der Abnahme des Abstandes zu dem zweiten Wirbelgebietes, so sieht man ein, dass die Curve $F^2 = \text{const}$ ausserhalb des Gebietes (1) durch eine andere Curve continuirlich fortgesetzt wird.

Es scheint in diesem Ergebniss eine Erklärung der Thatsache zu liegen, dass die vordere Seite einer Cyclone immer von viel längerer Dauer zu sein pflegt, als die hintere Seite derselben; einer Thatsache, welche Piddington für eine so ausgemachte Thatsache erklärt haben soll, dass sie keiner Belege bedürfe. In der That hat Piddington den vorderen Halbmesser einer Bengalischen Cyclone für den 12 und 13 October 1848 zu 140 und 115 Seemeilen bestimmt, den hinteren aber nur 90 respectiv 65 Seemeilen.*

Wenn $m < \frac{1}{2}$ ist, so dass die Luftgeschwindigkeit nach dem Wirbelcentrum zu in's unendliche zunimmt, so verhält sich die Sache etwas anderes. In dem Quadrante III herrscht nach wie vor das Maximum der Windstärke. Da aber $\varphi(\rho)$ wie $f(\rho)$ mit abnehmendem ρ in's Unendliche wächst, so wird die Gleichung (10a) in dem Quadrante der minimalen Windstärke durch ein kleineres ρ befriedigt, als in dem Quadrante III. Es folgt hieraus dass das Curvensystem $F^2 = \text{const}$ jetzt

* Reye. "Wirbelstürme etc." pag 93.

nach der linken Seite der Fortschreitungsrichtung hin kleinere Radien aufweist, als in der hinteren rechten Seite der Cyclone. In diesem Fall erscheint demnach das Sturmsgebiet eiförmig nach Hinten der Cyclone ausgedehnt, und der Punkt des minimalen Drucks das Centrum des Wirbelgebietes liegt somit nothwendig etwas *vor* dem Centrum des Sturmsgebietes, was mit der oben erwähnten Bemerkung Thom's, wie mit der Wahrnehmung Redfield's† zusammenfällt, dass das Barometer gewöhnlich kurz vor dem Windwechsel wieder zu steigen beginne.

Indessen muss ich mich hier ausdrücklich dagegen verwahren, als wenn ich glaube, diese Anomalien lediglich aus der Bewegung eines Wirbelgebietes erklären zu können, da es durchaus nicht ausgemacht ist, dass die Bewegung allein zureiche, diese Anomalien zu bewirken, und die Berücksichtigung der gasigen Natur der Luft zur Erklärung derselben nicht gerade nöthig sei.

Wenn das Wirbelgebiet (2) oder die beiden Wirbelgebiete anticyclonal sind, so begegnet man einer eigenen Schwierigkeit in Folge des Umstandes, dass das anticyclonale Wirbelgebiet nicht mit dem der vertical niedersteigenden Strömung zusammenfällt, sondern streng genommen in's Unendliche hinübergreift. Die in dem vorigen Paragraphen aufgestellten Ausdrücke für ein wirbelfreies Gebiet verlieren eigentlich ihre Gültigkeit, wenn eins der beiden Wirbelgebiete, oder beide anticyclonal sind. Indessen; es ist daran zu erinnern, dass die ausserhalb des Gebietes der vertical herabsteigenden Strömung existirenden Luftwirbel, wie wir bei dem kreisförmigen Querschnitte gesehen haben, ausserordentlich rasch verschwinden mit dem zunehmenden Abstand von der Grenze des Gebietes der verticalen Strömung.

† Redfield; Observations on the Hurricanes and Storms of the West-Indies and the coast of the United States. Sillmann's Journal Vol. 25 pag. 114.

dass jeder mässig entfernter Punkt des äusseren Gebietes schon als völlig wirbelfrei betrachtet werden kann. Wir können uns demnach, wenn die Begrenzungscurve des inneren Gebietes ein Kreis ist, einen dazu concentrischen Kreis denken, dessen Radius R' so gross ist, dass eine Function von der Form $e^{-\frac{\kappa}{\gamma}(\frac{\rho}{R'})^2}$ unendlich klein wird, aber noch unendlich klein gegen den Abstand der beiden Gebiete der verticalen Strömung. Dann gelten die für das äussere wirbelfreie Gebiet aufgestellten Ausdrücke für Windstärke, Windazimuth, and Druck in dem Punkt ausserhalb dieses Kreises. In den Ausdrücken, welche wir für einen Punkt innerhalb eines cyclonalen Wirbelgebietes abgeleitet haben, braucht man nur das Vorzeichen von μ_2 und P_2 zu wechseln, wenn das zweite Wirbelgebiet anticyclonal ist, da P_2 in diesem Fall nur sein Vorzeichen wechselt, wenn auch das Integrationsgebiet grösser ist, als das Gebiet der verticalen Strömung. Wir bilden, um dieses einzusehen

$$P_2 = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{R_2} (\zeta - 2\lambda \sin \theta) \rho d\rho d\theta \\ + \frac{1}{2\pi} \int_0^{2\pi} \int_{R_2}^{R'} (\zeta - 2\lambda \sin \theta) \rho d\rho d\theta$$

Das erste Integral ist ausgedehnt über das Gebiet der vertical herabsteigenden Strömung und das zweite Integral über das äussere ringförmige Gebiet bis zu dem mit R' beschriebenen Kreis. Für das erste Integral hat man zu setzen (pag 204 Vol. I dieses Journals)

$$\zeta - 2\lambda \sin \theta = -\frac{2\lambda \sin \theta}{\kappa + \gamma_2} \gamma_2$$

und für das zweite Integral

$$JW_a = \frac{1}{\rho} d\rho \frac{dW_a}{d\rho} = -\zeta + 2\lambda \sin \theta = C_e - \frac{\kappa}{\gamma_2} \left(\frac{\rho}{R_2}\right)^2$$

Die Integration ergibt hieraus

$$\frac{dW_a}{d\rho} = -\frac{R_2^2}{2} \frac{\gamma_2}{\kappa} \frac{C_e}{\rho} - \frac{\kappa}{\gamma_2} \left(\frac{\rho}{R_2} \right)^2 + \frac{C'}{\rho}$$

wo C' eine Constante ist. Es soll für $\rho=R'$ der Wirbel verschwinden, so dass

$$\frac{dW_a}{d\rho} = \frac{2\lambda \sin \theta}{\kappa} \frac{d\zeta_a}{d\rho} \quad \zeta_a = \frac{\gamma_2}{2} R_2^2 \log \rho + \text{Const.}$$

Diese Bedingung ergibt

$$C' = \frac{\lambda \sin \theta}{\kappa} \gamma_2 R_2^2$$

Es soll ferner an der Grenze der verticalen Strömung

$$\frac{dW_i}{d\rho} = \frac{dW_a}{d\rho} \quad \text{für } \rho = R_2$$

Dies giebt

$$C = \frac{2\lambda \sin \theta}{\kappa + \gamma_0} \gamma_2 e^{\frac{\kappa}{\gamma_2}}$$

Wir erhalten somit für das äussere Gebiet

$$\zeta - 2\lambda \sin \theta = -\frac{2\lambda \sin \theta}{\kappa + \gamma_2} \gamma_2 e^{\frac{\kappa}{\gamma_2}} \left[\left(\frac{\rho}{R_2} \right)^2 - 1 \right]$$

Es folgt hierdurch

$$p_2 = -\frac{2\lambda \sin \theta}{\kappa + \gamma_2} \frac{\gamma_2 R_2^2}{2} - \frac{2\lambda \sin \theta}{\kappa + \gamma_2} \gamma_2 \int_{R_2}^{R'} e^{-\frac{\kappa}{\gamma_2} \left(\frac{\rho}{R_2} \right)^2 - 1} \rho d\rho$$

d. h.

$$p_2 = -\frac{2\lambda \sin \theta}{(\kappa + \gamma_2)} \frac{\gamma_2 R_2^2}{2} \left(1 + \frac{\gamma_2}{\kappa} \right) = -\frac{2\lambda \sin \theta}{\kappa} \frac{\gamma_2 R_2^2}{2}$$

womit das oben Behauptete bewiesen ist.

Da in dem in Rede stehenden Fall

$$\mu_2 = -\frac{\tilde{\gamma}_2 R_2^2}{2}$$

ist, so bleibt nach wie vor

$$\frac{P_2}{\mu_2} = \frac{2\lambda \sin \theta}{\pi}.$$

Der immerhin willkürliche Radius R' kommt demnach bei der Betrachtung einer durch eine unendlich fernen Anticyclone bewegten Cyclone nicht weiter in Betracht.

Wohl würde die Bestimmung des Zeitpunktes, wo ein gegebener Ort in ein anticyclonales Wirbelgebiet eintritt, oder daraus hervortritt, davon abhängen, welchen Werth man R' beilegt. Allein; da die Componenten der Geschwindigkeit und die Function Φ so wohl für das äussere wirbelerfüllte Gebiet, wie für das Gebiet der vertical herabsteigenden Strömung kein Glied enthalten, das von dem Radius R' abhängig wäre, so sind die Ausdrücke für Windstärke, Windazimuth und Luftdruck in einem anticyclonalen Gebiet auch völlig frei von R' , selbst wenn das andere Wirbelgebiet auch anticyclonal sein sollte, und es hat somit keine Schwierigkeit, R' , σ , P für einen in ein anticyclonales Gebiet gerathenen Ort als Functionen von der Zeit darzustellen.

Es mag aber damit genügen, an den für ein cyclonales Wirbelgebiet entwickelten Ausdrücken nachgewiesen zu haben, dass sie im Stande sind, den Verlauf eines Wirbelsturms zu schildern. Wohl liegt der Gedanke nah, für die hier auftretenden Constanten willkürliche Zahlenwerthe anzunehmen und so die Veränderung der drei anemometrischen Elemente in einem gegebenen Ort numerisch zu verfolgen, und so näher zu prüfen, ob unsere in diesem und vorhergehenden Paragraphen entwickelten Ausdrücke einigermaßen richtig den wirklichen Verlauf eines Wirbelsturms darstellten. Der Haupt-

zweck dieser Entwicklungen liegt indessen darin, um zu zeigen, wie man hätte verfahren können, Windstärke, Windazimuth, und Luftdruck für einen gegebenen Ort, sowohl im Inneren, als Ausserhalb eines fortschreitenden Wirbelgebietes, als Functionen von der Zeit darzustellen, wenn man nur im Stande wäre, die hydrodynamischen Differentialgleichungen mit Annahmen zu integrieren, welche vielgenauer dem thatsächlichen Verhältnisse entsprechen würden, als diejenigen, von denen wir ausgegangen waren.

§ XIII. — *Verticale Luftströmungen in der Erdatmosphäre.*

Bisher war die Grösse γ einfach als eine Constante betrachtet worden, ohne näher die Bedeutung derselben festzustellen.* Wenn γ eine Constante ist, und von der Zeit unabhängig ist, so ist die Geschwindigkeit eines vertical auf—oder niedersteigenden Lufttheilchens durch eine Exponentialfunction ausgedrückt. Offenbar setzt ein solcher Bewegungszustand der Luft eine ganz bestimmte verticale Temperaturvertheilung in der Atmosphäre voraus, welche dazu noch sich mit der Zeit nicht ändert. Wenn nun die verticale Temperaturvertheilung eine andere und mit der Zeit veränderlich ist, so wird die Geschwindigkeit der verticalen Strömung einen anderen Ausdruck haben, als Exponentialfunction und die Wirbelbewegung, welche sie in der Atmosphäre hervorruft, einem anderen Gesetz gehorchen, und kann mit der Zeit veränderlich sein.

Wir wollen jetzt die Annahme fallen lassen, dass γ eine Constante sei, und den Fall zu behandeln suchen, wo γ eine gegebene Function von der Zeit ist. Da die Geschwindigkeit jeder geradlinigen Be-

* Bei der flüchtigen Feststellung der Grösse γ (pag. 167 Vol. I dieses Journals) ist ein Versehen uncorrectirt geblieben. Es ist überall für $g(\mu - \mu)$, $g \frac{(\mu' - \mu)}{\mu}$ zu setzen.

wegung immer in der Form γ , z. dargestellt werden kann, so behandeln wir damit Fälle, wo die Geschwindigkeit der verticalen Strömung durch andere Function ausgedrückt ist, als durch Exponentialfunction. Ehe wir aber zur Integration der Bewegungsgleichungen für diesen Fall schreiten, ist es nothwendig, uns über die physikalische Bedeutung der Grösse γ nähere Rechenschaft zu verschaffen.

Reye hat* schon näher die Bedingungen festgestellt, unter denen die atmosphärische Luft sich im labilen oder stabilen Gleichgewicht befindet, und einen Ausdruck für die Geschwindigkeit der verticalen Strömung abgeleitet, und zwar unter der Annahme, dass die verticale Abnahme der Temperatur eine lineare Function der Meereshöhe sei. Es geht aus seinen Untersuchungen hervor, dass, so lange die Vertheilung des Wasserdampfes in horizontalen Schichten eine gleichförmige ist, die Luft sich im indifferenten Gleichgewichte befindet und, wenn gleich die von der Erdoberfläche aus erwärmte Luft fortwährend aufsteigt, die Bildung einer vertical aufsteigenden Strömung unterbleibt, dass aber eine solche entstehen kann, wenn eine mit Wasserdampf gesättigte Luftmasse von einem beschränkten Gebiet der Erdoberfläche aus emporsteigt und die durch Condensation des Wasserdampfes frei gewordene Wärme den verticalen Temperaturgradienten modificiert. Der Ausdruck, den Reye für die Geschwindigkeit einer solchen verticalen Strömung abgeleitet hat, ist angenähert linear in Bezug auf die Meereshöhe, wenn wir festsetzen dass diese, soweit wir sie in Betracht ziehen, nur eine mässige sei, und dass die Geschwindigkeit an der Erdoberfläche verschwinde.

Wir wollen hier die Aufgabe etwas allgemeiner in Angriff nehmen, indem wir vor der Hand keinerlei Annahme über die Beziehung der Temperatur und der Meereshöhe machen. Es herrsche in der Meeres-

* Th. Reye. Die Wirbelstürme etc. 1880. Anhang pag. 221.

höhe z der Druck p , und die Temperatur θ . Man hat dann, wie man aus dem Boyle-Gay-Lussac'schen Gesetze leicht ableiten kann

$$\frac{p}{p_0} = e^{-\int_0^z \left(\frac{R}{z+R}\right)^2 \frac{dz}{H(a+b)}} \quad (11)$$

wo p_0 den Druck auf der Erdoberfläche, R den mittleren Erdradius, H die Constante 29.274, welche jedoch bei dampfhaltiger Luft von p und θ , darum von z auf gewisse Weise abhängt, und a endlich die Zahl 273 bedeutet. Wenn nun eine Masse Luft von der Dichte μ_1 emporsteigt, und die bis dahin im Gleichgewicht befindliche Luft durchbricht, deren Dichte in der Höhe z μ_a ist, so erhält sie einen Auftrieb, dessen Grösse durch

$$\frac{d^2z}{dt^2} = \frac{gR^2}{(R+z)^2} \left(\frac{\mu_a}{\mu_1} - 1 \right) \quad (12)$$

bestimmt ist, wo g die Schwere an der Erdoberfläche bedeutet.

Der Quotient $\frac{\mu_a}{\mu_1}$ ist offenbar nicht nur von z abhängig, sondern auch von den Coordinaten, welche einen Punkt der durch z gelegten Horizontalebene bestimmen, da die Temperaturvertheilung in horizontaler Richtung auch im allgemeinen als eine ungleichmässige vorausgesetzt werden muss. Wir wollen hier indessen den Querschnitt der verticalen Strömung so mässig annehmen, dass wir uns $\frac{\mu_a}{\mu_1}$ als eine Function von der Meereshöhe z , und eventuelle von der Zeit t allein vorstellen dürfen. Ist θ_a die Temperatur ausserhalb des Raumgebietes der verticalen Strömung, und θ_1 diejenige innerhalb desselben in der Meereshöhe z , so folgt nach dem Boyle-Gay-Lussac'schen Gesetz in dem wir ferner annehmen, dass die emporsteigende Luftmasse sich unter dem Druck ausdehne, den sie in der Meereshöhe z vorfindet

$$\frac{\mu_a}{\mu_i} = \frac{a + \theta_i}{a + \theta_a}$$

Wenn wir uns θ_i und θ_a als Functionen von z so dargestellt denken, dass

$$\theta_i = \theta_o - f(z)_i$$

$$\theta_a = \theta_o - f(z)_a$$

wo θ_o die Temperatur auf der Erdoberfläche bedeutet, so erhalten wir

$$\frac{d^2z}{dt^2} = g \frac{R^2}{(R+z)^2} \left(\frac{f(z)_a - f(z)_i}{a + \theta_o - f(z)_a} \right) \quad (13)$$

Die Geschwindigkeit $\frac{dz}{dt}$ ist hier positiv, und muss so bestimmt werden, dass sie für $z=0$ verschwindet. So lange demnach $f(z)_a - f(z)_i > 0$ ist, nimmt $\frac{dz}{dt}$ von 0 aus zu, bis in der Meereshöhe, wo $f(z)_a = f(z)_i$ und, $\frac{dz}{dt} = \text{constant}$ wird. Wenn nun $f(z)_a - f(z)_i$ von irgend einer Meereshöhe ab, oder zu irgend welcher Zeit sein Vorzeichen wechselt, was nur durch Null hindurch geschehen kann, so wird $\frac{dz}{dt}$, da $\frac{d^2z}{dt^2}$ negativ ist, entweder abnehmen, oder sein Vorzeichen wechseln, was wiederum nur durch Null hindurch geschehen kann. Da in der Meereshöhe, oder zu einer Zeit, wo $f(z)_a - f(z)_i = 0$ wird, $\frac{dz}{dt} = \text{constant}$ wird, und diese Constante, wenn der Strom niedersteigen soll, nur Null sein kann, so müsste $\frac{dz}{dt}$ von $z=0$ aus gegen den Werth Null *abgenommen* haben; mithin müsste $\frac{d^2z}{dt^2}$ negativ gewesen sein, als der Strom emporgeriegen war., was aber ein Widerspruch ist. Ich glaube hieraus schliessen zu dürfen, dass eine Luftmasse, wenn sie einmal eine vertical aufsteigende Strömung gebildet hat, nie wieder vertical abwärts zur Erdoberfläche zurückkehrt, und dass, wenn in einer

gewissen Meereshöhe oder zu irgend welcher Zeit $f(z)_n - f(z)_i = 0$ wird, die Luftmasse immer weiter emporsteigt aber mit einer in's unendliche abnehmenden Geschwindigkeit, da die Grösse $a + \theta_0 - f(z)_n$ mit wachsender Höhe in's Unendliche abnehmen kann.

Ist die Meereshöhe, so weit wir in Betracht ziehen, eine nur mässige, so kann man setzen

$$\theta_1 = \theta_0 - \tau \cdot z,$$

$$\theta_n = \theta_0 - \tau' \cdot z,$$

wo τ den Werth des verticalen Temperaturgradienten bedeutet, und im allgemeinen $< \frac{1}{100}$ ist. Man erhält dann

$$\frac{d^2 z}{dt^2} = g \left(\frac{\tau' - \tau}{a + \theta_0 - \tau' z} \right) z,$$

da für einen mässigen Werth von $z = \frac{R^2}{(R + z)^2} = 1$ gesetzt werden kann.

Weil $\frac{\tau}{a + \theta_0}$ ein sehr kleiner Bruch ist, so kann man auch so schreiben

$$\frac{d^2 z}{dt^2} = g \left(\frac{\tau' - \tau}{a + \theta_0} \right) \cdot z = K^2 z, \quad (14)$$

wo $g \frac{(\tau' - \tau)}{a + \theta_0} = K^2$ gesetzt ist. Die Beschleunigung ist demnach linear in Bezug auf z . Es ist leicht hieraus eine Gleichung für γ abzuleiten. Wir differentiren die Gleichung

$$w = \frac{dz}{dt} = \gamma \cdot z,$$

nach t , indem wir uns γ als eine Function von t vorstellen, so dass wir erhalten

$$\frac{d^2z}{dt^2} = z \left(\gamma^2 + \frac{d\gamma}{dt} \right)$$

Ein Vergleich dieses mit (14) ergibt sofort

$$\gamma^2 + \frac{d\gamma}{dt} = K^2 \quad (15)$$

Es erhellt zunächst aus dieser Beziehung, dass, wenn γ die Zeit nicht enthalten, d. h. die Bewegung eine stationäre sein soll, $\tau' - \tau$ immer positiv sein muss, da in diesem Fall

$$\gamma = \sqrt{g \frac{(\tau' - \tau)}{(a + \theta_0)}} = K$$

ist, wie schon W. Ferrel* und Th. Royet† abgeleitet haben. Der Fall $K = \text{const.}$ bedingt in dessen nicht nothwendig, dass auch γ von der Zeit unabhängig sei. Integriert man nämlich (15) unter der Annahme, dass $K = \text{const.}$ sei, so erhält man

$$\gamma = K \left(\frac{e^{Kt} (K + \gamma_0) - (K - \gamma_0) e^{-Kt}}{e^{Kt} (K + \gamma_0) + (K - \gamma_0) e^{-Kt}} \right)$$

wo γ_0 den Werth von γ für $t=0$ bedeutet. In diesem Fall verändert sich also γ von mit wachsender Zeit von γ_0 auf K . Die Integration der Gleichung

$$\frac{dz}{dt} = K z \cdot \left(\frac{e^{Kt} (K + \gamma_0) - (K - \gamma_0) e^{-Kt}}{e^{Kt} (K + \gamma_0) + (K - \gamma_0) e^{-Kt}} \right)$$

ergiebt

$$z = \frac{z_0}{K} \left(e^{Kt} (K + \gamma_0) + (K - \gamma_0) e^{-Kt} \right)$$

wo z_0 die Meereshöhe eines Lufttheilchens zur Zeit $t=0$ bedeutet. Aus dieser Gleichung fliesst weiter

* W. Ferrel: Recent Advances in Meteorology. Washington. 1886 pag. 293.

† Royet: Wirbelstürme &c. Hannover 1880 pag. 229.

$$\frac{dz}{dt} = z_0 \left(e^{Kt} (K + \gamma_0) - (K - \gamma_0) e^{-Kt} \right)$$

Das Lufttheilchen besass zur Zeit $t=0$ die Geschwindigkeit $\gamma_0 z_0$, als die vertical aufsteigende Strömung sich bildete. Dieser Fall lässt sich demnach dahin deuten, dass der verticale Temperaturgradient sich *plotzlich* geändert hat, dass daher die Geschwindigkeit der aufsteigenden Luftströmung, welche ursprünglich den Werth $\gamma_0 z$ besass, sich allmählig dem Werth nähert, der der neuen verticalen Temperaturvertheilung entspricht. Haben die Lufttheilchen zur Zeit $t=0$ keine durch Differenz des verticalen Temperaturgradienten erzeugten Geschwindigkeit, so hat man $\gamma_0=0$ zu setzen, und so erhält man

$$\gamma = K \left(\frac{e^{Kt} - e^{-Kt}}{e^{Kt} + e^{-Kt}} \right) \quad (16)$$

Dieses entspricht dem Fall, wo die Differenz des verticalen Temperaturgradienten sich gebildet und die bis dahin in Ruhe befindliche Luft *allmählig* empor steigt, und so auch *allmählig* eine verticale Luftströmung bildet.

Bei demselben Werthe von K sind demnach zwei verticale Strömungen von wesentlich verschiedenem Charakter möglich. In dem Fall, wo die verticale Strömung allmählig sich bildet, ist

$$\frac{dz}{dt} = z_0 K \left(e^{Kt} - e^{-Kt} \right)$$

sodass die Luft in jeder Schichte zur Zeit $t=0$ in Ruhe ist. Anderes hingegen, wenn $\gamma=K$ ist. In diesem Fall hat man für ein Lufttheilchen in der Höhe z_0

$$\frac{dz}{dt} = K z_0 e^{Kt}$$

so dass die Geschwindigkeit des Lufttheilchen zur Zeit $t=0$, den Werth $K z_0$ hat. Wenn wir nun näher die Gleichung

$$\gamma^2 + \frac{d\gamma}{dt} = K^2$$

betrachten, so sieht man, dass in dem Fall, wo die Bildung einer verticalen Strömung eine allmälige ist, $\frac{d\gamma}{dt}$ für $t=0$ einen endlichen Werth besitzt, aber für $t=\infty$ verschwindet, da γ gegen K convergirt. Ist γ hingegen für $t=0$ der Constante K gleich, so können wir uns γ und K als eine discontinuirliche Function denken, welche für $t=0$ verschwindet, aber für jedet andere t einer Constante gleich wird. Da in dem Fall, wo $K = \text{const.}$ und γ mit der Zeit veränderlich ist, γ erst nach dem Verlauf einer unendlich grossen Zeit den Werth K erreicht; da ferner in dem letzteren Fall nur eine verschwindend kleine Zeit dazu nöthig ist; so sehen wir, dass $\gamma=K$ für $t=0$ dem Fall entsprechen kann, wo eine Luftmasse *plötzlich* emporsteigt, welche bisher sich im labilen Gleichgewicht befunden hat; dass $\gamma=0$ für $t=0$ aber den Fall darstellt, wo die Luft beim Bilden der Differenz des verticalen Temperaturgradienten sofort zu steigen beginnt, und allmähig das Maximum ihrer Geschwindigkeit erreicht.

In den Fällen, wo K^2 eine gegebene Function von der Zeit ist, haben wir auch zwei Arten der verticalen Strömung zu unterscheiden je nachdem γ für $t=0$ einen endlichen Werth hat, oder verschwindet. In dem ersteren Fall entsteht die vertical aufsteigende Luftströmung plötzlich, während sie in dem letzteren Fall allmähig entsteht. Welche Form der vertical aufsteigenden Strömung eintritt, das hängt indessen weder von der Functionsform der Grösse K^2 nach von ihrem Werthe zur Zeit $t=0$. Die Wirbelbildung, die sie veranlasst, ist aber, wie wir weiter unten sehen werden, grundverschieden.

Es ist jedoch noch zu bemerken, dass der Fall, wo γ für $t=0$ einer endlichen Constante gleich ist, einer anderen Deutung fähig ist. Man kann sich auch die verticale Strömung als längst entstanden denken, und die Zeit von dem Augenblick zählen, wo γ den Werth erreicht, welchen wir der durch die Integration der Gleichung (15) auftretenden Constante für $t=0$ geben. Es ist daher in einem solchen Fall gleichgültig, ob wir uns die verticale Strömung im Momente $t=0$ plötzlich entstehend denken, oder längst entstanden.

Wir fassen jetzt eine Luftströmung in's Auge, welche vertical abwärts geht. In diesem Fall wird die Luft in dem Raumgebiet der verticalen Strömung schwerer sein müssen, als ausserhalb desselben, so dass

$$\mu_a < \mu_i$$

Da die Geschwindigkeit eines Lufttheilchens $\frac{dz}{dt}$ hier negativ ist und vermöge der an der Erdoberfläche zu erfüllenden Bedingung

$$\left(\frac{dz}{dt}\right)_{z=0} = 0$$

mit wachsender Zeit ihrem Maximum 0 zustreben muss, so muss $\frac{d}{dt} \frac{dz}{dt}$ essentiell negativ gesetzt werden. Mithin folgt

$$\frac{d^2z}{dt^2} = g \frac{R^2}{(R+z)^2} \left(1 - \frac{\mu_a}{\mu_i}\right)$$

Daher

$$\frac{d^2z}{dt^2} = g \frac{R^2}{(R+z)^2} \left(\frac{f'(z)_i - f'(z)_a}{\alpha + \theta_0 - f(z)_a}\right)$$

So lange demnach $f'(z)_i - f'(z)_a > 0$ ist, strebt $\frac{dz}{dt}$ seinem Maximum

0) zu mit wachsender Zeit. Da $\frac{d^2z}{dt^2}$ positiv ist und z seinem Minimum 0) zustrebt, ist $\frac{dz}{dt}$ also negativ. Kann $f(z)_h - f(z)_a$ in irgend einer Meereshöhe, oder zu irgend welcher Zeit Null werden *d. h.* sein Vorzeichen wechseln, so wird $\frac{dz}{dt}$ noch weiter abnehmen, oder sein Vorzeichen wechseln, so dass die ursprünglich niedersteigende Luft nun mehr emporsteigen müsste. Das erstere ist nun unmöglich, wenn $f(z) - f'(z)$ nur einmal und nicht für $z=0$ verschwindet, da ja für $\frac{dz}{dt}$ ein Ausdruck resultiren würde, der an der Erdoberfläche nicht verschwindet. In dem zweiten Fall wird $\frac{dz}{dt}$, da wo $f(z)_a - f(z)_h = 0$ wird, einer Constante gleich, und diese Constante muss $=0$ sein, weil die Richtung der Strömung eine umgekehrte wird. Da im weiteren Verlauf der Zeit $\frac{dz}{dt}$ von Null aus positiv wachsen muss, so müsste $\frac{d}{dt} \frac{dz}{dt}$ von nun an positiv sein, was wieder ein Widerspruch ist. Ich glaube hieraus schliessen zu dürfen, dass, wenn eine Luftmasse vertical gegen die Erdoberfläche herabsteigt und so eine niedersteigende Strömung bildet, sie nie wieder rückwärts in die Höhe steigt. Die Lufttheilchen strömen nur mit in's Unendliche abnehmender Geschwindigkeit gegen eine Schichte, wo der Temperaturunterschied verschwindet, deren Höhe jedoch sich mit der Zeit ändern muss, wenn $f'(z) - f(z) = 0$ noch die Zeit enthält.

Ist die Meereshöhe, so weit wir sie in Betracht ziehen, eine nur mässige, so hat man annähernd wieder

$$\frac{d^2z}{dt^2} = g \left(\frac{z - z_0}{a + \theta_0} \right) z.$$

Indem wir $w = \frac{dz}{dt} = -\gamma \cdot z$ nach t differentiren und das

Resultat mit dem obenstehenden Ausdruck vergleichen, erhalten wir

$$\dot{r}^2 - \frac{d\dot{r}}{dt} = K^2 \quad (17)$$

indem wir wieder

$$y\left(\frac{z-\tau'}{\alpha+\theta_0}\right) = h^2$$

setzen.

Es geht aus (17) zunächst hervor, dass, wenn eine stationäre vertical herabsteigende Strömung entstehen soll, $\tau > \tau'$ sein muss. Die Constanz der Grösse K^2 bedingt hier auch nicht nothwendig die Stationalität der verticalen Strömung. Integriert man nämlich (17) so erhält man

$$\dot{r} = K \left(\frac{e^{Kt+c} + e^{-Kt-c}}{e^{Kt+c} - e^{-Kt-c}} \right) \quad (18)$$

wo C die Integrationsconstante ist. \dot{r} ist also mit t veränderlich, und convergirt nach dem Verfluss einer unendlich grossen Zeit gegen K . Die Integration der Gleichung $\frac{dz}{dt} = -\dot{r}$ ergibt

$$z = \frac{C'}{e^{Kt+c} - e^{-Kt-c}} \quad (18)$$

wo C' wieder eine willkürliche Constante ist.

Es sei z_0 die Meereshöhe eines Lufttheilchens zur Zeit $t=0$, und w_0 seine Geschwindigkeit, so hat man

$$\begin{aligned} z_0 &= \frac{C'}{e^c - e^{-c}} \\ -w_0 &= -C'K \frac{(e^c + e^{-c})}{(e^c - e^{-c})^2} \end{aligned}$$

woraus folgt

$$\left(\frac{w_0 C'}{K z_0^2} \right)^2 - \left(\frac{z_0}{C'} \right)^2 = 4$$

aus welcher biquadratischen Gleichung C' zu bestimmen ist. Ist γ_0 der Werth von γ zur Zeit $t=0$, so dass

$$\gamma_0 = K \left(\frac{e^C + e^{-C}}{e^C - e^{-C}} \right)$$

so erhält man

$$C = \frac{1}{2} \log \left(\frac{K + \gamma_0}{\gamma_0 - K} \right)$$

Es muss hiernach zur Zeit $t=0$, als die verticale Strömung sich bildete, jedes Lufttheilchen bereits die Geschwindigkeit $W = -\gamma_0 z$ gehabt haben, und zwar eine solche, dass $\gamma_0 > K$ ist. Ist aber $\gamma_0 < K$ oder $=0$, so werden C' und C imaginär, d. h. die vertical herabsteigende Strömung erreicht eine endliche Meereshöhe erst nach dem Verfluss einer unendlich grossen Zeit, da γ gegen K , also gegen eine reelle Grösse doch convergirt, mochte C reell sein oder nicht. Wir denken uns: ein Lufttheilchen befinde sich zur Zeit $t=0$ in einer unendlich grossen Meereshöhe und habe eine unendlich grosse Zeit T gebraucht, um zur endlichen Meereshöhe z_0 zu gelangen. Nach (18) ist dann

$$z_0 = \frac{C'}{e^{KT+c} - e^{-KT-c}}$$

oder, da e^{-KT} verschwindet

$$C' = z_0 e^{KT+c}$$

Indem wir dieses in (18) einführen, erhalten wir

$$z = \frac{z_0}{e^{K(t-T)} - e^{-K(T+t)-2c}}$$

Nun verschwindet $e^{-K(T+t)}$, und $t-T$ ist die Zeit, welche das

Lufttheilchen gebraucht hat, um von einer Meereshöhe z_0 zur Meereshöhe z zu gelangen. Bezeichnet man diese Zeit mit t , so hat man,

$$z = z_0 e^{-kt}.$$

Wir sehen somit, dass in der endlichen Meereshöhe nur eine stationäre Strömung vertical abwärts entstehen kann, wenn k eine Constante ist, d. h. mit anderen Worten, dass, wenn eine Luftmasse allmählig niedersinkt, die so gebildete vertical herabsteigende Strömung nur als eine stationäre die endliche Meereshöhe erreichen kann, wenn die Differenz der Temperaturgradienten, die sie erzeugt, eine Constante ist.

Ich lasse hier einige Werthe von γ folgen, und für zwar für die Temperatur der untersten Luftschichte 0° und 30° , indem ich für g die Schwerkraft der Tokyo-Breite 9.798^m annehme.

$\gamma' - \gamma$ \ θ_0	$0^\circ C$	$30^\circ C$
0.000001	0.0001895 $\frac{1}{sec.}$	0.0001798 $\frac{1}{sec.}$
0.00001	0.0005991	0.0005068
0.00002	0.0008472	0.0007167
0.00003	0.001038	0.0008778
0.0001	0.0019164	0.0017973
0.001	0.0060602	0.0056866

Im Fall K^2 eine gegebene Function von t ist, lässt sich die Integration der Gleichungen (15) und (17) selten in geschlossener Form ausführen; sie lassen sich aber auf eine Gleichung zweiter Ordnung von einfacherer Form zurückführen. Man schreibe

$$\dot{r}^2 \pm \frac{d\dot{r}}{dt} = K^2$$

und setze $\dot{r} = \frac{\zeta}{\psi}$, und bestimme ψ so, dass

$$\frac{d\psi}{dt} = \pm \zeta,$$

so erhält man als Gleichung für ψ

$$K^2 \psi = \frac{d^2 \psi}{dt^2} \quad (19)$$

indem wir dieses integrieren, erhalten wir

$$\dot{r} = \pm \frac{\frac{d\psi}{dt}}{\psi} = \pm \frac{d \log \psi}{dt}$$

wo das obere Zeichen für die aufsteigende, und das untere für die niedersteigende Strömung gilt. Die Function ψ hat eine eigene Bedeutung. Es ist nämlich

$$\frac{dz}{dt} = \pm z \cdot \frac{d \log \psi}{dt}.$$

Hieraus folgt ohne weiteres

$$z = \text{Const. } \psi \quad (20)$$

oder

$$z = \frac{\text{Const}}{\psi} \quad (21)$$

Nicht jede Annahme über die Function K^2 führt zu einer möglichen Bewegungsform. Für ein imaginäres K hat man z. B.

$$\psi = A \cos kt + B \sin kt$$

wo A, B , zwei willkürliche Constanten sind. Es wird dann

$$\dot{r} = K \left(\frac{B \cos kt - A \sin kt}{A \cos kt + B \sin kt} \right)$$

Wohl kann $W = \gamma \cdot z$ an der Erdoberfläche verschwinden, so lange γ endlich ist. γ wird aber periodisch unendlich gross und w erhält an der Erdoberfläche die unbestimmte Form $0 \cdot \infty$, was endlich sein kann, und sonst unendlich gross. z wird aber $= 0$, d. h. jedes Lufttheilchen, welche Meereshöhe es zur Zeit $t=0$ gehabt haben mag, müsste in dieser Zeit zur Erdoberfläche angelangt sein. Eine solche Bewegung ist unmöglich.

Die Annahme $K^2 = \frac{1}{a+t}$ wo a eine Constante ist, führt zu einer möglichen Bewegung. Ein particuläres Integral der Gleichung

$$\frac{1}{a+t}, \psi = \frac{d^2 \psi}{dt^2}$$

ist

$$\psi = \sum_1^{\infty} \frac{(a+t)^n}{n^2! (n+1)}$$

eine unendliche Reihe, die für jedes endliche t convergirt. Es folgt hieraus

$$\gamma = \frac{\sum_1^{\infty} \frac{n(a+t)^{n-1}}{n^2! (n+1)}}{\sum_1^{\infty} \frac{(a+t)^n}{n^2! (n+1)}}$$

γ convergirt also gegen Null mit wachsender Zeit; die Geschwindigkeit der verticalen Strömung nimmt in jeder Luftschichte fortwährend ab. Im Fall der aufsteigenden Strömung ist nach (20)

$$z = z_0 \sum_1^{\infty} \frac{(a+t)^n}{n^2! (n+1)}$$

und im Fall der niedergehenden nach (21)

$$z = \frac{z_0}{\sum_1^{\infty} \frac{(a+t)^n}{n^2! (n+1)}}$$

In dem ersteren Fall nimmt die Meereshöhe eines Lufttheilchens fortwährend zu, in dem letzteren hingegen nimmt sie gegen 0 ab.

Zu einer solchen Transcendente führt die einfache Annahme, dass die Differenz der Temperaturgradienten sich wie $\frac{1}{(a+t)}$ verändere. γ kann daher nur durch eine äusserst complicirte Transcendente ausgedrückt werden, wenn man für K^2 eine Function annehmen würde welche dem wirklichen Verlaufe einer verticalen nicht stationären Strömung einigermaassen entsprechen könnte. Die Aufgabe wird indessen leicht, wenn wir uns γ als eine gegebene Function von t denken, und nach dem Gesetz fragen, gemäss dem die Differenz der verticalen Temperaturgradienten sich verändern muss, um die gegebene Geschwindigkeit der verticalen Strömung zu erzeugen.

Indess; auch hier führt nicht jede Annahme zu einem möglichen Ausdruck für die Differenz der verticalen Temperaturgradienten, da $\gamma^2 \pm \frac{d\gamma}{dt}$ für jeden Werth von t durchaus positiv und endlich sein muss.

Der Werth von γ ist, wie aus der oben stehenden Tabelle ersichtlich, sehr klein, und somit die Geschwindigkeit der verticalen Strömung; sie würde z. B. bei $\theta_0 = 30^\circ C$ und $\tau' - \tau = 0.001 C^\circ$ in der Meereshöhe 100 *m.* erst 6 *m.* betragen, und dies bei dem immer hin bedeutenden Unterschiede der Temperaturgradienten 0.1 C° pro 100 *m.* So geringfügig diese verticale Geschwindigkeit auch erscheint, so kann sie zu stärksten Wirbelstürmen Veranlassung geben. Nach der in pag. 180. Vol. I dieses Journals gegebenen Entwicklung ist die resultirende horizontale Geschwindigkeit des Windes an der Grenze des kreisförmigen Wirbelgebietes

$$\omega = \frac{\gamma R}{2} \sqrt{1 + \frac{4\lambda^2 \sin^2 \theta}{\kappa^2}}$$

wofür man, da $\frac{4\lambda^2 \sin^2 \theta}{\kappa^2}$ ungefähr gleich 1 gesetzt werden kann, schreiben kann

$$\omega = \frac{\gamma R}{\sqrt{2}}$$

Wenn der Durchmesser der Gebietes der verticalen Strömung 2 Kilometer ist, so ergibt sich z. B. für $\gamma = 0.0005686$ nur $0.402 \frac{\text{meter}}{\text{sec.}}$, was einen völlig unfehlbaren Wind bedeutet. Hat aber das Gebiet der verticalen Strömung einen Durchmesser von 200 Kilometer, so erreicht die Windgeschwindigkeit an der Grenze des Gebietes den Werth $40.2 \frac{\text{meter}}{\text{sec.}}$. Wenn die obenstehende Formel in der That einigermaßen der Wirklichkeit entspricht, so würde γ für einen grossen Wirbelsturm von mehreren Hundert Kilometer Durchmesser einen ausserordentlich kleinen Werth haben müssen, vorausgesetzt die Grenze der verticalen Strömung mit dem Ort der Maximalgeschwindigkeit zusammenfalle, was indess durchaus nicht ausgemacht ist, da die durch die obenstehende Gleichung gegebene Geschwindigkeit nur dann das Maximum ist, wenn $\gamma > 2\kappa$ ist, (vergleiche pag. 200 Vol. I dieses Journals).

Nun ist κ nach dem Werthe des beobachteten Deviationswinkels zu urtheilen (Randbemerkung pag. 148 Vol. I dieses Journals) eine Grösse von derselben Ordnung, wie 2λ , das Doppelte der Winkelgeschwindigkeit der Erdrotation, und entschieden kleiner, als diese. γ muss daher entschieden kleiner sein, als $4\lambda \sin \theta$, d. i. $0.000028988 \sin \theta \left(\frac{1}{\text{sec.}} \right)$, damit die Windgeschwindigkeit stetig nach dem Centrum des Wirbelgebietes zu abnehme. Dieses verlangt aber einen äusserst kleinen Werth der Differenz der verticalen Temperaturgradienten, und die Maximalgeschwindigkeit der Luft in dem ganzen Wirbelgebiete würde bei mässigem Durchmesser desselben so klein ausfallen,

dass die Bewegung der Luft so gut wie unfehlbar wäre. Nun ist die mechanische Kraft der kleineren Cyklonen, wie die americanischen Tornados, oft grösser, als diejenige der gewaltigen Wirbelstürme Westindiens. Zur Erklärung der Tornados und derartiger Wirbelbewegung der Luft müssen wir offenbar die Geschwindigkeit der verticalen Strömung grösser annehmen, als die Ungleichung $\gamma > 2\kappa$ sie giebt; also dass die Luftgeschwindigkeit statt nach dem Centrum des als kreisförmig gedachten Wirbelgebietes abzunehmen, in's unendliche zunimmt. Wie wir bereits (pag 190 Vol. I dieses Journals) gesehen haben, ist die resultirende horizontale Geschwindigkeit für $\gamma > 2\kappa$, in dem wir $\frac{\kappa}{\gamma} = m$, $\frac{2\lambda \sin \theta}{\gamma - \kappa} = K$ setzen

$$\omega^2 = \frac{\gamma^2 \rho^2}{4} \left\{ 1 + K^2 \left[\frac{1}{m} \left(\frac{R}{\rho} \right)^{2(1-m)} - 1 \right]^2 \right\}$$

oder

$$\omega^2 = \frac{\gamma^2 \rho^2}{4} + \frac{\gamma^2 K^2 R^2}{4} \left[\frac{1}{m} \left(\frac{R}{\rho} \right)^{1-2m} - \frac{\rho}{R} \right]^2 \quad (21a)$$

Das erste Glied in diesem Ausdruck nimmt stetig mit abnehmendem ρ ab, das zweite Glied aber in's Unendliche zu, und zwar um so schneller, je kleiner der echte Bruch m , d. h. je grösser γ gegen κ ist. Um ein Beispiel zu berechnen, setzen wir $\gamma = 0.001584$, was der Differenz der Temperaturgradienten $0.0000777^\circ \text{C}'$ bei $\theta_0 = 30^\circ \text{C}'$ entspricht, und schliesslich annäherungsweise $\frac{2\lambda \sin \theta}{\kappa} = 1$. Wenn wir ferner annehmen, dass der Halbmesser des Wirbelgebietes $1000m$, betrage, so erhalten wir, da $m = \frac{1}{10}$ ist, folgende Werthe für ω

	ω
$\rho = R$	2.24 $\frac{\text{meter}}{\text{sec.}}$
$\rho = \frac{R}{10}$	2.94
$\rho = \frac{R}{100}$	10.894
$\rho = \frac{R}{1000}$	26.66

Wenn wir weiter $\gamma = 0,01584$ setzen, so dass $m = \frac{1}{100}$ ist, dann erhalten wir

	ω
$\rho = R$	22.40 $\frac{\text{meter}}{\text{sec.}}$
$\rho = \frac{R}{10}$	79.50
$\rho = \frac{R}{100}$	800.
$\rho = \frac{R}{1000}$	8000.

Wie man sieht, steigt die Geschwindigkeit der Luft gegen das Centrum um so rascher, je kleiner der Bruch $\frac{\gamma}{r} = m$ ist. In dem ersteren Beispiel wächst sie von dem Grenzkreise aus in 900 m . von 2.24 m . auf nur 2.94 m . und in den weiteren 90 m . von 2.94 m . auf 10.88 m . um dann bei 1 m . Abstand von dem Centrum orkanartig zu werden. In dem letzteren Beispiele nimmt sie von dem Grenzkreis aus in 900 m . von 22.4 m . auf 79.50 m . um bei 90 m . Abstand vom Centrum die ungeheure Grösse 800 m . zu erreichen.

Der Begriff "Durchmesser" eines Wirbelstroms oder eines Tornado's ist ein höchst vager. Versteht man darunter den Durchmesser einer Luftgebietes, innerhalb dessen der Wind als orkanartig bezeichnet werden kann, so wird man dem Wirbel in dem ersten Beispiele, einen Durchmesser von nur ein Paar Dutzend Meter zuschreiben, ungeachtet der Luftwirbel durch eine verticale Strömung entstanden ist, deren Querschnitt mehrere Quadratkilometer beträgt. Der Wirbel in dem zweiten Beispiel hat hingegen schon einen Durchmesser von 1 Kilometer.

Wir sehen somit, dass der sogenannte Sturmdurchmesser desto grösser ausfällt, je grösser γ gegen 2κ ausfällt, dass derselbe mit dem Durchmesser des Gebietes der verticalen Strömung zusammenfallen muss, so bald $\gamma < 2\kappa$ wird. Denken wir uns sonach γ als eine Function, die stetig mit der Zeit abnimmt aber so langsam, dass sie während eines nicht all zu grossen Zeitintervalls als eine Constante angesehen werden kann; dass man daher innerhalb dieses Intervalls die Wirbelbewegung als eine stationäre ansehen kann. Wenn wir die Gleichung (15) für um ein grosses Zeitintervall auseinanderliegende Zeiten an einem und demselben Wirbel anwenden, so finden wir, dass der Sturmdurchmesser auch mit der Zeit abnimmt, so lange $\gamma > 2\kappa$ ist, und von dem Augenblick an, wo $\gamma > 2\kappa$ wird, wird der Ort der Maximalgeschwindigkeit sich in dem Umfang des Gebietes der verticalen Strömung herstellen, welche den Sturm veranlasst hat. Da diese Maximalgeschwindigkeit, wenn der Werth des R mässig ist, nur klein sein kann, so wird die Luftbewegung in dem Momente, wo $\gamma = 2\kappa$ wird, längst seinen sturmartigen Charakter eingeüsst haben. Anderes, wenn γ anfangs nicht allzu gross aber $> 2\kappa$, und R einen grossen Werth hat. So lange $\gamma > 2\kappa$ ist, wird der Sturmdurchmesser kleiner sein als R . Sinkt nun γ unter 2κ , so wird wieder das Maximum der Windgeschwindigkeit in dem Umfange der Gebietes der verticalen

Strömung stattfinden, und diese Maximalgeschwindigkeit, da R sehr gross ist, und K^2 sehr gross werden kann, noch eine Grösse haben, die noch als sturmartig bezeichnet werden kann. Denkt man sich hierzu noch die ganze wirbelnde Masse, durch irgend eine Ursache etwa von dem 7ten Breitengrade aus polwärts geführt und den Ausdruck (21a) noch angenähert gültig für eine niedrigere Breite, so wächst $2\lambda \sin \theta$ mit der höheren Breite etwa um das Achtefache des Werthes, welchen es beim 7ten Breitengrad gehabt hat, es kann daher ω selbst, wenn γ weit unter 2κ sinken würde, noch einen bedeutenden Werth haben.

Der Sturmdurchmesser wächst demnach mit der Zeit, während die Windgeschwindigkeit gleichzeitig abnimmt und der Deviationswinkel immer mehr wächst, wenn der Luftwirbel von der niederen Breite polwärts wandert. Ein Zahlenbeispiel möge dieses näher erläutern. Unter der Breite 8° sei ein Wirbel entstanden. Es sei dabei

$$\frac{2\lambda \sin 8^\circ}{\kappa} = 1$$

so dass
$$2\kappa = 0.0000403 \frac{1}{\text{sec.}}$$

und
$$\gamma = 0.0000453 \frac{1}{\text{sec.}}$$

sodass
$$\gamma > 2\kappa \quad \text{und} \quad \frac{\kappa}{\gamma} = m = 0.44$$

ist. Der Radius des Gebietes der verticalen Strömung sei 400,000 m.

Da in diesem Fall $\frac{2\lambda \sin \theta}{\kappa} = 1$ daher $K^2 = \left(\frac{2\lambda \sin \theta}{\kappa} \cdot \frac{\kappa}{\gamma \left(1 - \frac{\kappa}{\gamma} \right)} \right)^2$
 $= \frac{m^2}{(1-m)^2}$ ist, so hat man

$$\omega = 9.05 \sqrt{\left(\frac{\rho}{R} \right)^2 + (0.7857)^2 \left[\frac{1}{0.44} \left(\frac{R}{\rho} \right)^{0.12} - \frac{\rho}{R} \right]^2}$$

Hieraus berechnen sich folgende Werthe

ρ	$R = 400000 \text{ meter}$	ω
$\rho = 400000 \text{ m.}$	$\rho = R$	$12.81 \frac{\text{meter}}{\text{sec.}}$
$\rho = 40000 \text{ m.}$	$\rho = \frac{R}{10}$	20.68
$\rho = 4000 \text{ m.}$	$\rho = \frac{R}{100}$	27.9
$\rho = 400 \text{ m.}$	$\rho = \frac{R}{1000}$	37.03

Einem solchen Wirbelsturm würde man einen Durchmesser von 2×10 Kilometer zuschreiben. Wenn wir annehmen, dass, indem dieser Wirbel allmählig in den 70ten Breitengrad gelangt, γ etwa den Werth 0.00002 angenommen habe, so dass jetzt $\gamma < 2\kappa$ ist und das Maximum der Windgeschwindigkeit den Umfang des Gebietes der verticalen Strömung erreicht, hat so erhält man für dieses Maximum,

$$\begin{aligned} \omega &= \frac{400000 \times 0.00002}{2} \sqrt{1 + \left(\frac{2\kappa}{\pi} \sin 70^\circ \right)^2} \\ &= \frac{400000 \times 0.00002}{2} \sqrt{1 + \left(\frac{\sin 70^\circ}{\sin 8^\circ} \right)^2} = 20.355 \frac{\text{meter}}{\text{sec.}} \end{aligned}$$

Der Sturmdurchmesser beträgt denn jetzt ungefähr das Zehnfache von demjenigen unter dem 8ten Breitengrade, als wenn der Durchmesser des Wirbels sich um 2×360 Kilometer = 720 Kilometer ausgedehnt hätte.

Die Windgeschwindigkeit nimmt nach dem Centrum in's Unendliche zu, wenn $\gamma > 2\kappa$ ist; die von aussen her einströmende Luft gelangt gar nicht nach dem Centrum, sie wird, da der Druck nicht negativ in's Unendliche wachsen kann, in einer gewissen Entfernung von dem Centrum zerrissen. Da innerhalb dieser Entfernung

die Luft nur eine emporsteigende Bewegung haben und an der Wirbelbewegung ringsum nicht mehr theilnehmen kann, so muss, wenn $\gamma > 2\pi$ ist, ein Gebiet der Windstillen um das Centrum entstehen, welches aber verschwindet, sobald γ unter 2π sinkt. Indem wir dieses Gebiet der Windstillen näher zu bestimmen versuchen, wollen wir die Fläche, in der eine Flüssigkeit zerreisst, durch die Gleichung definiren

$$p = \text{Const.}$$

wo die Constante den Druck bedeutet, unter dem ein Gas aufhört, eine zusammenhängende Masse zu bilden. Für ideale incompressible Flüssigkeit könnten wir mit Von Helmholtz* diese Constante = 0 setzen, wenn bei der Bewegung, welche wir betrachten, das Geschwindigkeitspotential existirte; allein, da ein solches hier nicht existirt, wollen wir uns hier damit begnügen, anzunehmen, dass die Luft überall zerrissen werde, in der Fläche, wo die Druckverminderung einen gewissen durch den Zustand der Luft bestimmten Werth erreiche.

Wir haben als Druck für das innere wirbelerfüllte Gebiet gefunden [Gleichung (70) pag. 194. Vol. I dieses Journals]

$$\begin{aligned} \frac{p_i}{\rho} = & \text{Const} + G - \frac{\dot{r}^2}{2} r^2 + \dot{r}^2 \frac{(2m-1)}{8} \rho^2 \\ & + \frac{\dot{r}^2 \sin^2 \theta}{2m(m-1)^2} \left[m(2m-1) + \frac{2}{(2m-1)} \left(\frac{\rho}{R} \right)^{2m-4} - 3 \left(\frac{\rho}{R} \right)^{2m-2} \right]^\dagger \end{aligned}$$

Es sei p_0 der Druck an dem Umfange der kreisförmigen Wirbelgebietes. Wir erhalten dann durch Elimination der Grösse $\text{Const} + G$, indem wir zur Abkürzung setzen

* Helmholtz Über die discontinuirlichen Flüssigkeitsbewegungen. Gesammelte Abhandlungen Band I pag. 149.

† An dem angeführten Ort ist ein Druckfehler uncorrectirt geblieben. Der hier angegebene Ausdruck ist der richtige.

$$\frac{\gamma}{k \sin \theta} = n \quad k \sin \theta, R=N$$

$$\begin{aligned} \frac{p_o - p}{\mu N^2} = & \left[1 - \left(\frac{\rho}{R} \right)^2 \right] \left(\frac{2m-1}{2} \right) \left(\frac{n^2}{4} + \frac{1}{(m-1)^2} \right) + \frac{1}{m(m-1)^2(2m-1)} \times \\ & \times \left[1 - \left(\frac{\rho}{R} \right)^{2(2m-1)} \right] - \frac{3}{2m(m-1)^2} \left[1 - \left(\frac{\rho}{R} \right)^{2m} \right] + \frac{\gamma^2 z^2}{2 N^2} \end{aligned}$$

wobei $2m \leq 1$ und $m \geq 1$ sein muss. Im Fall $m=1$ oder $m=\frac{1}{2}$ ist, gilt der obige Ausdruck nicht mehr; sondern andere Ausdrücke, die auch leicht entwickelt werden können.

So lange $2m > 1$ ist, ist $p_o - p$ demnach überall positiv und endlich; denn das erste Glied, das den grössten Werth besitzt, ist positiv und endlich. Ist aber $2m < 1$, so verwandelt sich der obige Ausdruck in

$$\begin{aligned} \frac{p_o - p}{\mu N^2} = & - \left[1 - \left(\frac{\rho}{R} \right)^2 \right] \left(\frac{1-2m}{2} \right) \left(\frac{n^2}{4} + \frac{1}{(1-m)^2} \right) + \frac{1}{m(1-m)^2(1-2m)} \times \\ & \times \left[\left(\frac{R}{\rho} \right)^{2(1-2m)} - 1 \right] - \frac{3}{2m(1-m)^2} \left[1 - \left(\frac{\rho}{R} \right)^{2m} \right] + \frac{\gamma^2 z^2}{2 N^2} \quad (21a) \end{aligned}$$

$p_o - p$ wird hier in dem Centrum positiv unendlich gross d. h. p negativ unendlich gross. Es muss daher, da p von p bis $-\infty$ abnimmt, einen Werth von ρ geben, wo p den Werth annimmt, unter dem die Luft zerreiss. Bezeichnen wir diesen Druck mit P_o , so stellt die Gleichung

$$\begin{aligned} \frac{p_o - P_o}{\mu N^2} = & - \left[1 - \left(\frac{\rho}{R} \right)^2 \right] \left(\frac{1-2m}{2} \right) \left(\frac{n^2}{4} + \frac{1}{(1-m)^2} \right) + \frac{1}{m(1-m)^2(2m-1)} \times \\ & \times \left[\left(\frac{\rho}{R} \right)^{2(1-2m)} - 1 \right] - \frac{3}{2m(1-m)^2} \left[1 - \left(\frac{\rho}{R} \right)^{2m} \right] + \frac{\gamma^2 z^2}{2 N^2} \end{aligned}$$

die Fläche dar, an der die Luft zerreiss, und innerhalb deren die Luft

nur eine aufsteigende Bewegung hat.* Setzt man hierin $z = 0$, so erhält man den Radius eines Kreises, innerhalb dessen Windstille auf der Erdoberfläche herrscht. Dass ein solches Gebiet ganz im Inneren des Gebietes der verticalen Strömung liegt, ist leicht einzusehen. Da $\frac{p_o - P_o}{\mu N^2}$ positiv ist, und der rechter Hand stehende Ausdruck für $\frac{p}{R} = 1$, $z = 0$ verschwindet aber für $\left(\frac{p}{R}\right) = 0$ unendlich gross wird, so folgt, dass $\frac{p}{R}$ ein achter Bruch sein muss.

Die durch die Gleichung (21a) dargestellte Fläche ist ein Rotationshyperboloid höheren Grades; das Raumgebiet der Windstillen verbreitert sich mit der Meereshöhe, und erreicht die Grenz der verticalen Strömung, da wo

$$\frac{p_o - P_o}{\mu} = \frac{r^2 z^2}{2}$$

d. h.

$$rz = \sqrt{\frac{2(p_o - P_o)}{\mu}}$$

Die Geschwindigkeit der aufsteigenden Luft wäre dann diejenige, welche die Luft erhält, wenn sie aus einem Gefässe unter der Druckdifferenz $(p_o - P_o)$ strömt. Indessen, jede Schlussfolgerung hieraus ist werthlos, da, wenn auch $p_o - P_o$ ein Paar Centimeter Quecksilberdruck betragen sollte, z eine solche Meereshöhe darstellen würde, dass die Gültigkeit unserer Formeln als längst erloschen betrachtet werden muss.

Um ein Paar Beispiele berechnen zu können, setzen wir $p_o - P_o = 760 \text{ mm}$, Quecksilberdruck, was selbstredend in der Erdatmosphäre unmöglich ist. Im Fall

* William Ferrel (The Motions of fluids and solids on the earth's surface. Washington. 1882. pag. 26) hat durch eigenthümliche Behandlung der hydrodynamischen Gleichungen, als die isobarische Fläche für eine wirbelnde Luftmasse, eine Rotationsfläche vierten Grades abgeleitet und zwar ohne Berücksichtigung der verticalen Strömung.

$$\frac{2\lambda \sin \theta}{\kappa} = 1 \quad \frac{\kappa}{\tilde{r}} = \frac{1}{10} \quad R = 1000m.$$

haben wir dann für die Grenze der Windstillen.

$$273867 = 2.623 \left(\frac{\rho}{R} \right)^2 + 15.432 \left(\frac{R}{\rho} \right)^{\frac{5}{3}} \\ + 18.519 \left(\frac{\rho}{R} \right)^5$$

Hieraus findet man annäherungsweise

$$\frac{\rho}{R} = 0.00393. \quad d. i. \quad \rho = 3.93 m.$$

Die Geschwindigkeit der Luft ist dabei

$$\omega = 67.2 \frac{meter}{sec.}$$

In einem solchen Wirbel müsste der Wind in der Entfernung 4 Meter von dem Centrum plötzlich aufhören.

In dem Fall $\frac{\kappa}{\tilde{r}} = \frac{1}{100}$ hat man hingegen

$$276597 = 612.5 \left(\frac{\rho}{R} \right)^2 + 2004.07 \left(\frac{R}{\rho} \right)^{2-\frac{1}{5}} \\ + 150 \left(\frac{\rho}{R} \right)^{\frac{1}{5}}$$

Hieraus findet man angenähert

$$\frac{\rho}{R} = 0.0569 \quad d. h. \quad \rho = 56.9 m.$$

Die Geschwindigkeit der Luft ist dabei

$$131.4 \frac{meter}{sec.}$$

Das Gebiet der Windstille ist grösser, als in dem ersten Beispiel. In einem solchen Wirbel würde die Windgeschwindigkeit allmähig

gegen das Centrum zunehmen, bis sie bei 57 *m.* Entfernung den Werth $131. \frac{\text{meter}}{\text{sec.}}$ erreicht, und die Windstille plötzlich eintritt.

Wie man sieht, ist man im Stande, den ganzen Verlauf der kleineren Wirbelbildungen in der Erdatmosphäre, wie Wettersäulen, und Tornados, vollständig zu schildern, wenn man nur annimmt, dass $\gamma > 2\kappa$ sei. Bei den Wettersäulen ist so gut, wie sicher, dass die Geschwindigkeit der verticalen Strömung, die sie veranlasst, eine bedeutende Grösse hat, wie denn mitemporgerrissene Körper Schraubenbahnen beschreiben, deren Ganghöhe gegen die Windung oft so bedeutend ist, dass die letztere gegen die erstere fast verschwindet. Dasselbe gilt auch für die Tornados: denn die Windbahnen, wie sie Loomis† und Andere nach der Richtung der umgerissenen Bäumen gezeichnet haben, tragen ohne Ausnahme einen auffallend centripetalen Charakter, was auf eine grosse Geschwindigkeit der aufsteigenden Strömung hindeutet. Wenn die Annahme $\gamma > 2\kappa$ ferner den Umstand leicht und ungezwungen erklärt, dass die sturmartige Aufregung der Luft so wohl bei den Wellersäulen, als bei den Tornados, auf ein kleines ziemlich scharf markirtes Gebiet beschränkt ist, dass die Luft in einiger Entfernung von dem Centrum des Wirbels wenig oder gar nicht in Mitleidenschaft gezogen ist, so glaube ich mich zu dem Schluss berechtigt, dass nur die Zulassung der Grösse γ , welche grösser ist, als 2κ , im Stande ist, Wellersäulen, und Tornados in ihrem ganzen Habitus zu erklären.

Bei den eigentlichen Wirbelstürmen scheint der unter niederen Breitengraden fast immer beobachtete schroffe Übergang des Sturms zur sogenannten "Todtenstille" darauf hinzudeuten, dass man γ auch hier einen grösseren Werth beizulegen habe, als 2κ . Es lässt sich, wie wir oben sahen, nicht nur das Wachsen des Sturmdurchmessers mit

† z. B. der Fornado von Stow in Ohio am 20ten October 1837. (Reye, "die Wirbelstürme" pag. 69.

der höheren Breite ungezwungen erklären, sondern auch der Umstand, dass eine völlige Windstille im Innern eines Wirbelsturms fast nur unter den Tropen vorkommt; also da, wo er entstanden war, und r seine volle Grösse besitzt; dass in den höheren Breiten nur eine Lull, nicht eine Calm im Innern eines Wirbelsturms vorzukommen pflegt. Unter den Tropen, wo die aufsteigende Strömung noch seine volle Geschwindigkeit hat, wird die Fläche, innerhalb deren der Wind orkanartig ist, nur eine beschränkte sein und das Gebiet der Windstille um so grösser, je grösser r gegen 2κ ist. Wenn nun r allmählig abnimmt, während der Wirbel polwärts wandert, dehnte sich das Sturmgebiet weiter und weiter nach der Begrenzung des Gebietes der verticalen Strömung aus, die den Wirbel veranlasst hat, während die Windgeschwindigkeit in dem Sturmgebiete abnimmt, verschwindet das Gebiet der Windstille in dem Grade, wie r sich 2κ nähert. Sinkt r nun unter 2κ , so erreicht das Sturmgebiet seine Maximalausdehnung, vorausgesetzt, dass der Querschnitt der verticalen Strömung sich mit der Zeit nicht geändert habe; es findet nur eine allmähliche Abnahme der Windgeschwindigkeit nach dem Centrum statt, eine Lull, und das Gebiet der Windstillen ist verschwunden.

Die durch die Gleichung (12) bestimmte verticale Strömung der Luft erleidet auch eine Ablenkung durch die Erdrotation. Wenn auch die Bewegungsgleichungen eines vertical bewegten Lufttheilchens unter dem Einfluss der Erdrotation leicht entwickelt werden können, begnügen wir uns hier mit der Annahme, dass die Bewegung in höherer Breite vor sich gehe, dass daher der Einfluss der Erdrotation auf die verticale Strömung als verschwindend angesehen werden könne.

Er ist aber nöthig näher zu untersuchen, einen anderen Umstand, von dem die Zulässigkeit der Annahmen wesentlich abhängt, unter denen wir unsere Bewegungsgleichungen abgeleitet haben. Da die emporsteigende Luft sich ausdehnen muss, so kann, wie Reye* mit Recht bemerkt hat, die Gestalt des Raumgebietes der verticalen Strömung unmöglich eine Cylindrische sein, sondern eine konische wie man häufig an den Wettersäulen und auch an den Tornados beobachtet hat. Es fragt sich nun, wie weit man das Raumgebiet der verticalen Strömung als cylindrisch ansehen kann, wenn eine Luftmasse entweder theilweise oder völlig gesättigt von einem gegebenen Gebiete auf der Erdoberfläche aus emporsteigt und so eine verticale Strömung bildet.

Wir denken uns zu dem Ende das Gebiet auf der Erdoberfläche, von dem aus eine dampfgesättigte Luft emporsteigt, durch eine gegebene Curve begrenzt, deren Gleichung,

$$f(x, y, a, b) = 0 \quad (22)$$

wo a, b , zwei Längen sind, welche die Dimension dieses Gebietes bestimmen.

Es sei ϱ der Querschnitt des Raumgebietes in der Meereshöhe z und ϱ_0 ein solcher auf der Erdoberfläche. Man denke sich bei z einen um dz entfernten Querschnitt, so dass das Volumen zwischen diesen beiden Querschnitten $= \varrho dz$ ist. Wenn M das specifische Gewicht der Luft bei der Temperatur θ , welche in der Meereshöhe z herrscht, und unter dem Druck p bedeutet, und Q die Dampfmenge in 1 Cubikmeter Luft bei derselben Temperatur, so geht durch den Querschnitt z die Masse

$$(M + Q) \varrho \quad (23)$$

* Reye "die Wirbelstürme, etc." pag. 226.

Indem die Luft durch den Querschnitt $z + dz$ geht, verändert sich M um $\frac{dM}{dz} dz$, Q vermindert sich um $\frac{dQ}{dz} dz$ durch Condensation, und es verändert sich Ω um $\frac{d\Omega}{dz} dz$. Die Masse, welche durch den Querschnitt $z + dz$ geht, ist demnach

$$\left(\Omega + \frac{d\Omega}{dz} dz\right) \left(M + \frac{dM}{dz} dz + Q + \frac{dQ}{dz} dz\right) \quad (24)$$

wenn das condensirte Wasser mit emporrissen wird. Soll die Massenbewegung in jeder Luftschichte eine stetige sein, so muss die Differenz der Ausdrücke (23) und (24) = 0 sein d. h.

$$\Omega \left(\frac{dM}{dz} \frac{dQ}{dz}\right) dz + \left(M + Q\right) \frac{d\Omega}{dz} dz = 0$$

Hieraus folgt

$$\Omega = \frac{Const}{M + Q}$$

Bezeichnet der Index o den Werth der betreffenden Grösse an der Erdoberfläche, so hat man

$$\frac{\Omega}{\Omega_o} = \frac{M_o + Q_o}{M + Q} \quad (25)$$

Etwas anderes wird die Sache, wenn das condensirte Wasser in tropfbar-flüssiger Form niederfällt, und die Masse, welche durch den Querschnitt $z + dz$ geht, sich thatsächlich um die Menge Dampf vermindert hat, welcher condensirt wurde. In diesem Fall ist $\frac{dQ}{dz} dz$ in (24) negativ zu setzen und die Differenz der Ausdrücke (23) und (24) gleich der condensirten Dampfmenge, welche etwa als Regen niederfällt, d. h. $= \frac{dQ}{dz} \Omega dz$

Wir haben somit

$$- \Omega \left(\frac{dM}{dz} - \frac{dQ}{dz} dz - (M+Q) \frac{dQ}{dz} dz = \frac{dQ}{dz} \Omega dz \right.$$

$d, h,$

$$\Omega \frac{dM}{dz} + (M+Q) \frac{d\Omega}{dz} = 0$$

Hieraus folgt,

$$\int_0^z \frac{\frac{dM}{dz} dz}{M+Q} = \log \left(\frac{\Omega_0}{\Omega} \right) \quad (26)$$

Denkt man sich θ aus einer Beziehung zwischen θ und z eliminiert, so erhält man, so wohl aus (25) als aus (26) $\frac{\Omega}{\Omega_0}$ als eine Function von z , etwa $F(z)$. Da jedes Flächenelement $d\Omega = dx dy$ sich im Verhältniss $1 : F(z)$ verändert hat, so verändern sich a und b in (22) im Verhältniss $1 : \sqrt{F(z)}$. Mithin erhält man

$$f(x, y, a \sqrt{F(z)}, b \sqrt{F(z)}) = 0$$

als Gleichung der Gestalt der verticalen Strömung. Es findet eine Verbreiterung nach Oben zu statt, weil $M+Q$ gewöhnlich mit wachsender Meereshöhe abnimmt. Da Q immer sehr klein ist gegen M , wollen wir diese Grösse vernachlässigen. Es kommt dann sowohl aus (25), wie aus (26):

$$\frac{\Omega}{\Omega_0} = \frac{M_0}{M}$$

Nun ist

$$M = \frac{a \rho p}{760(a+\theta)}$$

oder nach (11)

$$M = \frac{a \rho p_0 e}{760(a+\theta)} - \int_0^z \left(\frac{R}{R+z} \right)^2 \frac{dz}{\pi(a+\theta)}$$

wo ρ das specifische Gewicht der Luft im Normalzustand und $a = 273$ ist, so folgt

$$\frac{\Omega}{\Omega_0} = \left(\frac{a + \theta}{a + \theta_0} \right) e^{-\int_0^z \left(\frac{R}{R+z} \right)^2 \frac{dz}{H(a + \theta)}}$$

H verändert sich mit dem Dampfgehalt der Luft und darum mit der Meereshöhe z . θ ist, wenn die Luft mit Dampf gerättigt ist, eine höchst complicirte Function von z .† Wir wollen uns indessen hier auf die Betrachtung einer mässigen Meereshöhe beschränken und setzen

$$\left(\frac{R}{z + R} \right)^2 = 1 \quad \theta_0 - \theta = \tau, z.$$

Wenn wir ferner für H den Werth geben, den es an der Erdoberfläche hat, so erhalten wir

$$\frac{\Omega}{\Omega_0} = \left(\frac{a + \theta_0}{a + \theta} \right)^{\frac{1}{\tau H} - 1} = \frac{1}{\left(1 - \frac{\tau, z}{a + \theta_0} \right)^{\frac{1}{\tau H} - 1}}$$

Da $\frac{1}{a + \theta_0}$ gewöhnlich ein sehr kleiner Bruch ist, so hat man angenähert

$$\frac{\Omega}{\Omega_0} = \frac{1}{(1 - \kappa z)} \quad (26a)$$

$$\text{wo} \quad \kappa = \frac{1}{a + \theta_0} \left(\frac{1}{H} - \tau \right) \text{ ist.}$$

Die Zunahme des Querschnittes der verticalen Strömung mit z erfolgt demnach sehr langsam, und zwar um so langsamer, je höher die Temperatur auf der Erdoberfläche ist und je geringer der Dampf-

† Vergleiche W. Thomson. Mathematical-physical Papers Vol. III. pag. 255.

gehalt, da τ mit der Feuchtigkeit im Allgemeinen abnimmt. So hat man z. B. für $\theta_0 = 0$ $H = 30$ $\tau = 0.0055$.

$$\text{für } \theta_0 = 0 \quad \kappa = 0,0001018$$

für $\theta_0 = 30$

$$\kappa = 0,0000918$$

Für $z = 1000 \text{ m.}$ würde sich als die mittlere Breite der verticalen Strömung ergeben

$$\text{für } \theta_0 = 0^\circ \quad \sqrt{\frac{Q}{Q_0}} = \sqrt{\frac{1}{1-0,1018}} = 1,0552$$

$$\text{für } \theta_0 = 30^\circ \quad \sqrt{\frac{Q}{Q_0}} = \sqrt{\frac{1}{1-0,0918}} = 1,0493$$

Wenn wir die Fläche als ein Kreiskegel denken, so erhält man als die Tangente des halben Öffnungswinkels

$$\text{für } \theta_0 = 0^\circ \quad 0,0000552$$

$$\text{für } \theta_0 = 30^\circ \quad 0,0000493$$

woraus sich so kleine Öffnungswinkel ergeben, dass man wohl das Raumgebiet der verticalen Strömung bis zur Meereshöhe 1000 m. als rein cylindrisch betrachten darf, wenn τ den Werth $0,0055$ hat, wie es meistens ungefähr der Fall ist.

Ganz ähnlich verhält sich die Sache, wenn eine kalte dampfarme Luftmasse niedersteigt und so eine vertical niedersteigende Strömung bildet. Sie kann dabei, indem sie sich nach unten zu erwärmt, Wasserdampf von der verdrängten Luft aufnehmen. Allein; wir sehen davon ab, und erhalten wieder als Bedingung der Continuität der bewegten Masse

$$\frac{dM Q}{dz} = 0$$

d. h. wieder

$$\varrho M \equiv M_0 \varrho_0$$

oder

$$\frac{\varrho}{\varrho_0} = \frac{M_0}{M}$$

wo ϱ_0 , M_0 den Werth der betreffenden Grösse in der Meereshöhe bedeutet, von der herab diese Luftmasse sinkt. Da M mit abnehmender Meereshöhe zunimmt, so nimmt $\frac{\varrho}{\varrho_0}$ auch nach der Erdoberfläche zu ab, d. h. das Raumgebiet der vertical herabsteigenden Strömung verengt sich nach Unten zu.

Ich bemerke noch, dass W. Ferrel* durch eine ihm eigenthümliche geistreiche Combination einfacher Gesetze für die Gestalt der Wettersäulen ein Rotationshyperboloid, wie unsere Gleichung (26a) ergeben würde, gefunden hat.

§ XIV.—Nichtstationäre Wirbelbildungen in der Erdatmosphäre.

Nach dem wir die physikalische Bedeutung der Grösse γ ermittelt haben, wenden wir uns zu der Aufgabe, Wirbelbewegung der Luft zu finden, wenn γ eine gegebene Function von der Zeit ist.

Die Bewegungsgleichungen für das äussere Gebiet sind ;

$$J\varphi_a = 0$$

$$\begin{aligned} \frac{\partial JW_a}{\partial t} + \frac{\partial JW_a}{\partial x} \left(\frac{\partial \varphi_a}{\partial x} + \frac{\partial W_a}{\partial y} \right) + \frac{\partial JW_a}{\partial y} \left(\frac{\partial \varphi_a}{\partial y} - \frac{\partial W_a}{\partial x} \right) \quad (27) \\ + \kappa JW_a = 0 \end{aligned}$$

und für das innere Gebiet

* W. Ferrel: Recent Advances in the Meteorology Washington. 1886 pag. 299.

$$\begin{aligned} \frac{\partial \mathcal{J}W_i}{\partial t} + \frac{\partial \mathcal{J}W_i}{\partial x} \left(\frac{\partial \varphi_i}{\partial x} + \frac{\partial W_i}{\partial y} \right) + \frac{\partial \mathcal{J}W_i}{\partial y} \left(\frac{\partial \varphi_i}{\partial y} - \frac{\partial W_i}{\partial x} \right) \\ + (\kappa \mp \gamma) \mathcal{J}W_i \pm 2\lambda \sin \theta \gamma = 0 \end{aligned} \quad (28)$$

$$\mathcal{J}\varphi_i = \mp \gamma.$$

wo das obere Vorzeichen für die vertical aufsteigende und das untere für die vertical niedersteigende Strömung gilt.

Da in diesen Gleichungen $\frac{\partial \varphi}{\partial t}$ nicht vorkommt, so beeinträchtigt die Abhängigkeit der Grösse γ von der Zeit auf keinerlei Weise die specialisirenden Annahmen, unter denen wir diese Gleichungen abgeleitet haben; sie gelten auch für ein mit der Zeit veränderliches γ ohne Weiteres. Da ferner die Gleichung der Massencontinuität für incompressible Flüssigkeit von t völlig unabhängig ist, so gilt sie auch hier in unveränderter Form, in so ferne als wir fortfahren, die Luft als incompressible zu betrachten.

Wir wollen zunächst näher untersuchen, ob die Bewegung der Luft auch eine nichtstationäre Form annehmen könne, wenn γ zu irgend welcher Zeit constant werden würde. Wir denken uns zu dem Ende das unendlich Luftgebiet begrenzt, theils durch einen Cylinder vom unendlich grossen Radius, theils in der Endlichkeit durch das cylindrische im übrigen beliebig gestaltete Raumbgebiet der verticalen Strömung. Es soll die Luft in der Unendlichkeit überall ruhen so dass

$$\begin{aligned} \frac{\partial \varphi_a}{\partial t} = 0 \quad \frac{\partial \varphi_a}{\partial z} = \frac{\partial \varphi_a}{\partial y} = 0 \quad \frac{\partial W_a}{\partial x} = \frac{\partial W_a}{\partial y} = 0 \\ \frac{\partial \mathcal{J}W_a}{\partial x} = \frac{\partial \mathcal{J}W_a}{\partial y} = 0 \quad \frac{\partial \mathcal{J}W_a}{\partial t} = 0 \end{aligned}$$

Die Function φ und ihre ersten Differentialquotienten sollen überall stetig endlich und eindeutig sein. Von der Function W

setzen wir voraus, dass sie überall eindeutig sei, dass ihre Differentialquotienten überall sonst stetig und endlich, aber in einem Punkt a, b innerhalb des Gebietes der verticalen Strömung unendlich gross werden könne, aber so, dass das Integral

$$\int \mathcal{J} W d\omega$$

über einen unendlich kleinen Cylinder um den Punkt a, b ausgedehnt entweder verschwindet oder einer endlichen Constante gleich wird, die aber noch die Zeit enthalten kann. Wir denken uns um diesen Punkt einen Kreis beschrieben mit dem verschwindendem Radius (ε). Da in diesem unendlich kleinen Kreisgebiete $\mathcal{J} W$ als eine Function von dem Radius ρ allein betrachtet werden kann, so hat man

$$\int \mathcal{J} W d\omega = \int_0^{2\pi} d\chi \int_0^\varepsilon \frac{1}{\rho} \frac{d}{d\rho} \left(\rho \frac{dW}{d\rho} \right) \rho d\rho = 2\pi \left[\left(\rho \frac{dW}{d\rho} \right)_\varepsilon - \left(\rho \frac{dW}{d\rho} \right)_0 \right]$$

Soll daher $\int \mathcal{J} W d\omega$ mit verschwindendem ε verschwinden, so kann dies geschehen, wenn $\rho \frac{dW}{d\rho}$ mit verschwindendem ε verschwindet, d. h. $\frac{dW}{d\rho}$ so unendlich gross in dem Punkt a, b wird, dass es gegen $\frac{1}{\varepsilon}$ unendlich klein ist.

Es lässt sich nun heraus schliessen, dass die Functionen $\mathcal{J} W$, $\frac{\partial \mathcal{J} W}{\partial t}$, $\frac{\partial^2 \mathcal{J} W}{\partial t^2}$ wenn sie in dem Punkte a, b unendlich werden, unendlich klein gegen $\frac{1}{\varepsilon^2}$ sein müssen, wo ε eine verschwindende Grösse ist, falls $\rho \frac{dW}{d\rho}$ sich überall nur stetig mit der Zeit ändert. Differentirt man die Gleichung (27) nach t , unter der Rücksichtnahme der Bedingung, dass r von t unabhängig sei

$$d. h. \quad \frac{\partial \varphi}{\partial t} = 0$$

so erhält man, indem wir $\frac{\partial W}{\partial t} = \psi$ setzen,

$$\begin{aligned} \frac{\partial \mathcal{J} \psi_i}{\partial t} + \frac{\partial W_i}{\partial x} \frac{\partial \psi_i}{\partial y} - \frac{\partial W_i}{\partial y} \frac{\partial \psi_i}{\partial x} + \left(\frac{\partial W_i}{\partial y} + \frac{\partial \psi_i}{\partial x} \right) \frac{\partial \mathcal{J} \psi_i}{\partial x} \\ + \left(\frac{\partial \varphi_i}{\partial y} - \frac{\partial W_i}{\partial x} \right) \frac{\partial \mathcal{J} \psi_i}{\partial y} + (\kappa \mp \gamma) \mathcal{J} \psi_i = 0 \end{aligned} \quad (29)$$

Innerhalb des unendlich kleinen um den Punkt a, b beschriebenen Cylinders verwandelt sich diese Gleichung in,

$$\frac{\partial \mathcal{J} \psi_i}{\partial t} + \frac{\partial \psi_i}{\partial \rho} \frac{\partial \mathcal{J} \psi_i}{\partial \rho} + (\kappa \mp \gamma) \mathcal{J} \psi_i = 0$$

Nun verschwindet $\frac{\partial \varphi_i}{\partial \rho}$ in einem solchen Gebiete, wie (ε). Soll daher diese Gleichung auch in dem Punkt a, b erfüllt sein, so müssen die Grössen $\frac{\partial \mathcal{J} \psi_i}{\partial t}$, $\frac{\partial \varphi_i}{\partial \rho}$, $\frac{\partial \mathcal{J} \psi_i}{\partial \rho}$, von derselben Grössenordnung sein, wie $\mathcal{J} \psi_i$. Da $\frac{\partial \mathcal{J} \psi_i}{\partial t}$ von derselben Ordnung ist, wie $\mathcal{J} \psi_i$, so muss $\frac{\partial \varphi_i}{\partial \rho}$, $\frac{\partial \mathcal{J} \psi_i}{\partial \rho}$ auch unendlich klein sein gegen $\frac{1}{\varepsilon^2}$. Mithin folgt, dass $\frac{\partial \mathcal{J} \psi_i}{\partial \rho}$ unendlich klein sein muss gegen $\frac{1}{\varepsilon^3}$, dass daher das Integral

$$\int \frac{\partial \mathcal{J} \psi_i}{\partial t} d\omega + \int \frac{\partial \psi_i}{\partial \rho} \frac{\partial \mathcal{J} \psi_i}{\partial \rho} d\omega + (\kappa \mp \gamma) \int \mathcal{J} \psi_i d\omega$$

ausgedehnt über die verschwindende Fläche um den Punkt a, b , verschwinden muss.

Für einen Punkt auf der Grenzcurve der verticalen Strömung kann man die Gleichung (28) in eine andere Form bringen. Be-

zeichnet man mit n die nach Innen gezogene Normale an der Grenzcurve und setzt man

$$\alpha = \cos (nx) \quad \beta = \cos (ny)$$

so ist

$$\begin{aligned} \frac{\partial W_i}{\partial x} &= \frac{\partial W_i}{\partial n} \alpha & \frac{\partial W_i}{\partial y} &= \frac{\partial W_i}{\partial n} \beta \\ \frac{\partial W_i}{\partial x} &= \frac{\partial W_i}{\partial n} \alpha & \frac{\partial W_i}{\partial y} &= \frac{\partial W_i}{\partial n} \beta \\ \frac{\partial \varphi_i}{\partial x} &= \frac{\partial \varphi_i}{\partial n} \alpha & \frac{\partial \varphi_i}{\partial y} &= \frac{\partial \varphi_i}{\partial n} \beta \end{aligned} \quad (30)$$

Wenn wir annehmen, dass die Grenzcurve der verticalen Strömung sich nicht ändere, was wir, wie ich glaube, annehmen können, da die Geschwindigkeit der verticalen Strömung sich nicht mit der Zeit ändern d. h. die Temperaturvertheilung in—und ausserhalb der verticalen Strömung dieselbe bleiben soll, so haben wir auch

$$\begin{aligned} \frac{\partial \psi_i}{\partial x} &= \frac{\partial \psi_i}{\partial n} \alpha & \frac{\partial \psi_i}{\partial y} &= \frac{\partial \psi_i}{\partial n} \beta \\ \frac{\partial \psi_i}{\partial x} &= \frac{\partial \psi_i}{\partial n} \alpha & \frac{\partial \psi_i}{\partial y} &= \frac{\partial \psi_i}{\partial n} \beta \end{aligned} \quad (31)$$

Die Gleichung (29) erhält daher für einen Punkt der Grenzcurve die Form

$$\frac{\partial \bar{\psi}_i}{\partial t} + \frac{\partial \bar{\varphi}_i}{\partial n} \frac{\partial \bar{\psi}_i}{\partial n} + (\kappa \mp \gamma) \bar{\psi}_i = 0 \quad (32)$$

Durch dieselbe Behandlung der Gleichung (27) erhält man für einen Punkt der Grenzcurve

$$\frac{\partial \mathcal{J} \bar{\psi}_n}{\partial t} + \frac{\partial \bar{\varphi}_n}{\partial n} \frac{\partial \mathcal{J} \bar{\psi}_n}{\partial n} + \kappa \mathcal{J} \bar{\psi}_n = 0 \quad (33)$$

Er sei $d\omega_i$ ein Querschnitelement der verticalen Strömung, und du ein Element des Umfangs der Grenzcurve. Indem wir die Gleichung mit $d\omega_i$ multipliciren, und uns der Formel erinnern

$$\frac{\partial V \frac{\partial U}{\partial x}}{\partial y} - V \frac{\partial^2 U}{\partial x \partial y} = \frac{\partial V}{\partial y} \frac{\partial U}{\partial x}$$

erhalten wir

$$\begin{aligned} 0 = & \int \frac{\partial \mathcal{J} \psi_i}{\partial t} d\omega_i + \int \frac{\partial \psi_i \frac{\partial \mathcal{J} W_i}{\partial x}}{\partial y} d\omega_i - \int \frac{\partial \psi_i \frac{\partial \mathcal{J} W_i}{\partial y}}{\partial x} d\omega_i \\ & + \int \frac{\partial W_i \frac{\partial \mathcal{J} \psi_i}{\partial x}}{\partial y} d\omega_i - \int \frac{\partial W_i \frac{\partial \mathcal{J} \psi_i}{\partial y}}{\partial x} k \omega_i \\ & + \int \frac{\partial \varphi_i \frac{\partial \mathcal{J} \psi_i}{\partial x}}{\partial x} d\omega_i + \int \frac{\partial \varphi_i \frac{\partial \mathcal{J} \psi_i}{\partial y}}{\partial x} d\omega_i \\ & + \int \varphi_i \mathcal{J} \mathcal{J} \psi_i d\omega_i + (\kappa \mp \gamma) \int \Delta \psi_i d\omega_i \end{aligned} \quad (33a)$$

Integrale von der Form $\int \frac{\partial \psi_i \frac{\partial \mathcal{J} W_i}{\partial x}}{\partial y} d\omega_i$ lassen sich nun leicht in die Integrale umsetzen, welche über den Umfang der Grenzcurve auszudehnen sind, wenn die Function wie $\psi_i \frac{\partial \mathcal{J} W_i}{\partial x}$ überall eindeutig, stetig und endlich wäre. Da wir der Allgemeinheit halber annehmen müssen, dass es innerhalb des Gebietes der verticalen Strömung einen- oder mehrere Unstetigkeitspunkte geben könne, so denken wir uns diese Unstetigkeitspunkte dadurch ausgeschieden, dass wir um sie Kreise mit einem unendlich kleinen Radius beschreiben. Dann sind alle

die in Frage kommenden Functionen überall endlich in dem übrig bleibenden Gebiete. Da aber der Betrag der Flächenintegrale der von den Unstetigkeitspunkten herrührt, mit verschwindendem Radius der umschriebenen Kreise verschwindet, so sieht man ein, dass die Flächenintegrale (33a) in Umfangsintegrale umgesetzt werden können ohne jede Rücksicht auf die Punkte, in denen die Functionen hätten unendlich gross werden können. Diese Bemerkung ergibt

$$\begin{aligned}
 0 = & -\frac{\partial}{\partial t} \int \frac{\partial \mathcal{J} \bar{\psi}_i}{\partial n} du - \int \bar{\psi}_i \frac{\partial \mathcal{J} \bar{W}_i}{\partial x} \beta du + \int \bar{\psi}_i \frac{\partial \mathcal{J} \bar{W}_i}{\partial y} \alpha du \\
 & - \int \bar{W}_i \frac{\partial \mathcal{J} \bar{\psi}_i}{\partial x} \beta du + \int \bar{W}_i \frac{\partial \mathcal{J} \bar{\psi}_i}{\partial y} \alpha du \\
 & - \int \bar{\psi}_i \frac{\partial \mathcal{J} \bar{\psi}_i}{\partial x} \alpha du - \int \bar{\psi}_i \frac{\partial \mathcal{J} \bar{\psi}_i}{\partial y} \beta du \\
 & - (\alpha \mp \gamma) \int \frac{\partial \bar{\psi}_i}{\partial n} du + \int \varphi_i \mathcal{J} \mathcal{J} \psi_i d\omega_i
 \end{aligned}$$

Da entlang der Grenzcurve die Gleichungen (30) und (31) gelten, so heben sich vier Glieder paarweise auf. Es bleibt nur, da $\alpha^2 + \beta^2 = 1$ ist

$$\begin{aligned}
 0 = & -\frac{\partial}{\partial t} \int \frac{\partial \mathcal{J} \bar{\psi}_i}{\partial n} du - \int \bar{\psi}_i \frac{\partial \mathcal{J} \bar{\psi}_i}{\partial n} du - (\alpha \mp \gamma) \int \frac{\partial \bar{\psi}_i}{\partial n} du \\
 & + \int \varphi_i \mathcal{J} \mathcal{J} \psi_i d\omega_i
 \end{aligned}$$

Es ist nun

$$- \int \varphi_i \frac{\partial \mathcal{J} \bar{\psi}_i}{\partial n} du + \int \varphi_i \mathcal{J} \mathcal{J} \psi_i d\omega_i = \int \left(\frac{\partial \varphi_i}{\partial x} \frac{\partial \mathcal{J} \psi_i}{\partial x} + \frac{\partial \varphi_i}{\partial y} \frac{\partial \mathcal{J} \psi_i}{\partial y} \right) du_i$$

und ferner

$$\frac{\partial \mathcal{J} \psi_i \frac{\partial \varphi_i}{\partial x}}{\partial x} - \mathcal{J} \psi_i \frac{\partial^2 \varphi_i}{\partial x^2} = \frac{\partial \mathcal{J} \psi_i}{\partial x} \frac{\partial \varphi_i}{\partial x}$$

$$\frac{\partial \mathcal{I} \psi_i \frac{\partial \varphi}{\partial y}}{\partial y} - \mathcal{I} \psi_i \frac{\partial^2 \varphi}{\partial y^2} = \frac{\partial \mathcal{I} \psi_i}{\partial y} \frac{\partial \varphi_i}{\partial y}$$

Hieraus folgt

$$\begin{aligned} & - \int \overline{\varphi_i} \frac{\partial \Delta \overline{\psi_i}}{\partial n} du - \int \overline{\varphi_i} \mathcal{I} \mathcal{I} W_i d\omega_i \\ & = \int \frac{\partial \mathcal{I} \psi_i \frac{\partial \varphi_i}{\partial x}}{\partial x} d\omega_i + \int \frac{\partial \mathcal{I} \psi_i \frac{\partial \varphi_i}{\partial y}}{\partial y} d\omega_i - \int \mathcal{I} \psi_i \mathcal{I} \psi_i d\omega_i \end{aligned}$$

Da in dem Gebiete der verticalen Strömung

$$\mathcal{I} \varphi = \mp \gamma$$

ist, so ist dieses Integral

$$= - \int \mathcal{I} \overline{\psi_i} \frac{\partial \overline{\varphi_i}}{\partial n} du \mp \gamma \int \mathcal{I} \psi_i d\omega_i$$

Er kommt schliesslich

$$0 = - \frac{\partial}{\partial t} \int \frac{\partial \mathcal{I} \overline{\psi_i}}{\partial n} du - \kappa \int \frac{\partial \psi_i}{\partial n} du \mp \gamma \int \frac{\partial \psi_i}{\partial n} du - \int \mathcal{I} \psi_i \frac{\partial \varphi_i}{\partial n} du \pm \int \mathcal{I} \psi_i d\omega_i$$

da $\int \frac{\partial \overline{\psi_i}}{\partial n} du = - \int \mathcal{I} \psi_i d\omega_i$ ist, so hat man

$$0 = - \int \left(\frac{\partial^2 \mathcal{I} \psi_i}{\partial n \partial t} + \mathcal{I} \psi_i \frac{\partial \varphi_i}{\partial n} + \kappa \frac{\partial \psi_i}{\partial n} \right) du$$

d. i., weil diese Gleichung in jedem Punkt des Umfangs bestehen muss

$$\frac{\partial^2 \mathcal{I} \psi_i}{\partial n \partial t} + \mathcal{I} \psi_i \frac{\partial \varphi_i}{\partial n} + \kappa \frac{\partial \psi_i}{\partial n} = 0 \quad (34)$$

Wenn wir die Gleichung (27) auf dieselbe Weise behandeln, so erhalten wir, da hier überall

$$\mathcal{I} \varphi_a = 0$$

ist,

$$0 = - \int \frac{\partial^2 \mathcal{J} \bar{\psi}_a}{\partial n \partial t} du - \int \mathcal{J} \bar{\psi}_a \frac{\partial \bar{\psi}_a}{\partial n} du - \kappa \int \frac{\partial \bar{\psi}_a}{\partial n} du$$

Die Integrale sind auszudehnen, theils über die Grenzcurve der verticalen Strömung, theils über einen mit unendlich grossem Radius beschriebenen Kreis. Der Umfang der Integration ist also hier unendlich gross, wie R' , welches den unendlich grossen Radius bezeichnet. Da aber die Luft in der Unendlichkeit überall und fügen wir hinzu, immer ruhen soll, so dass

$$\mathcal{J} W_a = \text{Const} = 0 \quad \frac{\partial W_a}{\partial n} = \text{Const} = 0$$

Da ferner $\frac{\partial \varphi_a}{\partial n}$ wie $\frac{1}{R}$ verschwindet, so bleibt von dem über R' ausgedehnten Integralen nur

$$\kappa \int \frac{\partial \psi_a}{\partial n} du,$$

was dasselbe ist

$$2\pi \kappa R' \frac{\partial \psi_a}{\partial n}$$

Da aber W_a in der Unendlichkeit von der Zeit unabhängig ist dass daher gesetzt werden kann

$$W_a = \frac{2\lambda \sin \theta}{\kappa} \cdot \varphi_a$$

(Vol. I dieses Journals pag. 171) und ferner

$$\varphi_a = \mp \frac{\tilde{r}}{2} R'^2 \log \rho + \text{Const.}$$

so dass

$$\left(\frac{dW_a}{dn} \right)_{\rho=R'} = \left(\frac{dW_a}{d\rho} \right)_{\rho=R'} = \mp \frac{\tilde{r} R'^2}{2R'}$$

so folgt.

$$2\pi \kappa R' \frac{\partial \bar{\psi}_a}{\partial n} = 2\pi \kappa \frac{\partial}{\partial t} \left(\mp \frac{\tilde{r} R'^2}{2} \cdot \frac{2\lambda \sin \theta}{\kappa} \right) = 0$$

Mithin verschwindet das über den Umfang des unendlich grossen Kreises ausgeführte Integral. Es kommt daher

$$0 = - \int \left(\frac{\partial^2 \Delta \bar{\psi}_a}{\partial n \partial t} + J \bar{\psi}_a \frac{\partial \bar{\zeta}_a}{\partial n} + \kappa \frac{\partial \bar{\psi}_a}{\partial n} \right) dn$$

wo die Integration nur über den Umfang der Grenzcurve der verticalen Strömung auszuführen ist. Da diese Gleichung wieder in jedem Punkt der Grenzcurve stattfinden muss, erhalten wir

$$\frac{\partial^2 J \bar{\psi}_a}{\partial n \partial t} + J \bar{\psi}_a \frac{\partial \bar{\zeta}_a}{\partial n} + \kappa \frac{\partial \bar{\psi}_a}{\partial n} = 0 \quad (35)$$

Da nun an der Grenzcurve $\frac{\partial \zeta_a}{\partial n} = \frac{\partial \zeta_i}{\partial n}$ ist, so folgt aus (34) und (35), da diese identisch erfüllt sein müssen

$$\frac{\partial \psi_i}{\partial n} = \frac{\partial \psi_a}{\partial n} \quad J \psi_i = J \psi_a \quad \frac{\partial \frac{\partial J W_i}{\partial n}}{\partial t} = \frac{\partial \frac{\partial J W_a}{\partial n}}{\partial t}$$

Geht man mit diesen Gleichungen in die Gleichungen (32) und (33), so sieht man, dass diese nicht anderes gleichzeitig erfüllt werden können, als durch

$$\frac{\partial \bar{\psi}_i}{\partial n} = \frac{\partial \bar{\psi}_a}{\partial n} = 0 \quad J \bar{\psi}_i = J \bar{\psi}_a = 0 \quad \frac{\partial \frac{\partial J \bar{\psi}_i}{\partial n}}{\partial t} = \frac{\partial \frac{\partial J \bar{\psi}_a}{\partial n}}{\partial t} = 0$$

Es ist dem demnach

$$\int J \psi_i d\omega_i = - \int \frac{\partial \bar{\psi}_i}{\partial n} dn = 0$$

$$\int J \psi_a d\omega_a = - \int \frac{\partial \bar{\psi}_a}{\partial n} dn = 0$$

Hieraus folgt, da $JW = \xi - 2\lambda \sin \theta$ nirgends und niemals sein

Vorzeichen wechseln soll, dass in dem ganzen unendlichen Luftraum

$$\mathcal{J}\psi_i = \mathcal{J}\psi_a = 0$$

sein muss d. h., dass

$$\frac{\partial \mathcal{J}W_i}{\partial t} = \frac{\partial \mathcal{J}W_a}{\partial t} = 0 \quad (36)$$

zu setzen ist, wenn γ sich nicht mit der Zeit ändert, oder zu irgend einer Zeit einen endlichen constanten Werth erhält; dass, wenn zur Zeit $t=0$ γ constant und endlich ist, der Bewegungszustand der entstandenen Wirbel zur Zeit $t=0$, durch die Gleichungen (36) gegeben ist.

Wir sehen somit, dass wir wohl berechtigt waren, als wir mit der Annahme der Unveränderlichkeit des γ mit der Zeit, ohne Weiteres $\frac{\partial \mathcal{J}W}{\partial t} = 0$ gesetzt haben. In den nicht stationären Fällen, wo γ eine gegebene Function von der Zeit t ist, muss der Anfangszustand gegeben sein, der aber verschieden ist, je nach dem die verticale Strömung, welche einmal entstanden war, von $t=0$ ab, sich mit der Zeit ändert, oder von diesem Momente an zu entstehen beginnt. In dem ersteren Fall ist γ eine endliche Constante für $t=0$, es ist dabei gleichgiltig, ob diese verticale Strömung plötzlich entsteht, oder längst entstanden war. In dem letzteren Fall verschwindet γ mit t .

Bezeichnet man mit γ_0 den Werth des γ für $t=0$ so handelt sich darum, eine Function W zu finden, welche in dem Gebiete der verticalen Strömung die Gleichung (28), und ausserhalb desselben die Gleichung (27) befriedigt, und an der Grenzcurve der verticalen Strömung die Bedingung

$$\frac{\partial W_i}{\partial n} = \frac{\partial W_a}{\partial n}$$

für jedes t erfüllt, und $\frac{\partial W_a}{\partial x}, \frac{\partial W_a}{\partial y}$ in der Unendlichkeit für jedes

t verschwinden. Wenn die verticale Strömung zur Zeit $t=0$, die Geschwindigkeit $\mp \gamma_0 z$ hat oder keine, so hat W_i^* ferner in dem ersteren Fall die Bedingung zu erfüllen

$$\begin{aligned} \frac{\partial \Delta W_i}{\partial x} \left(\frac{\partial \zeta_{i0}}{\partial x} + \frac{\partial W_i^*}{\partial y} \right) + \frac{\partial \Delta W_i^*}{\partial y} \left(\frac{\partial \zeta_{i0}}{\partial y} - \frac{\partial W_i^*}{\partial x} \right) \\ + (\kappa \mp \gamma_0) \Delta W_i \mp 2\lambda \sin \theta \gamma_0 = 0 \end{aligned} \quad (37)$$

wo $\zeta_{i0} = -\frac{\gamma_0}{4\pi} \int \log r \, d\omega$ ist,

und in dem letzteren Fall, wo $\gamma_0=0$ ist, soll

$$\frac{\partial W_i^*}{\partial x} = \frac{\partial W_i^*}{\partial y} = 0 \quad \Delta W_i = 0$$

indem wir annehmen, dass die Luft zur Zeit $t=0$ überall geruht habe.

W_a^* muss dabei der Gleichung genügen

$$\frac{\partial \Delta W_a^*}{\partial x} \left(\frac{\partial \zeta_{a0}}{\partial x} + \frac{\partial W_a^*}{\partial y} \right) + \frac{\partial \Delta W_a^*}{\partial y} \left(\frac{\partial \zeta_{a0}}{\partial y} - \frac{\partial W_a^*}{\partial x} \right) + \kappa \Delta W_a^* = 0 \quad (38)$$

falls γ_0 nicht verschwindet. Verschwindet γ_0 , so soll W_a^* wieder der Bedingung genügen

$$\frac{\partial W_a^*}{\partial x} = \frac{\partial W_a^*}{\partial y} = 0 \quad \Delta W_a^* = 0$$

Die Differentialgleichungen für die Isodynamen [(44) und (45) pag. 170 Vol. I dieses Journals] bleiben unverändert, da hierin ein Differentialquotient nach t nicht vorkommt. Man kann sie durch Integration der Gleichungen finden und damit den Druck, wenn W bestimmt ist, wie in dem Fall der stationären Wirbelbildung. Die Gleichung für den Deviationswinkel i (46) pag. 170 Vol. I dieses Journals gilt aber nur im Fall der stationären Wirbelbewegung

Für den vorliegenden Fall finden wir leicht aus (9c) (pag. 132 Vol. I dieses Journal.)

$$\operatorname{tag} i = \frac{1}{\kappa \omega} \sqrt{\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + 2\kappa \omega \frac{\partial w}{\partial t} + \kappa^2 \omega^2 + 2\kappa \left(u \frac{\partial v}{\partial t} - v \frac{\partial u}{\partial t}\right)} \quad (38)$$

wo

$$u^2 + v^2 = \omega^2$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + 2\lambda \sin \theta = \kappa$$

gesetzt worden ist.

Es lässt sich die Function W für das wirbelfreie äussere Gebiet auch hier ganz allgemein finden. Da auch hier $\mathcal{L}\varphi_a = 0$ ist, so lässt sich die Gleichung (27) auch, wie folgt, schreiben

$$\begin{aligned} \mathcal{L}\left(\frac{\partial W_a}{\partial t} + \kappa W_a - 2\lambda \sin \theta \varphi_a\right) = & -\frac{\partial \mathcal{L}W_a}{\partial x} \left(\frac{\partial \varphi_a}{\partial x} + \frac{\partial W_a}{\partial y}\right) \\ & -\frac{\partial \mathcal{L}W_a}{\partial y} \left(\frac{\partial \varphi_a}{\partial y} - \frac{\partial W_a}{\partial x}\right) \end{aligned}$$

Die Grösse $\mathcal{L}W_a$ ist für das wirbelfreie Gebiet $= \text{Const} = a$, und da ferner $\frac{\partial \varphi_a}{\partial x} + \frac{\partial W_a}{\partial y}$, und $\frac{\partial \varphi_a}{\partial y} - \frac{\partial W_a}{\partial x}$ im äusseren Gebiete überall und immer stetig und endlich bleiben sollen, so folgt

$$\mathcal{L}\left(\frac{\partial W_a}{\partial t} + \kappa W_a - 2\lambda \sin \theta \varphi_a\right) = 0$$

was befriedigt wird, wenn wir setzen

$$\frac{\partial W_a}{\partial t} + \kappa W_a - 2\lambda \sin \theta \varphi_a = T \varphi_a \quad (39)$$

wo T eine willkürliche Function von t ist. Setzt man nun

$$W_a = e^{-\kappa t} F(x, y, t)$$

wo F eine gewisse Function von den eingezeichneten Argumenten ist, so dass

$$\frac{\partial W_a}{\partial t} = -\kappa e^{-\nu t} F(x, y, t) + e^{-\nu t} \frac{\partial F}{\partial t}$$

Einführung dieses in (39) ergibt

$$e^{-\nu t} \frac{\partial F}{\partial t} (T + 2\lambda \sin \theta) \zeta_a$$

Hieraus findet man

$$F = \int e^{\nu t} (T + 2\lambda \sin \theta) \zeta_a dt + C$$

wo C eine Function von x, y allein sein kann, die aber der Gleichung $\Delta C = 0$ genügen muss, da F einer solchen Gleichung genügen muss, was die Function $\int e^{\nu t} (T + 2\lambda \sin \theta) \zeta_a dt$ es in der That thut,

$$W_a = e^{-\nu t} \int e^{\nu t} (T + 2\lambda \sin \theta) \zeta_a dt + e^{-\nu t} C. \quad (40)$$

kann als eine allgemeine Lösung für das äussere wirbelfreie Gebiet betrachtet werden, in so ferne, als zwei willkürliche Functionen T und C auftreten, mit denen man jeder der Bedingungen, denen $\frac{\partial W_a}{\partial \rho}$ genügen muss, gerecht werden kann.

Es soll z, B , zur Zeit $t=0$ die Luftbewegung eine stationäre sein, so dass, wie wir gefunden haben

$$W_{a0} = \frac{2\lambda \sin \theta}{\kappa} \zeta_{a0} \quad \zeta_{a0} = \mp \frac{\gamma_0}{2\kappa} \int \log \rho d\omega.$$

Man genügt in diesem Fall durch die Annahme

$$C = \frac{2\lambda \sin \theta}{\kappa} \zeta_{a0}$$

so dass die Bedingung $\Delta C = 0$ erfüllt ist, und durch die Ausdehnung des Zeitintegrals von 0 bis t , so dass wir erhalten,

$$W_a = e^{-\nu t} \int_0^t (T + 2\lambda \sin \theta) \zeta_a e^{\nu t} dt + \frac{2\lambda \sin \theta}{\kappa} e^{-\nu t} \zeta_{a0}$$

Die zweite willkürliche Function T bestimmt sich aus der Bedingung, die an der Grenze der verticalen Strömung zu erfüllen ist, und wird daher durch den Bewegungszustand des inneren Gebietes zur Zeit t bestimmt. Soll γ zu jeder Zeit den Werth haben, den es zur Zeit $t=0$ gehabt, d. h. soll die Bewegung eine stationäre sein, so hat man $T=0$ zu setzen und $\varphi_a = \varphi_{a0}$. Man erhält dann

$$W_a = e^{-\kappa t} \frac{2\lambda \sin \theta}{\kappa} (e^{\kappa t} - 1) \varphi_{a0} + \frac{2\lambda \sin \theta}{\kappa} e^{-\kappa t} \varphi_{a0}$$

d. h.

$$W = \frac{2\lambda \sin \theta}{\kappa} \varphi_{a0}$$

wie es sein müsste.

Für das Gebiet, wo der Wirbel nicht verschwindet, lässt sich eine solche allgemeine Lösung nicht finden; wohl aber, wenn wir uns das Gebiet der verticalen Strömung kreisförmig begrenzt denken.

Wir fassen zuerst den Fall in's Auge, wo die verticale Strömung eine aufsteigende ist. Wenn wir setzen

$$\varphi_a = -\frac{\gamma}{2} R^2 \log \rho + \text{Const.}$$

$$\varphi_i = -\frac{\gamma}{4} \rho^2 + \text{Const.} \quad \rho = \sqrt{x^2 + y^2}$$

so erhalten wir für das äussere Gebiet aus (27)

$$\frac{\partial JW_a}{\partial t} - \frac{\gamma R^2}{2\rho} \frac{\partial JW_a}{\partial \rho} + \kappa JW_a = 0$$

oder, was dasselbe ist

$$\frac{\partial JW_a}{\partial t} - \gamma \frac{\partial JW_a}{\partial \left(\frac{\rho}{R}\right)^2} + \kappa JW_a = 0$$

Setzen wir hierin

$$JW_a = e^{-\kappa t} \varphi$$

wo φ eine gewisse Function von t und $\left(\frac{\rho}{R}\right)^2$ ist, so erhalten wir

$$\frac{\partial \varphi}{\partial t} = r \frac{\partial \varphi}{\partial \left(\frac{R}{r}\right)^2}$$

was gegnügt wird durch

$$\varphi = f \left[\int r dt + \left(\frac{\rho}{R}\right)^2 \right]$$

wo f eine willkürliche Function des Argumentes $\int r dt + \left(\frac{\rho}{R}\right)^2$ bedeutet. Wir erhalten somit

$$W_a = e^{-\nu t} f \left[\int r dt + \left(\frac{\rho}{R}\right)^2 \right] \quad (41)$$

Eine Lösung, die als allgemein bezeichnet werden kann, da hierin eine willkürliche Function auftritt. Die Rotationsgeschwindigkeit der Lufttheilchen ist somit

$$\zeta = 2\lambda \sin \theta - e^{-\nu t} f \left[\int r dt + \left(\frac{\rho}{R}\right)^2 \right]$$

Das Eigenthümliche bei der also bestimmten Form der Wirbelbewegung ist, dass sie mit gewisser mit der Zeit variabler Geschwindigkeit sich *fortpflanzt*. ζ erhält nämlich, wenn $\nu=0$ wäre, denselben Werth, so oft die Grösse $\int r dt + \left(\frac{\rho}{R}\right)^2$ denselben Werth erhält. Denkt man sich die Function f so bestimmt, dass sie für einen reellen Werth C verschwindet und dergleichen für jedes Argument, das kleiner ist als C , so pflanzt sich die Wirbelbewegung von Aussen her nach dem Gebiet der verticalen Strömung fort, denn die Fortpflanzungsgeschwindigkeit

$$\frac{d\rho}{dt} = - \frac{rR}{\sqrt{C - \int r dt}}$$

ist negativ. Wenn demnach eine vertical aufsteigende Strömung entsteht, so gerathen die entfernten Lufttheilchen zuerst in Drehung, und diese Drehung ergreift ein Lufttheilchen nach dem anderen allmählig nach dem Centrum des inneren Gebietes zu, während die Lufttheilchen, deren Entfernung zur Zeit t kleiner ist, als $R\sqrt{C-f\gamma dt}$ gerathen noch nicht in Drehung, sondern bewegen sich nur centripetal nach dem inneren Gebiete mit der Geschwindigkeit, deren Componenten $\frac{\partial \varphi_a}{\partial x}$, und $\frac{\partial \varphi_a}{\partial y}$ sind.

Wenn die verticale Strömung eine niedersteigende ist, so erhält man, indem man $-\gamma$ für γ setzt

$$\Delta W_a = e^{-\gamma t} f \left[\left(\frac{\rho}{R} \right)^2 - \int \gamma dt \right]$$

Mitthin als Rotationsgeschwindigkeit eines Lufttheilchens

$$\zeta = 2k \sin \theta = e^{-\gamma t} f \left[\left(\frac{\rho}{R} \right)^2 - \int \gamma dt \right] \quad (42)$$

Auch in diesem Fall pflanzt sich die Wirbelbewegung fort und zwar von Innen nach Aussen mit der Geschwindigkeit

$$\frac{d\rho}{dt} = \frac{\gamma R}{\sqrt{C + f\gamma dt}}.$$

vorausgesetzt, dass der Anfangszustand der Bewegung so beschaffen ist, dass die Function für $\left(\frac{\rho^2}{R} \right) - \int \gamma dt = C$ verschwindet. Die entfernten Lufttheilchen bewegen sich hier aufangs nur centripetal nach Aussen hin gleichsam, als wenn sie zuerst durch die aus dem inneren Gebiet herausfahrende Luft verschoben würden, ehe sie in Drehung gerathen.

Es lässt sich ΔW ebenso leicht für dass innere Gebiet finden. Im Falle die kreisförmige Strömung aufsteigt, erhält die Gleichung (28) die Form

$$\frac{\partial JW_i}{\partial t} - \frac{r\rho}{2} \frac{\partial JW_i}{\partial \rho} + (x-r) JW_i + 2\lambda \sin \theta \gamma = 0$$

oder, indem wir dieses etwas umformen

$$\frac{\partial JW_i}{\partial t} - r \frac{\partial JW_i}{\partial \log \left(\frac{\rho}{R} \right)^2} + (x-r) JW_i + 2\lambda \sin \theta \gamma = 0$$

Wenn wir hierin setzen

$$JW_i = f(\rho, t) - 2\lambda \sin \theta e^{\int (x-r) dt} \int r e^{\int (x-r) dt} dt$$

wo f eine Function von den eingeschriebenen Variablen bedeutet, so kommt

$$\begin{aligned} \frac{\partial JW_i}{\partial t} &= \frac{\partial f}{\partial t} - 2\lambda \sin \theta (x-r) e^{\int (x-r) dt} \int r e^{\int (x-r) dt} dt - 2\lambda \sin \theta r \\ &- r \frac{\partial JW_i}{\partial \log \left(\frac{\rho}{R} \right)^2} = - r \frac{\partial f}{\partial \log \left(\frac{\rho}{R} \right)^2} \end{aligned}$$

$$\begin{aligned} (x-r) JW_i &= (x-r) f - 2\lambda \sin \theta (x-r) e^{\int (x-r) dt} \int r e^{\int (x-r) dt} dt \\ 2\lambda \sin \theta \gamma &= 2\lambda \sin \theta r \end{aligned}$$

Die Addition dieser Gleichungen ergibt

$$0 = \frac{\partial f}{\partial t} - r \frac{\partial f}{\partial \log \left(\frac{\rho}{R} \right)^2} + (x-r) f$$

Setzt man hierin weiter

$$f = e^{-\int (x-r) dt} \varphi$$

wo φ eine Function von t und ρ ist, so erhält man als Bestimmungsgleichung für φ

$$0 = \frac{\partial \varphi}{\partial t} - r \frac{\partial \varphi}{\partial \log \left(\frac{\rho}{R} \right)^2}$$

was befriedigt wird durch

$$\varphi = F \left[\int \gamma dt + \log \left(\frac{\rho}{R} \right)^2 \right]$$

wo F eine willkürliche Function des Argumentes $\int \gamma dt + \log \left(\frac{\rho}{R} \right)^2$ ist. Wir erhalten somit als die allgemeine Lösung

$$\Delta W_i = e^{-\gamma t + \int \gamma dt} \left[F \left\{ \int \gamma dt + \log \left(\frac{\rho}{R} \right)^2 \right\} - 2\lambda \sin \theta \int \gamma e^{\gamma t - \int \gamma dt} dt \right] \quad (43)$$

Auch hier pflanzt sich die Wirbelbewegung von Aussen nach dem Centrum zu mit der Geschwindigkeit

$$\frac{d\rho}{dt} = -\gamma R e^{\frac{1}{2}(C - \int \gamma dt)}$$

gleichfalls unter derselben Voraussetzung, wie oben. Für ein Gebiet der vertical herabsteigenden Strömung findet man hieraus durch Vertauschung des γ mit $-\gamma$, so dass

$$\Delta W_i = e^{-\gamma t - \int \gamma dt} \left[F \left\{ \log \left(\frac{\rho}{R} \right)^2 - \int \gamma dt \right\} + 2\lambda \sin \theta \int \gamma e^{\gamma t + \int \gamma dt} dt \right] \quad (44)$$

Unter der nämlichen Voraussetzung, wie oben, pflanzt sich die Wirbelbewegung von Innen nach Aussen fort mit der Geschwindigkeit

$$\frac{d\rho}{dt} = \gamma R e^{\frac{1}{2}(C + \int \gamma dt)}$$

Die in den Lösungen (41) (42) (43) und (44) auftretenden willkürlichen Functionen sind nun gemäss des anfänglichen Bewegungszustandes zu bestimmen, welcher daher gegeben sein muss. Wenn wir nun annehmen, dass die Wirbelbewegung zur Zeit $t = 0$ eine stationäre gewesen sei, also dass γ zur Zeit $t = 0 = \text{Const} = \gamma_0$ gewesen sei. Es ist sonach für $t = 0$

$$\frac{\partial JW}{\partial t} = 0$$

Für das Gebiet der verticalaufsteigenden Strömung ist demnach zur Zeit $t = 0$

$$-\frac{\gamma_o \rho}{2} \frac{\partial JW_{io}}{\partial \rho} + (\kappa - \gamma_o) JW_{io} + 2\lambda \sin \theta \gamma_o = 0$$

d. h.,

$$JW_{io} = -\frac{2\lambda \sin \theta}{m-1} \left[1 - \left(\frac{\rho}{R} \right)^{2(m-1)} \right] \quad m = \frac{\kappa}{\gamma_o}$$

was sich auch so schreiben lässt

$$JW_{io} = -\frac{2\lambda \sin \theta}{(m-1)} \left(1 - e^{(m-1) \log \left(\frac{\rho}{R} \right)^2} \right) \quad (44a)$$

Die Lösung (43) lässt sich auch so schreiben

$$JW_i = e^{-\kappa t + \int_0^t \gamma dt} \left[F \left\{ \log \left(\frac{\rho}{R} \right)^2 + \int_0^t \gamma dt \right\} - 2\lambda \sin \theta \int_0^t \gamma e^{-\int_0^t (\kappa - \gamma) dt} \right]$$

Setzt man hierin $t = 0$, so hat man

$$JW_{io} = F \left[\log \left(\frac{\rho}{R} \right)^2 \right]$$

Ein Vergleich dieses mit (44a) ergibt sofort

$$F \left[\log \left(\frac{\rho}{R} \right)^2 + \int_0^t \gamma dt \right] = -\frac{2\lambda \sin \theta}{m-1} \left[1 - \left(\frac{\rho}{R} \right)^{2(m-1)} e^{-(m-1) \int_0^t \gamma dt} \right]$$

Somit erhalten wir

$$JW_i = -\frac{2\lambda \sin \theta}{(m-1)} e^{-\kappa t + \int_0^t \gamma dt} \left[(m-1) \int_0^t \gamma dt e^{\int_0^t (\kappa - \gamma) dt} + 1 - \left(\frac{\rho}{R} \right)^{2(m-1)} e^{-(m-1) \int_0^t \gamma dt} \right] \quad (44b)$$

Wenn γ nun für jeden Werth von t denselben Werth γ_o behalten

sollte, so müsste auch W_i denselben Werth behalten, den es bei $t=0$ gehabt hat. In der That verwandelt sich (44b) in diesem Fall in

$$\begin{aligned} \Delta W_i &= -\frac{2\lambda \sin \theta}{(\kappa - \gamma_0)} e^{-\kappa t + \gamma_0 t} \left[\frac{(\kappa - \gamma_0)}{(\kappa - \gamma_0)} \gamma_0 \left(e^{(\kappa - \gamma_0)t} - 1 \right) \right. \\ &\quad \left. + \gamma_0 \left\{ 1 - \left(\frac{\rho}{R} \right)^2 \left(\frac{\kappa}{\gamma_0} - 1 \right) e^{\left(\frac{\kappa}{\gamma_0} - 1 \right) \gamma_0 t} \right\} \right] \\ &= -\frac{2\lambda \sin \theta}{(\kappa - \gamma_0)} \left[\gamma_0 - \gamma_0 e^{-\kappa t + \gamma_0 t} + \gamma_0 e^{-\kappa t + \gamma_0 t} + \gamma_0 \left(\frac{\rho}{R} \right)^2 \left(\frac{\kappa}{\gamma_0} - 1 \right) \right] \\ &= -\frac{2\lambda \sin \theta}{(m-1)} \left[1 - \left(\frac{\rho}{R} \right)^2 (m-1) \right] \end{aligned}$$

d. h.

$$W_i = \Delta W_{i0}$$

Wir schreiben zur Abkürzung

$$2\lambda \sin \theta = a$$

$$A = e^{\int_0^t \gamma dt} \int_0^t \gamma e^{\int (\kappa - \gamma) dt} dt \quad B = \frac{\gamma_0}{(\kappa - \gamma_0)} e^{\int_0^t \gamma dt}$$

$$C = \frac{\gamma_0}{\kappa - \gamma_0} e^{m \int_0^t \gamma dt}$$

so dass wir haben

$$W_i = -a e^{-\kappa t} \left[A + B - C \left(\frac{\rho}{R} \right)^2 (m-1) \right]$$

Hieraus erhält man weiter

$$\begin{aligned} \frac{dW_i}{d\rho} &= -\frac{a e^{-\kappa t}}{\rho} \int \rho d\rho \left[A + B - C \left(\frac{\rho}{R} \right)^2 (m-1) \right] \\ &= -\frac{a e^{-\kappa t} \rho}{2} \left[A + B - \frac{C}{m} \left(\frac{\rho}{R} \right)^2 (m-1) \right] \end{aligned} \quad (45)$$

Ist $\gamma = \text{Const} = \gamma_0$ für jedes t , so erhält man hieraus

$$\begin{aligned}\frac{dW_i}{d\rho} &= -\frac{ae^{-\gamma t}\rho}{2}\left[\frac{e^{\gamma_0 t}\tilde{\gamma}_0}{(\kappa-\tilde{\gamma}_0)}\left(e^{-\gamma t-\gamma_0 t}-1\right)+\frac{\tilde{\gamma}_0 e^{\gamma_0 t}}{\kappa-\tilde{\gamma}_0}\right. \\ &\quad \left.-\frac{\tilde{\gamma}_0 e}{\kappa-\tilde{\gamma}_0}\frac{\rho}{\gamma_0}\gamma_0 t\left(\frac{1}{m}\right)\left(\frac{\rho}{R}\right)^{2(m-1)}\right] \\ &= -\frac{a\rho}{2(m-1)}\left[1-\frac{1}{m}\left(\frac{\rho}{R}\right)^{2(m-1)}\right]\end{aligned}$$

d. h.

$$\frac{dW_i}{d\rho} = \frac{dW_{i0}}{d\rho}$$

wie es sein sollte.

Es handelt sich jetzt darum $\frac{dW}{d\rho}$ für das äussere Gebiet zu bilden. Auch in diesem Fall können Wirbel nicht vorhanden sein, da $\left(\frac{dW}{d\rho}\right)$ für $t=0$ sonst in der Unendlichkeit unendlich gross sein müsste. Die Lösung (40) gilt dann in diesem Fall.

Wir setzen in (40) $t=0$, und

$$\zeta_a = -\frac{\gamma R^2}{2} \log \rho + \text{Const}$$

so dass wir haben

$$W_a = -\frac{R^2 \log \rho}{2} e^{-\gamma t} \int (T + 2\lambda \sin \theta) \gamma e^{\gamma t} dt$$

Es ist daher

$$\frac{dW_a}{d\rho} = -\frac{R^2}{2\rho} e^{-\gamma t} \int (T + 2\lambda \sin \theta) \gamma e^{\gamma t} dt$$

Die Grenzbedingung $\frac{dW_i}{d\rho} = \frac{dW_a}{d\rho}$ für $\rho=R$ ergibt

$$-\frac{e^{-\gamma t} R}{2} \int (T + 2\lambda \sin \theta) \gamma e^{\gamma t} dt = -\frac{ae^{-\gamma t} R}{2} \left(A + B - \frac{C}{m}\right)$$

d. h.

$$\int (T + 2\lambda \sin \theta) \gamma e^{\gamma t} dt = a \left(A + B - \frac{C}{m}\right)$$

womit T bestimmt worden ist.

Wir erhalten somit

$$\frac{dW_a}{d\rho} = -\frac{aR^2e^{-\gamma t}}{2\rho} \left(A + B - \frac{C}{m} \right) \quad (46)$$

Für $\gamma = \text{Const} = \gamma_0$ wird dieses

$$\begin{aligned} \frac{dW_a}{d\rho} &= -\frac{aR^2e^{-\gamma_0 t}}{2\rho} \left[\frac{\gamma_0 e^{\gamma_0 t}}{\kappa - \gamma_0} \left(e^{(\kappa - \gamma_0)t} - 1 \right) + \frac{\gamma_0 e^{\gamma_0 t}}{\kappa - \gamma_0} - \frac{\gamma_0 e^{\gamma_0 t}}{\kappa - \gamma_0} \frac{\kappa}{\gamma_0} \frac{\gamma_0 t}{\kappa} \right] \\ &= -\frac{aR^2}{2\rho} \left(\frac{\gamma_0}{\kappa - \gamma_0} - \frac{\gamma_0}{\kappa - \gamma_0} \frac{\gamma_0}{\kappa} \right) = -\frac{a}{\kappa} \frac{\gamma_0}{2} \frac{R^2}{\rho} \end{aligned}$$

d. h.

$$\frac{dW_a}{d\rho} = \frac{dW_{a0}}{d\rho}$$

wie es sein sollte.

Hiermit ist die Aufgabe für den vorliegenden Fall vollständig gelöst. Ausdrücke für die Geschwindigkeitscomponenten und Druck können ohne jede Schwierigkeit aufgestellt werden; die Differentialgleichung für die Windbahn ist aber auch bei der einfachsten Annahme über die Function γ äusserst verwickelt.

Wir beschäftigen uns mit der vertical herabsteigenden Strömung von kreisförmigem Querschnitte unter der Voraussetzung, dass die Wirbelbewegung zur Zeit $t = 0$ eine stationäre gewesen sei, d. h.

$$\frac{\gamma_0 \rho}{2} \frac{dJW_{i0}}{d\rho} + (\kappa + \gamma_0) JW_{i0} - 2\lambda \sin \theta \gamma_0 = 0$$

woraus sich ergibt

$$JW_{i0} = \frac{a\gamma_0}{\kappa + \gamma_0}$$

da JW_{i0} für $\rho = 0$ nicht unendlich sein kann. Soll die Lösung (42) für $t = 0$ diesem gleich sein, so sieht man, dass man dieser Bedingung genügt, wenn man setzt

$$R' \left[\log \left(\frac{\rho}{R} \right)^2 - \int \gamma dt \right] = \text{Const} = \frac{\alpha \tilde{\gamma}_0}{\kappa + \tilde{\gamma}_0}$$

und

$$\Delta W_i = \alpha e^{-\kappa t - \int_0^t \gamma dt} \left(\frac{\tilde{\gamma}_0}{\kappa + \tilde{\gamma}_0} + \int_0^t \tilde{\gamma} e^{\int_0^t (\gamma + \kappa) dt} dt \right) \quad (47)$$

Wird γ constant und $= \tilde{\gamma}_0$, so muss für jedes t

$$\Delta W_i = \Delta W_{i0}$$

sein, was in der That der Fall ist. Es ist nämlich für $\gamma = \tilde{\gamma}_0$

$$\begin{aligned} \Delta W_i &= \alpha e^{-\kappa t - \gamma_0 t} \left[\frac{\tilde{\gamma}_0}{\kappa + \tilde{\gamma}_0} + \frac{\tilde{\gamma}_0}{\kappa + \tilde{\gamma}_0} \left(e^{(\kappa + \gamma_0)t} - 1 \right) \right] \\ &= \alpha e^{-\kappa t - \gamma_0 t} \left(\frac{\tilde{\gamma}_0}{\kappa + \tilde{\gamma}_0} e^{(\kappa + \gamma_0)t} \right) = \frac{\alpha \tilde{\gamma}_0}{\kappa + \tilde{\gamma}_0} \end{aligned}$$

d. h.

$$\Delta W_i = \Delta W_{i0}$$

Setzen wir zur Abkürzung

$$F = e^{-\int_0^t \gamma dt} \left(\frac{\tilde{\gamma}_0}{\kappa + \tilde{\gamma}_0} + \int_0^t \tilde{\gamma} e^{\int_0^t (\gamma + \kappa) dt} dt \right)$$

so dass

$$\Delta W_i = \alpha F e^{-\kappa t} \quad (48)$$

Für das äussere Gebiet gilt die Lösung

$$\Delta W_a = e^{-\kappa t} f \left[\left(\frac{\rho}{R} \right)^2 - \int \gamma dt \right] \quad (49)$$

und es soll f so bestimmt werden, dass

$$\text{für } t = 0 \quad \Delta W_{a0} = \Delta W_{i0}$$

$$\text{für } \rho = R \quad \frac{dW_a}{d\rho} = \frac{dW_i}{d\rho}$$

erfüllt sind. Es ist nun für $t = 0$ $\gamma = \tilde{\gamma}_0$.

$$\frac{\gamma_0 R^2}{2\rho} \frac{d\Delta W_{a0}}{d\rho} + \kappa \Delta W_{a0} = 0$$

d. h.

$$\Delta W_{a0} = \frac{a\gamma_0}{\kappa + \gamma_0} e^{-m \left[\left(\frac{\rho}{R} \right)^2 - 1 \right]}$$

Ein Vergleich dieses Ausdrucks mit (49) ergibt

$$f \left[\left(\frac{R}{\rho} \right)^2 - \int_0^t \gamma dt \right] = \frac{a\gamma_0}{\kappa + \gamma_0} e^{-m \left[\left(\frac{\rho}{R} \right)^2 - 1 - \int_0^t \gamma dt \right]}$$

folglich erhält man

$$\Delta W_a = \frac{a\gamma_0}{\kappa + \gamma_0} e^{-\gamma t - m \left[\left(\frac{\rho}{R} \right)^2 - 1 - \int_0^t \gamma dt \right]} \quad (50)$$

Für $\gamma = \text{Const} = \gamma_0$ kommt

$$\begin{aligned} \Delta W_a &= \frac{a\gamma_0}{\kappa + \gamma_0} e^{-\gamma_0 t - m \left[\left(\frac{\rho}{R} \right)^2 - 1 \right]} + \frac{\gamma_0}{\gamma_0} \gamma_0 t \\ &= \frac{a\gamma_0}{\kappa + \gamma_0} e^{-m \left[\left(\frac{\rho}{R} \right)^2 - 1 \right]} \end{aligned}$$

wie es sein sollte.

Nun ergibt sich aus (48) durch Integration

$$\frac{\partial W_a}{\partial \rho} = \frac{a R e^{-\gamma_0 t} \rho}{2} \quad (50a)$$

weil $\frac{\partial W_a}{\partial \rho}$ nie für $\rho = 0$ unendlich gross werden darf. Es ergibt sich ferner aus (50)

$$\frac{\partial W_a}{\partial \rho} = - \frac{a R^2 e^{-\gamma_0 t + m \int_0^t \gamma_0 dt}}{2m(1+m)\rho} e^{-m \left[\left(\frac{\rho}{R} \right)^2 - 1 \right]} + \frac{C}{\rho}$$

wo C hier eine Function von t sein kann.

Die Bedingung

$$\frac{\partial W_a}{\partial \rho} = \frac{\partial W_i}{\partial \rho} \quad \rho = R$$

ergibt dann

$$- \frac{\alpha R}{2m(1+m)} e^{-\gamma t + m \int_0^t \gamma dt} + \frac{C}{R} = \frac{\alpha R}{2} e^{-\gamma t} A'$$

Hieraus folgt

$$C = \frac{\alpha R^2}{2} e^{-\gamma t} \left(A' + \frac{e^{-m \int_0^t \gamma dt}}{m(1+m)} \right)$$

folglich erhalten wir

$$\frac{\partial W_a}{\partial \rho} = \frac{\alpha R^2 e^{-\gamma t}}{2\rho} \left[A' + \frac{e^{-m \int_0^t \gamma dt}}{m(1+m)} \left(1 - e^{-m \left[\left(\frac{\rho}{R} \right)^2 - 1 \right]} \right) \right] \quad (51)$$

womit die Aufgabe auch für den Fall der herabsteigenden Strömung vollständig gelöst worden ist

Im Fall γ für jedes t constant, $= \gamma_0$ ist, hat man

$$\begin{aligned} \frac{\partial W_a}{\partial \rho} &= \frac{\alpha R^2 e^{-\gamma_0 t}}{2\rho} \left[\frac{\gamma_0}{\alpha + \gamma_0} e^{-\gamma_0 t} + \frac{\gamma_0}{\alpha + \gamma_0} e^{-\gamma_0 t} \left(e^{\gamma_0 t + \gamma_0 t} - 1 \right) \right. \\ &\quad \left. + \frac{e^{-m \gamma_0 t}}{m(1+m)} \left(1 - e^{-m \left[\left(\frac{\rho}{R} \right)^2 - 1 \right]} \right) \right] \\ &= \frac{\alpha R^2 e^{-\gamma_0 t}}{2\rho} \left[\frac{\gamma_0}{\alpha + \gamma_0} e^{\gamma_0 t} + \frac{e^{\frac{\gamma_0 \gamma_0 t}{\alpha}}}{m(1+m)} \left(1 - e^{-m \left[\left(\frac{\rho}{R} \right)^2 - 1 \right]} \right) \right] \\ &= \frac{\alpha R^2}{2\rho} \left[\frac{\gamma_0}{\alpha + \gamma_0} + \frac{1}{m(1+m)} \left(1 - e^{-m \left[\left(\frac{\rho}{R} \right)^2 - 1 \right]} \right) \right] \end{aligned}$$

d. h.

$$= \frac{\alpha R^2}{2\rho} \left(1 - \frac{\gamma_0}{\alpha + \gamma_0} e^{-\frac{\gamma_0}{\gamma} \left[\left(\frac{\rho}{R} \right)^2 - 1 \right]} \right)$$

wie es hätte sein müssen.

Wir wollen die Rechnung in einigen einfachen Fällen durchführen. Wir nehmen zunächst an, dass

$$r = \frac{1}{a+t}$$

sei, wo a eine Zeit ist und $= \frac{1}{r_0}$. Es ist dies ein Fall, der in so ferne vom Interesse ist, als es ungefähr der Wirbelbewegung der Luft entspricht, welche entsteht, wenn eine Luftmasse von einem kreisförmig begrenzten Gebiet auf der Erdoberfläche aus durch irgend eine Ursache emporgeschleudert wird und so eine vertical aufsteigende Strömung veranlasst. Denn es ist

$$\frac{dr}{dt} = - \frac{1}{(a+t)^2}$$

Mithin

$$\frac{dr}{dt} + r^2 = 0$$

Die Beschleunigung ist demnach Null; mithin verschwindet die Differenz der verticalen Temperaturgradienten $\tau' - \tau$.

Da

$$w = \frac{z}{a+t} = \frac{dz}{dt}$$

ist, folgt

$$z = C(a+t)$$

wo C eine Constante ist. Nennt man die anfängliche Geschwindigkeit der emporgeschleuderten Luftmasse w_0 ,

so ist $C = w_0$

mithin $w = w_0$

Jedes Lufttheilchen steigt mit constanter Geschwindigkeit in die Höhe, während die Geschwindigkeit, womit die Luft durch die Schichte $z = \text{Const}$ passirt, in's Unendliche wie $\frac{1}{a+t}$ abnimmt.

Es ist nun in diesem Fall,

$$I = \left(\frac{a+t}{a}\right) a \int_0^t \frac{e^{\nu t} dt}{(a+t)^2} \quad B = \frac{1}{(m-1)} \left(\frac{a+t}{a}\right)$$

$$C = \frac{1}{(m-1)} \left(\frac{a+t}{a}\right)^m.$$

wo bei die Function A durch theilweise Integration in die Form gebracht werden kann

$$A = \left(\frac{a+t}{a}\right) - e^{\nu t} + \kappa(a+t) \int_0^t \frac{e^{\nu t} dt}{(a+t)}$$

Diese Functionen wachsen in's Unendliche mit wachsendem t . Die Functionen aber

$$B e^{-\nu t}, \quad C e^{-\nu t}$$

verschwinden mit wachsendem t . Nicht so unmittelbar einleuchtend ist es, ob die Function $A e^{-\nu t}$ mit wachsendem t verschwindet. Bei dieser Frage handelte es sich nur darum, ob

$$\lim \left[\kappa(a+t) e^{-\nu t} \int_0^t \frac{e^{\nu t} dt}{(a+t)} \right]$$

gegen Einheit convergirt, da

$$\lim \left(\frac{(a+t)}{a} - e^{\nu t} \right) e^{-\nu t} = -1$$

ist. Weil nun $\int_0^t \frac{e^{\nu t} dt}{(a+t)}$ mit in's Unendliche wachsendem t gegen

$\frac{1}{\kappa(a+t)}$ convergirt, so sieht man, dass

$$\lim \left[x(a+t) e^{-\nu t} \int_0^t \frac{e^{\nu t} dt}{(a+t)} \right] = 1$$

ist, dass daher auch

$$\lim e^{-\nu t} A = 0$$

ist.

Man findet aus der Formel

$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial \Pi^*}{\partial y} \quad v = \frac{\partial \varphi}{\partial y} - \frac{\partial \Pi^*}{\partial x}$$

als die Componenten der Geschwindigkeit für das äussere wirbelfreie Gebiet, da

$$\varphi_a = -\frac{R^2}{2(a+t)} \log \rho + \text{Const.} \quad \rho = \sqrt{x^2 + y^2}$$

ist

$$u = -\frac{R^2}{2\rho^2} \left[\frac{a}{a+t} + ay e^{-\nu t} \left(A + B - \frac{C}{m} \right) \right]$$

$$v = -\frac{R^2}{2\rho^2} \left[\frac{y}{a+t} - ax e^{-\nu t} \left(A + B - \frac{C}{m} \right) \right]$$

Hieraus erhält man als die resultirende Geschwindigkeit

$$w = \frac{R^2}{2\rho} \sqrt{\frac{1}{(a+t)^2} + a^2 e^{-2\nu t} \left(A + B - \frac{C}{m} \right)^2}$$

Die also durch eine emporgeschleuderte Luftmasse entstehende Wirbelbewegung ist demnach eine Cyclonale, und die Windgeschwindigkeit nimmt dabei mit wachsender Zeit in's Unendliche ab.

Um den Deviationwinkel zu finden, könnte man die gefundenen Geschwindigkeitscomponenten in die Gleichung (38) substitution. Man gelangt aber kürzer dazu durch die Überlegung, dass die Isodynamen, und auch Isobaren auch hier nur concentrische Kreise sein können, so dass

$$xu + yv = \omega \rho \cos i$$

ist. Hieraus findet man für das äussere Gebiet

$$\operatorname{tag} i = a(a+t)e^{-\nu t} \left(A + B - \frac{C}{m} \right)$$

Im Anfang der Bewegung ist

$$\operatorname{tag} i = aa \left(\frac{1}{m-1} - \frac{1}{m(m-1)} \right) = \frac{aa}{m}$$

d. h.

$$\operatorname{tag} i = \frac{2\lambda \sin \theta}{\kappa}$$

da $m = \frac{\kappa}{\gamma_0}$, und $a = \frac{1}{\gamma_0}$ ist

Für $t = \infty$, wo die Wirbelbewegung verschwunden ist, ist, da $Ae^{-\nu t}(a+t)$ gegen $\frac{1}{\kappa}$, und $(a+t)e^{-\nu t}B$ und $(a+t)e^{-\nu t}C$ gegen 0 convergirt, wieder

$$\operatorname{tag} i = \frac{2\lambda \sin \theta}{\kappa}$$

d. h. der Deviationswinkel erreicht einmal ein Maximum oder Minimum im Verlauf der Zeit, um dann demselben Werthe wie am Anfang der Bewegung allmählig zuzustreben. Es lässt sich nun ohne Weiteres übersehen, dass diese Veränderlichkeit des Deviationswinkels durch den Werth von γ_0 bestimmt wird. Wenn γ_0 grösser ist als κ , so trägt der Wirbel einen mehr centripetalen Charakter; i muss hier zuerst mit der Zeit abnehmen. Ist aber γ_0 kleiner als κ , so nährt sich der Wirbel einer kreisförmigen Drehung; i muss daher zuerst mit der Zeit wachsen, um dann mit verschwindendem Wirbel wieder zu dem anfänglichen Werthe allmählig abzunehmen.

Als Componenten der Geschwindigkeit im inneren Gebiete findet man

$$u = -\frac{x}{2(a+t)} - \frac{ay}{2}e^{-\nu t} \left[A + B - \frac{C}{m} \left(\frac{a}{t} \right)^{2(m-1)} \right]$$

$$v = -\frac{y}{2(a+t)} + \frac{ax}{2} e^{-\kappa t} \left[A + B - \frac{C}{m} \left(\frac{\rho}{R} \right)^{2(m-1)} \right]$$

Hieraus folgt als die Resultirende

$$w = \frac{\rho}{2} \sqrt{\frac{1}{(a+t)^2} + \alpha^2} e^{-2\kappa t} \left[A + B - \frac{C}{m} \left(\frac{\rho}{R} \right)^{2(m-1)} \right]^2$$

Als Deviationswinkel findet man weiter

$$\text{tag } i = a(a+t) e^{-\kappa t} \left[A + B - \frac{C}{m} \left(\frac{\rho}{R} \right)^{2(m-1)} \right]$$

Die Wirbelbewegung der Luft ist auch im inneren Gebiete eine cyclonale und ihre Geschwindigkeit nimmt mit der Zeit in's Ueendliche ab. Der Deviationswinkel ist zur Zeit $t=0$

$$\text{tag } i = \frac{2\lambda \sin \theta}{(\kappa - \gamma_0)} \left[1 - \frac{1}{m} \left(\frac{\rho}{R} \right)^{2(m-1)} \right]$$

aber für $t=\infty$ wieder

$$\text{tag } i = \frac{2\lambda \sin \theta}{\kappa}$$

da $C(a+t) e^{-\kappa t}$ für $t=\infty$ verschwindet.

Nach dem Verlauf einer sehr grossen Zeit stellt sich in dem inneren Gebiet derselbe Bewegungszustand her, wie in dem äusseren Gebiete; die Windgeschwindigkeit vermindert sich dabei unausgesetzt, bis die ganze Wirbelbewegung verschwunden ist.

Es ist leicht Ausdrücke für den Luftdruck im inneren und äusseren Gebiete aufzustellen, was wir aber unterlassen wollen, da wir dabei nichts mehr kennen lernen, als die Thatsache, dass die Druckdepression mit der verticalen Strömung verschwindet.

Wir wollen einen anderen Fall in's Auge fassen den Fall nämlich, wo die vertical aufsteigende Strömung allmählig entsteht, so dass

$$\gamma = K \left(\frac{e^{Kt} - e^{-Kt}}{e^{Kt} + e^{-Kt}} \right)$$

wird. Weil in diesem Fall für $t=0$, $\gamma=0$ ist, so hat man auch

$$\Delta W_i = 0 \text{ für } t = 0$$

Dieser Bedingung wird genügt, wenn man in (43) $t'=0$ setzt, und schreibt

$$\Delta W_i = -a e^{-\nu t + \int_0^t \gamma dt} \int_0^t \gamma e^{\int_0^t (\nu - \gamma) dt} dt$$

Hieraus findet man durch Integration

$$\frac{\partial W_i}{\partial \rho} = -\frac{a\rho}{2} e^{-\nu t + \int_0^t \gamma dt} \int_0^t \gamma e^{\int_0^t (\nu - \gamma) dt}$$

d.h. indem wir für γ seinen Ausdruck einsetzen

$$\frac{\partial W_i}{\partial \rho} = -\frac{aK\rho}{2} e^{-\nu t} \left(e^{Kt} + e^{-Kt} \right) \int_0^t \frac{e^{\nu t} (e^{Kt} - e^{-Kt}) dt}{(e^{Kt} + e^{-Kt})^2}$$

Für das äussere Gebiet hat man wieder

$$\frac{\partial W_a}{\partial \rho} = -\frac{R^2}{2\rho} e^{-\nu t} \int (T+a) \gamma e^{\nu t} dt$$

Die Grenzbedingung ergibt

$$-\frac{R}{2} e^{-\nu t} \int (T+a) \gamma e^{\nu t} dt = -\frac{aK}{2} R e^{-\nu t} \left(e^{Kt} + e^{-Kt} \right) \int_0^t \frac{e^{\nu t} (e^{Kt} - e^{-Kt}) dt}{(e^{Kt} + e^{-Kt})^2}$$

d. h.

$$\int (T+a) \gamma e^{\nu t} dt = aK \left(e^{Kt} + e^{-Kt} \right) \int_0^t e^{\nu t} \frac{(e^{Kt} - e^{-Kt}) dt}{(e^{Kt} + e^{-Kt})^2}$$

Mithin folgt

$$\frac{\partial W_a}{\partial \rho} = - \frac{\alpha K R^2}{2\rho} e^{-Kt} \left(e^{Kt} + e^{-Kt} \right) \int_0^t e^{Kt} \frac{(e^{Kt} - e^{-Kt}) dt}{(e^{Kt} - e^{-Kt})^2}$$

Oder kürzer, indem man setzt

$$\varphi(t) = e^{-Kt} \left(e^{Kt} + e^{-Kt} \right) \int_0^t e^{Kt} \frac{(e^{Kt} - e^{-Kt}) dt}{(e^{Kt} + e^{-Kt})^2}$$

$$\frac{\partial W_i}{\partial \rho} = - \frac{\alpha K \rho}{2} \varphi(t)$$

$$\frac{\partial W_a}{\partial \rho} = - \frac{\alpha K R^2}{2\rho} \varphi(t)$$

Man findet somit als die Componenten der Geschwindigkeit für das innere Gebiet

$$u = - \frac{K}{2} \left(\frac{e^{Kt} - e^{-Kt}}{e^{Kt} + e^{-Kt}} \right) x - \frac{\alpha K y}{2} \varphi(t)$$

$$v = - \frac{K}{2} \left(\frac{e^{Kt} - e^{-Kt}}{e^{Kt} + e^{-Kt}} \right) y + \frac{\alpha K x}{2} \varphi(t)$$

als die Resultirende

$$w = \frac{K \rho}{2} \sqrt{\left(\frac{e^{Kt} - e^{-Kt}}{e^{Kt} + e^{-Kt}} \right)^2 + \alpha^2 [\varphi(t)]^2}$$

Für das äussere Gebiet

$$u = - \frac{K}{2} \frac{R^2}{\rho^2} \left[\left(\frac{e^{Kt} - e^{-Kt}}{e^{Kt} + e^{-Kt}} \right) x + \alpha y \varphi(t) \right]$$

$$v = - \frac{K}{2} \frac{R^2}{\rho^2} \left[\left(\frac{e^{Kt} - e^{-Kt}}{e^{Kt} + e^{-Kt}} \right) y - \alpha x \varphi(t) \right]$$

als die Resultirende

$$w = \frac{K}{2} \frac{R^2}{\rho} \sqrt{\left(\frac{e^{Kt} - e^{-Kt}}{e^{Kt} + e^{-Kt}} \right)^2 + \alpha^2 [\varphi(t)]^2}$$

Die Function φ lässt sich auch so schreiben

$$\varphi(t) = \frac{1}{K} \left[\frac{e^{-(\nu-K)t} + e^{-(\nu+K)t}}{2} + \nu(e^{-(\nu+K)t} + e^{-(\nu-K)t}) \times \right. \\ \left. \times \int_0^t \frac{e^{\nu t} dt}{(e^{\nu t} + e^{-Kt})} - 1 \right]$$

Wenn $\nu > K$ ist, so verschwindet das erste Glied in der Klammer mit wachsender Zeit und das zweite Glied nimmt dabei die unbestimmte Form $0 \cdot \infty$ an. Da nun

$$(e^{-(\nu-K)t} + e^{-(\nu+K)t}) \int_0^t \frac{e^{\nu t} dt}{(e^{\nu t} + e^{-Kt})} = \left(1 + e^{-\frac{(\nu+K)}{(\nu-K)}t} \right) \int_0^t \frac{e^{\nu t} dt}{(e^{\nu t} + e^{-Kt})}$$

und

$$\lim \int_0^t \frac{e^{\nu t} dt}{(e^{\nu t} + e^{-Kt})} = \frac{1}{\nu - K}$$

ist, wie man nach der gewöhnlichen Methode finden kann, so hat man

$$\varphi(t)_{t=\infty} = \frac{1}{K} \left(\frac{\nu}{\nu - K} - 1 \right) = \frac{1}{\nu - K}$$

Nach dem Verlaufe einer unendlich grossen Zeit, wo $\tau = K$ wird, werden die Componenten der Geschwindigkeit im inneren Gebiete

$$u = -\frac{K}{2}x - \frac{\alpha K}{2(\nu - K)}y$$

$$v = -\frac{K}{2}y + \frac{\alpha K}{2(\nu - K)}x$$

und im äusseren Gebiete

$$u = -\frac{K R^2}{2 \rho^2} \left(x + \frac{\alpha}{2(\nu - K)} y \right)$$

$$v = -\frac{K R^2}{2 \rho^2} \left(y - \frac{\alpha}{2(\nu - K)} x \right)$$

Also stationär, aber eine andere stationäre Wirbelbildung, als in dem Fall, wo die verticale Luftströmung etwa plötzlich entstand, und sich mit der Zeit nicht ändert.

Wenn $\kappa = K$ oder $\kappa < K$ wird, wächst $\frac{\partial W}{\partial \rho'}$ so wohl im inneren, als im äusseren Gebiet mit wachsender Zeit in's Unendliche, während $\frac{\partial \varphi}{\partial \rho'}$ gegen einen Grenzwert convergirt, der durchaus endlich ist. Da nun die Grösse K eigentlich keinerlei Einschränkung unterworfen ist und daher eine physikalische Bedeutung auch diesen Fällen $\kappa = K$ oder $K > \kappa$ zukommen muss, so ist entweder eine solche Wirbelbildung, wie die Behandelte, unmöglich oder K ist bei der Wirbelbildung solcher Art durch die Vorgänge in der höheren Schichte der Erdatmosphäre eingeschränkt, um welche wir uns bei der Bestimmung der Grösse γ eigentlich gar nicht bekümmert haben. Da aber die Annahme $\kappa > K$ zu keinerlei Absurdität führt, so halte ich es für wahrscheinlich, dass die Annahme von der Constanz des K unzulässig ist, wenn K bei der Wirbelbildung, wie die in Rede stehende, einen gewissen Werth übersteigt. Es ist indessen nicht zu vergessen, dass ein solcher Fall, wo die Geschwindigkeit der vertical aufsteigenden Strömung mit dem Verfluss einer unendlich grossen Zeit sich zu einem Maximum steigern soll, nichts mehr ist, als eine mathematische Fiction, dass, wenn die für eine solche Wirbelbildung entwickelten Ausdrücke besser einer solchen in der Natur angepasst werden sollen, γ eine Function sein muss, welche mit wachsender Zeit zu einem Maximum steigt, um dann wieder zu verschwinden.

Als Deviationswinkel findet man für die beiden Gebiete eine und dieselbe Function

$$\tan i = 2\lambda \sin \theta e^{-\nu t} \frac{(e^{Kt} + e^{-Kt})^2}{(e^{Kt} - e^{-Kt})} \int_0^t e^{\nu t} \frac{(e^{Kt} - e^{-Kt})}{(e^{Kt} + e^{-Kt})^2} dt$$

Es ist für $t = 0$

$$\operatorname{tag} i = \frac{0}{0}$$

da

$$\lim_{t \rightarrow 0} \frac{\int_0^t e^{\kappa t} \frac{(e^{\kappa t} - e^{-\kappa t})}{(e^{\kappa t} - e^{-\kappa t})^2} dt}{(e^{\kappa t} - e^{-\kappa t})} = 0$$

ist, so folgt

$$\operatorname{tag} i = 0$$

Alle Lufttheilchen befinden sich zur Zeit $t = 0$ in centripetaler Bewegung. Für $t = \infty$, falls $\kappa > K$ ist, wird $\operatorname{tag} i$ wieder unbestimmt. Man erhält aber nach der gewöhnlichen Methode

$$\operatorname{tag} i = \frac{2\lambda \sin \theta}{\kappa - K}$$

Der Deviationswinkel wächst demnach von 0 aus mit wachsender Zeit fort und fort zu dem durch die obige Gleichung bestimmten Werthe. Die Windbahn verwandelt sich daher aus einer Geraden allmählig in eine logarithmische Spirale, welche die Grenze des inneren Gebietes continuirlich durchsetzt und in unendlicher Anzahl der Windungen das Centrum des inneren Gebietes erreicht.

Wir wollen als das letzte Beispiel mit einem Fall der vertical herabsteigenden Strömung beschäftigen, indem wir annehmen

$$\gamma = -\frac{1}{a+t} \quad a = \frac{1}{\gamma_0}$$

In diesem Fall ist die Beschleunigung nicht Null; denn es ist

$$\gamma^2 - \frac{d\gamma}{dt} = \frac{2}{(a+t)^2}$$

so dass demnach

$$K = \frac{\sqrt{2}}{(a+t)}$$

d. h. die Differenz der verticalen Temperaturgradienten sich wie $\frac{1}{(a+t)^2}$ verändert. Aus der Gleichung $\frac{dz}{dt} = rz$ ergibt sich ferner

$$z = \frac{C}{(a+t)}$$

was eine mögliche Bewegung anzeigt, da z mit wachsender Zeit gegen 0 convergirt. Die Geschwindigkeit eines Lufttheilchens ist

$$\frac{dz}{dt} = - \frac{C}{(a+t)^2}$$

Sie nimmt also mit wachsender Zeit ab und verschwindet für $t = \infty$, wo die Lufttheilchen erst die Erdoberfläche erreichen. Es ist nun

$$\int r dt = \int \frac{dt}{a+t} = \log \left(\frac{a+t}{a} \right)$$

Geht man mit diesem Ausdruck in (50a) ein, so hat man für das innere Gebiet

$$\frac{\partial W_i}{\partial \rho} = \frac{a}{2} \rho e^{-rt} \left(\frac{a}{a+t} \right) \left[\frac{1}{m+1} + \frac{1}{na} (e^{rt} - 1) \right]$$

und

$$\frac{\partial \zeta_i}{\partial \rho} = \frac{\rho}{2(a+t)}$$

und aus (51) für das äussere Gebiet

$$\frac{\partial W_a}{\partial \rho} = \frac{aR^2 e^{-rt}}{2\rho} \left(\frac{a}{a+t} \right) \left[\frac{1}{m+1} + \frac{e^{rt}-1}{a\alpha} + \frac{1}{m+1+m} \left(\frac{a+t}{a} \right)^m \left(1 - e^{-m \left(\frac{\rho^2}{R^2} - 1 \right)} \right) \right]$$

und

$$\frac{\partial \zeta_a}{\partial \rho} = \frac{R^2}{2\rho(a+t)}$$

Hieraus erhält man als Componenten der Geschwindigkeit im inneren Gebiet, indem wir zur Abkürzung setzen

$$\zeta(t) = \frac{1}{1+m} + \frac{e^{-\nu t} - 1}{a\nu}$$

$$u = \frac{x}{2(a+t)} + \frac{a}{2(a+t)} e^{-\nu t} \zeta(t) y$$

$$v = \frac{y}{2(a+t)} - \frac{a}{2(a+t)} e^{-\nu t} \zeta(t) x$$

und für das äussere Gebiet, indem wir setzen

$$\frac{1}{m(1+m)} \left(\frac{a+t}{a} \right)^{m+1} = \psi(t)$$

$$u = \frac{R^2 x}{2\rho^2(a+t)} + \frac{a R^2 e^{-\nu t}}{2\rho^2} \left(\frac{a}{a+t} \right) \left[\zeta(t) + \psi(t) \left(1 - e^{-m \left(\frac{\rho^2}{R^2} - 1 \right)} \right) \right] y$$

$$v = \frac{R^2 y}{2\rho^2(a+t)} - \frac{a R^2 e^{-\nu t}}{2\rho^2} \left(\frac{a}{a+t} \right) \left[\zeta(t) + \psi(t) \left(1 - e^{-m \left(\frac{\rho^2}{R^2} - 1 \right)} \right) \right] x$$

Die Function $e^{-\nu t} \zeta(t)$ verschwindet mit wachsender Zeit und dergleichen $e^{-\nu t} \psi(t)$, was für einen Werth a auch haben mag. Die Windgeschwindigkeit nimmt daher mit der Zeit in's Unendliche ab und für sehr grosses t werden ihre Componenten im inneren Gebiete

$$u = \frac{x}{2(a+t)} + \frac{a}{2(a+t)\nu} y$$

$$v = \frac{y}{2(a+t)} - \frac{a}{2(a+t)\nu} x$$

und im äusseren Gebiete

$$u = \frac{R^2 x}{2\rho^2(a+t)} + \frac{a R^2 y}{2\nu \rho^2(a+t)} = \frac{R^2}{2\rho^2(a+t)} \left(x + \frac{a}{\nu} y \right)$$

$$v = \frac{R^2 y}{2\rho^2(a+t)} - \frac{a R^2 x}{2\nu \rho^2(a+t)} = \frac{R^2}{2\rho^2(a+t)} \left(y - \frac{a}{\nu} x \right)$$

Da nun so wohl im inneren, als im äusseren Gebiete für grosses t

$$\frac{v}{u} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx} = \frac{y - \frac{ax}{\kappa}}{x + \frac{ay}{\kappa}}$$

so sieht man, dass die Windbahn nach dem Verlauf einer grossen Zeit eine logarithmische Spirale wird, die nach Aussen anticyclonal gewunden ist und den Grenzkreis der verticalen Strömung continuirlich durchsetzt.

Für Deviationswinkel findet man im inneren Gebiete, indem wir $a = \frac{1}{\gamma_0}$ einführen

$$\tan i = \frac{a}{\gamma_0} e^{-\gamma t} \left[\frac{1}{1+m} + \frac{\gamma_0}{\kappa} (e^{\gamma t} - 1) \right]$$

d. h.,

$$\tan i = \frac{2\lambda \sin \theta}{\kappa} \left(1 - \frac{\gamma_0 e^{-\gamma t}}{\kappa + \gamma_0} \right)$$

und in dem äusseren Gebiete

$$\tan i = \frac{2\lambda \sin \theta}{\kappa} \left[1 - \frac{\gamma_0 e^{-\gamma t}}{\kappa + \gamma_0} + e^{-\gamma t} \frac{\gamma_0 (1 + \gamma_0 t)^{m+1}}{(\gamma_0 + \kappa)} \left(1 - e^{-m \left(\frac{\rho^2}{L^2} - 1 \right)} \right) \right]$$

In dem inneren Gebiete wird demnach für $t = \infty$

$$\tan i = \frac{2\lambda \sin \theta}{\kappa}$$

und auch in dem äusseren Gebiet, aber mit dem Unterschied, dass dieser Bewegungszustand früher in dem inneren Gebiete eintritt, als in dem äusseren Gebiete, da $e^{-\gamma t}$ schneller verschwindet, als $(1 + \gamma_0 t)^{m+1} e^{-\gamma t}$.

Viel complicirter werden die Ausdrücke für die Geschwindigkeitscomponenten, wenn wir über den anfänglichen Zustand der Bewegung

in dem kreisförmigen Gebiete, wo die verticale Strömung entsteht, eine andere Annahme machen als diejenigen, für die wir die hier auftretenden willkürlichen Functionen ermittelt haben. Dass wir über den Aufanszustand andere Annahmen machen können, erhellt daraus, dass für $t=0$ $\gamma_0=0$ ΔW_i nicht nothwendig zu verschwinden braucht. Denn in diesem Fall fallen die Gleichungen (27) und (28) in die eine zusammen

$$\frac{\partial \Delta W}{\partial t} + \frac{\partial \Delta W}{\partial x} \frac{\partial W}{\partial y} - \frac{\partial \Delta W}{\partial y} \frac{\partial W}{\partial x} + \kappa \Delta W = 0 \quad \text{für } t = 0$$

was anzeigt, dass ΔW eine Function von x, y sein kann und zwar eine willkürliche, da sie keiner anderen Bedingung zu genügen hat, als der, dass sie in der Unendlichkeit verschwindet. Es ist aber einleuchtend, dass, was wir auch für eine Annahme über den anfänglichen Bewegungszustand machen mögen, die Aufgabe auf die Quadratur zurückgeführt ist, wenn das Gebiet der verticalen Strömung kreisförmig oder, was wir hier hinzufügen wollen, geradlinig begrenzt ist.

Wenn in der Erdatmosphäre mehr als ein Gebiet der veränderlichen verticalen Strömungen vorhanden sind, so bewegen sich die also gebildeten Gebiete der nicht stationären Wirbel und die Differentialgleichungen für diese Bewegung können auf dieselbe Weise, wie wir sie für stationäre Wirbelbewegung aufgestellt haben, entwickelt werden; sie aber sind selbst im Fall, wo nur zwei unendlich von einander entfernte Wirbelgebiete vorhanden sind, nicht integrirbar da die Functionen φ und W die Zeit explicite enthalten. Indessen; in dem Falle, wo nur zwei um Unendliches von einander entfernte Wirbelgebiete vorhanden sind, und eine von denselben stationäre ist, oder die durch dieses Wirbelgebiet allein in der Unendlichkeit verursachte Luftgeschwindigkeit gegeben ist, lässt sich die Luftbewegung in- und ausserhalb des anderen Wirbelgebietes finden, da wie wir

gesehen haben, der Einfluss des unendlich fernen Wirbelgebietes auf das in Rede stehende Wirbelgebiet jederzeit durch die Hinzufügung einer gewissen linearen Function zu $4W$ berücksichtigt wird.

Wir haben somit durch consequente Behandlung der unter specialisirenden Annahmen vereinfachten hydrodynamischen Differentialgleichungen die wesentlichsten Eigenschaften der Cyclonen und Anticyclonen abgeleitet und die Bewegung eines Wirbelgebietes als eine Folge des Vorhandenseins eines anderen Wirbelgebietes erkannt. Wir haben ferner den Weg gezeigt, die zeitliche Veränderung der Windstärke des Windazimuths und des Luftdrucks in einem gegebenen Ort bei zweifacher Wirbelbildung aus der Eigenbewegung der Wirbelgebiete abzuleiten und sind dabei zu manchen Thatsachen geführt worden, die in der Wirklichkeit beobachtet worden sind. Für die von manchen Meteorologen mit Aufwand von vielem Scharftinn erklärte Ausdehnung der Wirbelstürme in den höheren Breiten haben wir wenigstens den Weg gezeigt, sie als eine Folge der Verlangsamung der verticalen Strömung und ihrer Wanderung nach der höheren Breite abzuleiten. Diese Resultate unserer bisherigen Entwicklungen dürften demnach wohl als Beweise gelten, dass die vereinfachenden Annahmen, von denen wir ausgegangen waren, nicht allzu unser Streben beeinträchtigt haben. Einblick in den verwickelten Mechanismus der Luftwirbel in der Erdatmosphäre auf rein analytischem Wege wenigstens für die untere Partie der Erdatmosphäre zu gewinnen.

Indessen muss ich hier auf zwei Ungenauigkeiten aufmerksam machen, welche aus den eben erwähnten Grundannahmen fliessen und die Anwendbarkeit unserer bisher entwickelten Formeln eigentlich auf die unterste Schichte der Erdoberfläche beschränken mögen. Die

Annahme, dass die Luft sich in dem Raum ausserhalb des cylindrischen Raumgebietes der verticalen Strömung überall parallel zu der als eine unendliche Ebene gedachten Erdoberfläche bewege, wäre nur dann zulässig, wenn die Reibung an der Erdoberfläche in jeder Luftschichte dieselbe Verzögerung der bewegten Luft hervorbrächte, was wohl nicht der Fall ist. Um daher Ausdrücke zu erhalten, die sich den Vorgängen in der höher gelegenen Luftschichte bei Wirbelbildung besser anpassen lassen, müsste man die Reibung erst in der auf der Erdoberfläche zuerfüllenden Bedingung berücksichtigen, indem wir die innere Reibung der Luft in Rechnung ziehen. Wie weit aber die in Rede stehende Annahme zulässig ist, das lässt sich leider nicht beurtheilen. Als eine Folge der Annahme, dass die Wirbelaxe überall senkrecht zur als eine unendliche Ebene gedachten Erdoberfläche stehe, haben wir die Gleichung erhalten

$$\frac{\partial w}{\partial z} = \begin{array}{l} \text{einer Constante} \\ \text{oder einer Functions von } t \text{ allein} \end{array}$$

Wie wir gesehen haben, entsteht aus diesem Gesetz der verticalen Strömung keine analytische Schwierigkeit, welche die Unzulässigkeit der Annahme, deren Folge das Gesetz ist, irgend wie darthun würde. Nichtsdestoweniger resultirt eine Bewegungsform, die wenig wahrscheinlich ist. Jedes Lufttheilchen, welches sich ausserhalb des Gebietes der vertical aufsteigenden Strömung horizontal bewegt hat, wirbelt z. B. in einer Spiralbahn hinauf, so bald es den Mantel des cylindrischen Raumgebietes der verticalen Strömung durchsetzt. Da aber die Geschwindigkeit der verticalen Strömung nicht etwa, wie es in der Natur sicher der Fall ist, nach dem Umfang des Cylindrischen Gebietes zu allmähig abnimmt, sondern nur von der verticalen Coordinate abhängt, so wird jedes Lufttheilchen beim Durchsetzen der Grenzfläche der verticalen Strömung völlig unvermittelt emporgerissen und die Strömungslinie erscheint dort nach der Höhe geknickt.

während ihre horizontale Projection völlig continuirlich aus dem äusseren Raum in den inneren hinein verläuft, und der Luftdruck ändert sich dabei sprüingsweise um den Betrag $\frac{\gamma^2 z^2}{2}$.

So sehen wir denn, dass die Annahme, auf der jenes Gesetz der verticalen Strömung beruht, streng genommen nicht zulässig ist, dass wir γ auch als eine Function von x, y , zu denken haben, um Ausdrücke zu erhalten, welche genauer die Vorgänge in einiger Höhe über dem Meeresniveau darstellen würden. Die Bewegungsgleichungen werden aber dann so verwickelt, dass sie sich bei dem jetzigen Zustande der Analysis schwerlich integriren lassen würden, auch wenn wir wieder die verticale Strömung kreisförmig oder geradlinig begrenzt denken. Es ist indessen dagegen zu erinnern, dass γ im Gewöhnlichen eine sehr kleine Grösse ist, dass, wenn γz für $\gamma = 0,000158$ also vielleicht grösser als der Reibungscoefficient auf der Erdoberfläche selbst in der Meereshöhe 1000 *m.* erst den kleinen Werth $0,158 \frac{\text{meter}}{\text{sec}}$ erreicht, dass die Discontinuität des Luftdrucks an der Grenzfläche der verticalen Strömung in dieser ziemlich grossen Höhe erst 0,000158 mal dem äusseren Quecksilberdruck in dieser Höhe betragen würde. Wenn daher der Durchmesser der verticalen Strömung nicht unbedeutend und in Folge dessen die horizontale Luftgeschwindigkeit an der Grenze derselben bedeutend ist, so wird die Neigung der Strömungslinie an der Grenze gegen die Horizontalebene selbst in dieser Meereshöhe so klein sein können, dass wir wohl berechtigt sind, die Annahme bis dieser Höhe als zulässig anzusehen, welche zu jenem Gesetz der verticalen Strömung geführt hat.



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